

Peter Ove Christensen

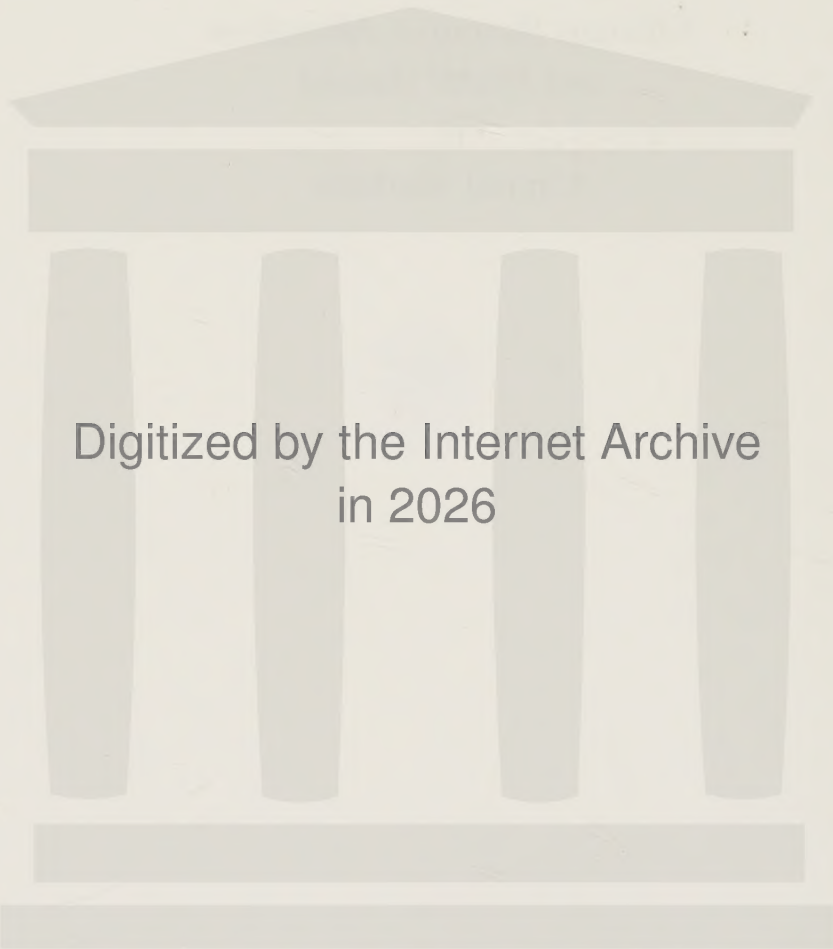
Asymmetric Information,
Efficient Resource Allocation
and Moral Hazard
in
Capital Markets

ODENSE UNIVERSITY PRESS

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by

Peter Ove Christensen

Odense University Press

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Preface

The purpose of this dissertation is to give an economic analysis of the demand for firm-specific information in efficient capital markets. The analysis integrates the theory of finance, the theory of the firm and accounting theory.

We consider an economy in which the managers of firms are better informed than the investors about the production conditions of their own firms but not about the general economic conditions in the economy. The question asked is then whether a more efficient allocation of the resources in the economy can be achieved if the managers report their superior information to the investors.

The answer to the question depends of the model considered. Hence, the issue is not whether preferred allocations can be achieved or not. The interesting issue is the reasons why these preferred allocations do occur in a model with rational economic agents. This gives insight into the problem of determining the type of information to report and into the problem of organizing the managers' reporting activities.

The dissertation falls into two parts. Part I is a general equilibrium analysis of efficient resource allocation in an economy with asymmetric information. In Part II we use the results of Part I to make a partial equilibrium analysis of the value of external accounting information reported by the managers when there are moral hazard problems. The dissertation is concluded with a discussion of the implications of the analysis for accounting.

Many of the ideas in this dissertation were formed while I was visiting The University of British Columbia in 1985/86. I would like to use this opportunity to thank those people who supported me during my studies at UBC. They include Professors Amin Amershi, Michael Brennan, Gerald Feltham, Alan Kraus, Eduardo Schwartz and Rex Thompson. I am especially indebted to Gerald Feltham, both as an excellent teacher and as the solely most important source of inspiration for my work with this dissertation. I also thank him for allowing me to use the material in our joint paper, "Firm-Specific Information and Efficient Resource Allocation", in this dissertation.

My stay at The University of British Columbia was made financially possible through a scholarship from Odense University and a grant from The Danish Social Science Foundation. This support is very much appreciated.

Discussions with my friends and colleagues at Odense University, Professors John Christensen and Peter Bogetoft greatly improved this study, and helpful comments from Professors Joel Demski and John Hughes are also gratefully acknowledged.

Thanks are also due to Mona Andersen for typing assistance and for trying to teach me the basics of the written American language.

A special thanks goes to my wife Else for her patience with me and for taking her disproportionate share in taking care of our son Kasper during the completion of this dissertation.

Odense, 1990.

PART I

Efficient Resource Allocation in Capital Markets

with

Asymmetric Information.

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CHAPTER 1.

Introduction and Resumé of Part I.

Several recent papers have analyzed the role of external accounting reports in capital markets. One branch of this research has focused on the impact of information on risk sharing among investors and the allocation of investment capital over time and among firms (see Ohlson and Buckman (1981), Hakansson, Kunkel and Ohlson (1982), Kunkel (1982), Feltham (1983), Feltham (1985) and Ohlson (1988)). A main result of this research is the identification of a set of sufficient conditions for financial reporting to have zero social value relative to reporting only current dividends (see Feltham (1985)):

- (a) Time-additive preferences
- (b) Homogeneous beliefs
- (c) Pareto-efficient consumption given the production plan
- (d) Fixed production.

A necessary condition for financial reports to have positive social value is that at least one of the above conditions is violated. We assume in Part I that the conditions (a) to (c) are satisfied, and allow for endogenous production choice. In this literature, the events and information that characterize the economy are quite general, but they are most readily interpreted as referring to events that influence a number of firms. We retain this general formulation but extend it (along the lines in Feltham (1983)) to consider a large economy, in which we explicitly recognize both economy-wide and firm-specific events.

The distinction used between firm-specific and economy-wide events is that firm-specific events are those events which only affect the dividends of individual firms, and which are independent conditional on the economy-wide events. We use this formulation for two reasons. First, one of the most significant developments in the theory of finance during the last thirty years has been the results on the gains to diversification (see e.g. Markowitz (1959) and Samuelson (1967)) and the implications for the equilibrium in capital markets. Various capital asset pricing models have been developed based on those results.¹ The main idea in these models is that equilibrium prices are such that the investors can insure themselves against firm-specific risk by diversification without paying a premium. It turns out that this has a significant impact on investors' demand for firm-speci-

fic information in exchange economies as well as in production economies. Secondly, based on these theoretical developments empirical accounting research has treated accounting information as firm-specific information (see Beaver (1972)).

In the literature referred to above, managers of firms play a somewhat artificial role. They are assumed to act in the interest of the investors, and their production decisions are based on information available to investors. Much research effort has been devoted to finding conditions under which investors are unanimous with respect to production decisions, and under which production responsibility can be delegated to managers without social loss. These conditions are the assumptions for a perfectly competitive capital market. All economic agents are price-takers and they all have the same information. The production decision criterion unanimously supported is maximizing the market value of the firm net of investments (see Nielsen (1976), Baron (1979), and Ohlson (1988)). Although delegation of production responsibility is a central issue in this literature there are no gains to delegation. Managers can only do what investors can do themselves. We provide an alternative model where managers still act in the best interests of investors. However, the managers have more information about firm-specific events than investors. This leads to social gains from delegating production responsibility.²

The perfectly competitive capital market analysis is not capable of giving a satisfactory analysis of the value of external accounting information. Financial reporting is a process where the managements of firms communicate information they already have to less informed investors. Therefore, it seems that a necessary ingredient in the analysis is that the managers, at least at the outset, have more information than the investors have. We analyze the role of external accounting reports in a setting where the managers have superior information about the firm-specific events of their own firms, and where the managers' information does not depend on the external reporting system used. However, managers and investors are equally informed about the general economic conditions determined by the economy-wide events.

In Chapter 2 we describe the economy in section 2.1 and characterize Pareto

efficient consumption/production plans in section 2.2. Previous analyses have identified the importance of endogenous production choice in providing a basis for information to be of value.³ It is shown that Pareto-efficient consumption plans do not depend on firm-specific events, and that Pareto-efficient production plans maximize the intrinsic values of the dividends of firms given the managers' superior information. Furthermore, this is preferred to merely maximizing the perceived market values. The basis for these results is that in a large economy the basic state-contingent prices (i.e. the cost of capital) pertain to economy-wide events and not to firm-specific events. Each manager can be characterized as taking these state-contingent prices as given. At these prices he evaluates the expected dividends conditional on each economy-wide event and on his own information. As a consequence of this production decision rule, reporting firm-specific information is of no social value, since the production plan does not depend on the amount of firm-specific information available to the investors.

Only a few papers in the literature provide analyses of the characteristics of information that give the information value in a production economy.⁴ In this chapter we explicitly consider information about productivity (the marginal state-contingent returns from alternative production choices), and compare it with information about windfall gains and losses. It is demonstrated in Proposition 2.7 that Pareto-preferred consumption/production plans can be obtained with more information about productivity, whether the variations in productivity are due to economy-wide or to firm-specific events, whereas information about windfall gains and losses is only valuable if it pertain to economy-wide events. Prior knowledge of which firms are going to gain and which are going to lose next period, cannot be used to make well-diversified investors better off even though it will influence the current market values of the firms. Information about firm-specific productivity events influences the choice of Pareto efficient production plans and, therefore, it is useful that the managers have this information, but it is not necessary that the managers report this information to the investors in order to obtain these production gains.

Our initial analysis of the economy in Chapter 2 characterizes Pareto efficient consumption/production plans. In Chapter 3 we define an equilibrium concept for a large market economy in section 3.1. In a market setting,

there are two basic reasons why the resulting plans may not be efficient. First, there may be an insufficient variety of claims to provide an efficient risk sharing, and secondly, managers may not choose the Pareto efficient production plan. We consider these two issues in section 3.2 and in Chapter 4, respectively.

Our economy has two rounds of trading, one before and one after the release of information. The first round of trading is to allow the investors to insure themselves before any information is released. Otherwise, public information might be of negative value (see Hirshleifer (1971)). The key to efficiently sharing risks is a sufficient variety of tradeable claims which permit a trading strategy that can implement any desired state-contingent consumption plan. A market in which any financially feasible state-contingent consumption plan can be implemented is defined to be dynamically quasi-complete. We identify sufficient conditions for a dynamically quasi-complete market in an economy in which preferences are time-additive and beliefs are homogeneous. The means to achieve a dynamically quasi-complete market in a large and seemingly incomplete market are diversification and dynamic trading.

If preferences are time-additive and beliefs are homogeneous then efficient consumption plans depend only on the level of expected aggregate consumption conditional on the economy-wide event (Proposition 2.3). Hence, only these consumption plans have to be implementable. A market with this characteristic is defined to be dynamically investment wealth quasi-complete. The sufficient condition for the market to be of this type is that there exists a partition of firms into well-diversified portfolios with pay-off vectors contingent on the economy-wide events which span the linear subspace of desired investment wealths. This linear subspace is determined by the level of current conditional expected aggregate consumption and the probability distribution of the future level of conditional expected aggregate consumption.

In order to achieve Pareto efficiency, managers must be precluded from personally trading on their private information and they must be motivated to maximize the intrinsic values of the dividends. These issues are analyzed in Chapter 4, section 4.1 and section 4.2, respectively. The insider

trading problem can be avoided by only allowing the managers to trade in well-diversified portfolios. Market values of firms are only based on public information and, therefore, some firms will be overvalued while others are undervalued relative to the managers' information. Well-diversified portfolios, however, will be accurately valued. Provided that a sufficiently varied set of well-diversified portfolios exists, this restriction on the managers' portfolio choice still allows an efficient sharing of risks between investors and managers, but it precludes the managers from exploiting their private information at the expense of the investors. Moreover, it is shown that investors as well as managers unanimously support this restriction on the managers' trading opportunities.

The managers must be motivated to maximize the intrinsic values of the dividends. This is done by offering them suitable compensation contracts. Such contracts can be created if the managers have no nonpecuniary returns from production decisions. The form of the equilibrium compensation contracts is a fixed state-contingent term and a proportional share of the average dividend of all firms. This type of contract provides both the optimal incentives with respect to production decisions and the optimal risk sharing.

The compensation contracts motivate the managers to implement the Pareto-efficient production plan, such that the managers act as if they maximize the intrinsic values of the dividends. The intrinsic value of a firm is equal to the intrinsic value of the dividends minus the value of the management compensations. It is shown in Proposition 4.2 that the Pareto-efficient production plan also maximizes the intrinsic values of firms, given the form of the equilibrium compensation contracts. Moreover, the investors unanimously support this production plan.

The form of the equilibrium compensation contracts is a case against compensation contracts which are more closely related to the performance of the individual firms. There are two reasons why this type of compensation contracts is not optimal in the model we consider. First, a compensation based on the market value of a firm motivates the manager to make production decisions in order to maximize the market value of that firm. In this case, the manager does not use his private information appropriately to

obtain an efficient allocation of investment capital among firms. Secondly, the market value of a firm depends on firm-specific events. Hence, the managers will be exposed to firm-specific risk and they will require a risk premium to bear that risk. Since investors can diversify all firm-specific risk in the capital market, it is not optimal to pay the managers a risk premium for bearing this type of risk.

Firm-specific events have been defined as those events which only influence the dividends of individual firms and which are independent conditional on the economy-wide events. Otherwise, the events are included in the economy-wide events. Many economy-wide events may therefore be irrelevant to the investors' consumption and portfolio plans. The previous analysis has been concerned with an economy where investors and managers are equally informed about all the economy-wide events. In Chapter 5, section 5.1., we compare this economy to an otherwise identical economy in which the investors only receive information about the current level and the probability distribution of the future level of conditional expected aggregate consumption. We identify conditions under which the equilibrium obtained in the two economies are identical. In general, only information relevant for the composition and the market values of well-diversified portfolios needs to be reported to the investors in order to obtain a Pareto efficient allocation of investment capital among firms and over time. There are no additional gains associated with reporting the managers' additional private information.⁵ That is, more information is needed to implement efficient production plans than to implement efficient consumption plans.

We conclude Part I in section 5.2 with an informal analysis of one potential source of demand for the managers to report firm-specific information to the investors. The managers' and the investors' information structures for firm-specific events have been taken as exogenously given in the previous analysis. However, the managers' information structures are a result of the accounting systems and other decision support systems used internally in firms. Thus, they are part of the managers' decision problem. The no-insider trading assumption and the equilibrium compensation schemes offered to the managers imply that the managers' preferences are aligned with those of investors not only with respect to alternative production plans but also with respect to alternative costly information structures

for firm-specific events. The managers' information structures can therefore be obtained as a result of the equilibrium in the economy.

The investors' information structures for firm-specific events, however, cannot be sustained as equilibrium information structures. It is of no social value that investors receive information about firm-specific events, but the investors have strong private incentives to acquire such information. If an investor obtains private information, then he will clearly not restrict his portfolio plan to well-diversified portfolios. On the contrary, he is willing to take some firm-specific risk from trading based on that information, and he is willing to spend resources in order to get the information.

All investors realize that these potential gains from private information exist. In the extreme, all available information relevant to the intrinsic values of firms will be reflected in the market values of firms through the changes in the supply and the demand for shares in firms. Then, no investor will be able to obtain abnormal gains from private information acquisition. This leads, however, to a social loss both due to an inefficient risk sharing and due to the waste of resources spent on that activity. It is possible that it will be both socially and privately preferred that managers precommit to reporting their superior information about firm-specific events to the investors. Then no investor has any private incentives to acquire additional information. In that case, no resources are used to acquire private information, and the risk sharing is optimal. In this sense, it is of social value that managers report firm-specific information to the investors through, for example, external accounting reports.

If it is costly to report firm-specific information to investors, then it is socially inferior to report that information, compared to the case where managers are not reporting firm-specific information and the investors are able to precommit themselves not to acquire private information. However, the investors might not be able to make such a precommitment.

Our analysis in Part I focuses on the decision-facilitating role of information in the implementation of efficient consumption/production plans. The decision-influencing role of information is not an issue in the economy we

consider.⁶ Hence, our analysis considers only one aspect of the demand for external accounting information. Agency analyses that explore the decision-influencing role of information, largely do so in the context of partial equilibrium analyses, focusing on a single firm. Our analysis in Part I provides a general equilibrium analysis of the setting in which agency problems arise. Explicitly introducing agency issues, creates additional demands for information. This is the issue in Part II. However, it does not change the conclusions obtained in Part I about the decision-facilitating role of information.

Footnotes for Chapter 1.

1. These models include the one-period capital asset pricing model by Sharpe (1964), Lintner (1965) and Mossin (1966), the intertemporal capital asset pricing models by Merton (1973), Rubinstein (1976), Breeden and Litzenberger (1978), Breeden (1979), and the arbitrage pricing theory by Ross (1976). The model used in this dissertation is similar to the one by Breeden and Litzenberger (1978), in that it recognizes the crucial role of aggregate consumption. Also, the model has similarities to the arbitrage pricing theory, which considers the possibility of diversifying idiosyncratic risks.
2. Leland (1978) gets a similar result in a model, where managers have better information than investors about project-specific risks.
3. See, for example, Ohlson and Buckman (1981), Hakansson, Kunkel and Ohlson (1982), Kunkel (1982), Feltham (1985) and Ohlson (1988)
4. One such analysis is Kanodia (1980).
5. This implies that even if the market is only "semistrong efficient" in terms of the information reflected in market prices, the resource allocation can be "strong efficient". That is, a strong efficient capital market is not required to have a fully efficient resource allocation.
6. The decision-facilitating/decision influencing distinction is discussed in Demski and Feltham (1976, pp. 8-9)

CHAPTER 2.

The Structure of the Economy and Efficient Allocations.

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In this chapter we set out the assumptions that describe the economy in section 2.1. We define firm-specific events as well as economy-wide events, and we define the information structures for those events. Finally, feasible consumption/production plans are defined for a large economy.

In section 2.2 we characterize full Pareto efficient consumption/production plans given that these plans do not have to be individually rational. This gives the highest degree of efficiency of the allocations and, hence, the analysis in this section provides a useful benchmark for the analysis in later chapters, where we analyze equilibrium allocations which are individually rational.

2.1 Description of the economy.

Our analysis considers a two-period, single-good economy in which there are N investors, J firms and one manager for each firm. The sequence of events is as follows.

$t=0$: Investors are endowed with shares in the firms and trade to an equilibrium.

$t=1$: Information is revealed, production choices are made by managers, investors and managers consume and trade to an equilibrium.

$t=2$: Investors and managers consume the final output from the firms.

All uncertainty is depicted by the set of states $S = \{s\}$, about which investors and managers at $t=0$ have homogeneous beliefs, with density $f(s) > 0$ for all s . The set of states is composed of two types of events, economy-wide events s_0 and firm-specific events s_j , $j = 1, \dots, J$. These events are such that

$$S = S_0 \times S_1 \times \dots \times S_J$$

$$s = \{s_0, s_1, \dots, s_J\}, \quad s_0 \in S_0, s_j \in S_j, j = 1, \dots, J$$

$$f(s) = f(s_1|s_0) \cdot \dots \cdot f(s_J|s_0) \cdot f(s_0)$$

and such that the dividends of firm j only depend on the economy-wide event and on the event specific to firm j . The firm-specific events are conditionally independent given s_0 , and therefore, knowledge of s_j tells nothing about s_k if s_0 is known. That is,

(A1) Beliefs are homogeneous, strictly positive and conditionally independent given the economy-wide event:

$$f(s) = f(s_1|s_0) \cdot \dots \cdot f(s_J|s_0) \cdot f(s_0)$$

and

$$d_j = d_j(s_0, s_j, a_j)$$

$$x_j = x_j(s_0, s_j, a_j)$$

where d_j and x_j are the dividends from firm j at $t=1$ and $t=2$, respectively, and a_j is the production choice by the manager of firm j .

Prior to trading and production decisions at $t=1$, the investors' and the managers' knowledge of the state depends on the information structures. Since S represents all uncertainty in the economy, the information structures define a partition of S for each information recipient. We assume that the managers are at least as well-informed about their respective firms as the investors. Therefore, the partition Y_j^m of S_j for the manager of firm j is at least as fine a partition of S_j as the investors' partition Y_j^I . However, the partition Y_0 on the set of economy-wide events S_0 are common to all investors and managers:

(A2) Information is revealed at $t=1$ according to the partitions $Y_0, Y_1^I, \dots, Y_J^I$ common to all investors and managers. Furthermore, it is common-knowledge that the manager of firm j gets information according to Y_j^m which is at least as fine a partition of S_j as Y_j^I .

The information about firm-specific events received by investors can be viewed as an external accounting report issued by the manager and the information conveyed by the dividend at $t=1$, as the managers' production choices may depend on their private information. The information about economy-wide events could be general economic information from other sources than external accounting reports.

We make the additional assumption that the firm-specific information provides no additional information about the economy-wide events given $y_0 \in Y_0$:

$$(A3) \quad f(s_0|y) = f(s_0|y_0), \quad s_0 \in S_0, \quad y_0 \in Y_0$$

for

$$y \in Y_0 \times Y^I = Y_0 \times Y_1^I \times \dots \times Y_J^I$$

$$y \in Y_0 \times Y^m = Y_0 \times Y_1^m \times \dots \times Y_J^m$$

This assumption and the conditional independence of firm-specific events imply that firm-specific information for one firm provides no additional information about any other firm, i.e.,¹

$$f(s_j|y^I) = f(s_j|y_0, y_j^I), \quad y^I \in Y_0 \times Y^I, \quad y_j^I \in Y_j^I$$

$$f(s_j|y^{mj}) = f(s_j|y_0, y_j^m), \quad y^{mj} \in Y_0 \times Y^{mj}, \quad y_j^m \in Y_j^m$$

where Y^{mj} is the manager of firm j 's information structure for firm-specific events:

$$Y^{mj} \equiv Y_1^I \times \dots \times Y_j^m \times \dots \times Y_J^I.$$

The signal-beliefs are²

$$f(y^I) = f(y_1^I|y_0) \cdot \dots \cdot f(y_J^I|y_0) \cdot f(y_0)$$

$$f(y^{mj}) = f(y_1^I|y_0) \cdot \dots \cdot f(y_j^m|y_0) \cdot \dots \cdot f(y_J^I|y_0) \cdot f(y_0)$$

for investors and for the manager of firm j , respectively.

The cash flows from the firms are generated as follows. At date $t=1$ a non-negative cash flow from the operations of firm j in the previous period, x_{0j} , is observed by the manager of firm j . Therefore, it is $(Y_0 \times Y_j^m)$ -measurable, i.e.,

$$x_{j0}(s_0, s_j) = x_{j0}(y_0, y_j^m) \geq 0, \quad \forall s_0 \in y_0, \quad \forall s_j \in y_j^m.$$

Furthermore, a production choice $a_j \in A_j$, also $(Y_0 \times Y_j^m)$ -measurable, is selected by the manager according to a criterion known by the investors. The capital investment required to implement the production choice is denoted by $q_j(y_0, y_j^m, a_j)$. The cash flow of the previous period minus the capital investment is paid to the investors as dividends (or if negative, paid by the investors to the firm), i.e.,³

$$d_j(y_0, y_j^I, a_j) = x_{j0}(y_0, y_j^m) - q(y_0, y_j^m, a_j(y_0, y_j^m)).$$

Since dividends are observed by investors, and since the production decision rule is known by the investors, d_j must be $(Y_0 \times Y_j^I)$ -measurable to reflect that the investors form rational beliefs. However, even though investors know the production function and the manager's decision rule, the dividends do not necessarily reveal the manager's information completely. More than one signal to the manager may result in the same dividend. Observe also that even if the manager has superior information about the cash flow of the previous period, he is not able to use it for his own benefit. That is, we can think of cash flows as being audited but not necessarily reported to investors.

At $t=2$, the economy-wide event and the firm-specific events are revealed to investors and managers. Firms are liquidated and pay non-negative dividends according to the production functions

$$x_j = x_j(s, s_j, a_j) \geq 0$$

Let $DX_j(Y_0, Y_j^m)$ denote the production set for firm j given information structures Y_0 and Y_j^m :⁴

$$DX_j(Y_o, Y_j^m) = \{ d_j, x_j \mid d_j = x_{jo}(y_o, y_j^m) - q(y_o, y_j^m, a_j(y_o, y_j^m)),$$

$$x_j \leq x_j(s_o, s_j, a_j),$$

$$a_j(y_o, y_j^m) \in A_j, s_o \in y_o \in Y_o, s_j \in y_j^m \in Y_j^m \}$$

To secure a well-behaved problem (see Debreu (1959)) we make the assumption that⁵

(A4) For each firm the production functions are differentiable and the production set $DX_j(Y_o, Y_j^m)$ is closed, convex, permits free disposal, and contains a "null" vector (d_j^o, x_j^o) that occurs if the manager does "nothing" to reflect the fact that there is an existing production plan in place prior to the production decision at $t=1$. Furthermore, there is no free aggregate production relative to the "null" aggregate production. Finally, the optimal production plan is unique and interior for given information structures.

The assumption that the optimal production plan is unique and interior is only made to simplify the characterizations of optimal production plans and to simplify the comparisons of optimal production plans for different information structures. Observe also that there are no externalities, i.e., the production decision of one firm has no impact on the cash flow of any other firm. Of course, this is a critical assumption but it is standard in general equilibrium analyses.

Investor i 's consumption plan is denoted by $c_i = \{c_{i1}(s), c_{i2}(s)\}_{s \in S}$ and the consumption plan of the manager of firm j is denoted by $c_j^m = \{c_{j1}^m(s), c_{j2}^m(s)\}_{s \in S}$, where $c_{it}(s)$ ($c_{jt}^m(s)$) is the non-negative amount to be consumed by investor i (the manager of firm j) at time t if state s occurs.

Investors' and managers' preferences are characterized by the following assumption:

(A5) Investors' and managers' preferences are represented by time-additive utility functions over consumption:

$$U_i(c_i) = \sum_{s \in S} \{ u_{i1}(c_{i1}(s)) + u_{i2}(c_{i2}(s)) \} \cdot f(s)$$

$$U_j^m(c_j^m) = \sum_{s \in S} \{ u_{j1}^m(c_{j1}^m(s)) + u_{j2}^m(c_{j2}^m(s)) \} \cdot f(s)$$

for investors and managers respectively. Furthermore, $u_{it}(\cdot)$ and $u_{jt}^m(\cdot)$ are differentiable, strictly increasing, strictly concave, and have infinite marginal utility for zero consumption.

In order to be consistent with the information available at $t=1$, $c_{i1}(s)$ must be $(Y_0 \times Y^I)$ -measurable and $c_{j1}^m(s)$ must be $(Y_0 \times Y^{mj})$ -measurable. That is, the investors' consumption plans at $t=1$ can be written as

$$c_{i1}(s) = c_{i1}(y^I), \quad y^I \in Y_0 \times Y^I$$

while the managers' consumption plans at $t=1$ can be written as

$$c_{j1}^m(s) = c_{j1}^m(y^{mj}), \quad y^{mj} \in Y_0 \times Y^{mj}$$

Investors (and managers) are endowed with $\hat{c}_{it}$ ($\hat{c}_{jt}^m$) units of consumption at time t . We do not consider personal risks about endowments, since an efficient allocation of this type of risk is more related to the available insurance-instruments (such as fire-insurance) than to the function of capital markets.

Before trading takes place at $t=0$, investors (and managers) are endowed with investment portfolios $\hat{z}_i = \{\hat{z}_{i1}, \dots, \hat{z}_{iJ}\}$ ($\hat{z}_j^m = \{\hat{z}_{j1}^m, \dots, \hat{z}_{jJ}^m\}$), where $\hat{z}_{ik}$ ($\hat{z}_{jk}^m$) is the endowed ownership of firm k . The investment portfolio after trading at time t ($t=0$ and $t=1$) is denoted z_{it} (z_{jt}^m), which at time $t=1$ can be written as a function of the information available, i.e.

$$z_{i1} = z_{i1}(y^I), \quad y^I \in Y_0 \times Y^I$$

and

$$z_{j1}^m = z_{j1}^m(y^{mj}), \quad y^{mj} \in Y_0 \times Y^{mj}$$

In order to motivate the managers, they are paid management compensation at $t=1$ and at $t=2$ according to compensation schemes determined at $t=0$. To be enforceable the compensation at $t=1$ can only depend on information common to both investors and managers at $t=1$:

$$M_{j1} = M_{j1}(y^I), \quad y^I \in Y_0 \times Y^I.$$

Similarly, the compensation at $t=2$ depends on information available to both investors and managers at that date:

$$M_{j2} = M_{j2}(s)$$

The net-dividend available to the owners of firm j prior to trading at $t=1$ is therefore

$$d_j(y_0, y_j^I, a_j) - M_{j1}(y^I), \quad y^I \in Y_0 \times Y^I$$

and at $t=2$ the net-dividend is

$$x_j(s_0, s_j, a_j) - M_{j2}(s)$$

The market value of firm j at $t=0$ is denoted by V_{j0} and at $t=1$ the market value cum net-dividend is denoted by V_{j1} and is measurable with respect to public information at $t=1$:

$$V_{j1} = V_{j1}(y^I), \quad y^I \in Y_0 \times Y^I$$

i.e., the market is "semi-strong efficient" in terms of information reflected in market prices. The market value at $t=1$ of firm j ex net-dividend is

$$v_{j1}(y^I) = V_{j1}(y^I) - [d_j(y_0, y_j^I, a_j) - M_{j1}(y^I)], \quad y^I \in Y_0 \times Y^I$$

At $t=2$ the market value of firm j is simply equal to the net-dividend to the owners of the firm:

$$V_{j2}(s) = x_j(s_0, s_j, a_j) - M_{j2}(s)$$

The economy is assumed to be large with many investors and firms, such that given the conditional independence of firm-specific events, these events have very little impact on aggregate dividends for a given production plan. More formally, given that no firm is large relative to the total economy and considering a sequence of economies in which J is increasing, the strong law of large numbers for independent random variables implies that for all $y_0 \in Y_0$:

$$\text{Prob} [\{ y^I \in Y_0 \times Y^I \mid \lim_{J \rightarrow \infty} |1/J \cdot \sum_j (d_j(y_0, y_j^I, a_j) - \bar{d}_j(y_0, a_j))| = 0 \} \mid y_0] = 1$$

and for all $s_0 \in S_0$:

$$\text{Prob} [\{ s \in S \mid \lim_{J \rightarrow \infty} |1/J \cdot \sum_j (x_j(s_0, s_j, a_j) - \bar{x}_j(s_0, a_j))| = 0 \} \mid s_0] = 1$$

where

$$\bar{d}_j(y_0, a_j) = \sum_{y_j^I \in Y_j^I} d_j(y_0, y_j^I, a_j) \cdot f(y_j^I \mid y_0)$$

$$\bar{x}_j(s_0, a_j) = \sum_{s_j \in S_j} x_j(s_0, s_j, a_j) \cdot f(s_j \mid s_0).$$

Furthermore, since the managers' consumption plans at $t=1$, $c_{j1}^m(y^{mj})$, are independent conditional on public information at $t=1$, we assume that there is sufficient regularity such that for all $y^I \in Y_0 \times Y^I$:

$$\text{Prob} [\{ y^m \in Y_0 \times Y^m \mid \lim_{J \rightarrow \infty} |1/J \cdot \sum_j (c_{j1}^m(y^{mj}) - \bar{c}_{j1}^m(y^I))| = 0 \} \mid y^I] = 1$$

where

$$\bar{c}_{j1}^m(y^I) = \sum_{y_j^m \subseteq y_j^I} c_{j1}^m(y_0, y_1^I, \dots, y_j^m, \dots, y_j^I) \cdot f(y_j^m | y^I)$$

The regularity conditions include the condition that no private signal to any manager gives the manager perfect information about the cash flow of his firm at $t=2$. If that was the case then the manager might be able to take an infinite arbitrage position based on his superior information, and in that case the strong law of large numbers would not apply.

As in Berninghaus (1977) we define on this basis an alternative concept of feasible consumption/production plans:⁶

Definition 2.1: Quasi (Y_0, Y^I, Y^m) -feasible plans.

The consumption/production plan (c, a) is quasi (Y_0, Y^I, Y^m) -feasible if

(a) (c, a) is consistent with the information system (Y_0, Y^I, Y^m) .

(b) $c_{it}(s) \geq 0$ for all i, t and s

$c_{jt}^m(s) \geq 0$ for all j, t and s

(c) $a_j(y_0, y_j^m) \in A_j$ for all j , $y_0 \in Y_0$ and $y_j^m \in Y_j^m$

(d) $\sum_i c_{i1}(y^I) + \sum_j \bar{c}_{j1}^m(y^I) \leq \sum_i \hat{c}_{i1} + \sum_j \hat{c}_{j1}^m$

$+ \sum_j \bar{d}_j(y_0, a_j)$ for all $y^I \in Y_0 \times Y^I$

$\sum_i c_{i2}(s) + \sum_j c_{j2}^m(s) \leq \sum_i \hat{c}_{i2} + \sum_j \hat{c}_{j2}^m$

$+ \sum_j \bar{x}_j(s_0, a_j)$ for all $s \in S$

where

$$\bar{c}_{j1}^m(y^I) = \sum_{y_j^m \in Y_j^I} c_{j1}^m(y_0, y_1^I, \dots, y_j^m, \dots, y_J^I) \cdot f(y_j^m | y^I)$$

$$\bar{d}_j(y_0, a_j) = \sum_{y_j^I \in Y_j^I} d_j(y_0, y_j^I, a_j) \cdot f(y_j^I | y_0).$$

$$\bar{x}_j(s_0, a_j) = \sum_{s_j \in S_j} x_j(s_0, s_j, a_j) \cdot f(s_j | s_0)$$

The definition recognizes that firm-specific events when aggregated over firms and managers have almost no effect on "per capita supply", such that the demand-supply restriction can be formulated in terms of expected aggregate consumption possibilities conditional on the economy-wide events.

Observe also, that dividends at $t=1$ are not required to be measurable with respect to the public information at $t=1$, $y^I \in Y_0 \times Y^I$. It implies that the production plan can be selected without consideration to the information that is revealed through the dividends. This gives the highest degree of efficiency of the allocations. However, equilibrium plans, as defined in Chapter 3, are such that the dividends are measurable with respect to public information at $t=1$.

We make the assumption that:

(A6) A consumption/production plan (c, a) is feasible in our large economy, if it is quasi (Y_0, Y^I, Y^m) -feasible.

This completes the description of the basic elements of the economy. The sequence of the allocation problem is summarized in Figure 2.1 and the notation is summarized in Figure 2.2.



Figure 2.1: Sequence of allocation problem

	t=0	t=1	t=2
Uncertain events:			
Economy-wide events			s_o
Firm-specific events			s_j
Economy-wide information		y_o	
Firms			
Firm-specific manager info.		y_j^m	
Cash flow of the previous period		$x_{jo}(y_o, y_j^m)$	
Production choice		$a_j(y_o, y_j^m)$	
Capital investment		$q(y_o, y_j^m, a_j)$	
Dividend to owners		$d_j(y_o, y_j^I, a_j)$	$x_j(s_o, s_j, a_j)$
Comp. to manager		$M_{j1}(y^I)$	$M_{j1}(s)$
Manager's consump. endowments		$\hat{c}_{j1}^m$	$\hat{c}_{j2}^m$
Manager's actual consumption		$c_{j1}^m(y^{mj})$	$c_{j2}^m(s)$
Manager's endowed portf.	$\hat{z}_j^m$		
Manager's investm. portf.	z_{jo}^m	$z_{j1}^m(y^{mj})$	
Cum net-dividend market value	V_{jo}	$V_{j1}(y^I)$	$V_{j2}(s)$
Ex net-dividend market value		$v_{j1}(y^I)$	
Investors			
Firm-specific information		y_j^I	
Endowed consumption		$\hat{c}_{i1}$	$\hat{c}_{i2}$
Actual consumption		$c_{i1}(y^I)$	$c_{i2}(s)$
Endowed portfolio	$\hat{z}_i$		
Investment portfolio	z_{io}	$z_{i1}(y^I)$	

Figure 2.2: Summary of notation.

2.2 Unconstrained Pareto efficient allocations.

In this section we define and give characterizations of unconstrained Pareto efficient consumption/production plans with respect to the information structures Y_0 , Y^I and Y^m . An unconstrained Pareto efficient allocation is an allocation in which the distribution of initial endowments is ignored. Moreover, the resulting consumption/production plans do not have to be individually rational. The solution concept is that of optimality which we can think of as being the decision problem of a central planner, who has access to all information in the economy. This gives the highest degree of efficiency of the allocations, and is therefore a useful benchmark for the analysis in later chapters, where we define and characterize equilibrium allocations that are individually rational. In the sections 2.2.1 to 2.2.3 we analyze the characteristics of information that give the information value in a large production economy.

We begin by considering Pareto-optimal allocations given a particular production plan, i.e., allocations in an exchange economy.

Definition 2.2: (Y_0, Y^I, Y^m) -efficient consumption plans given the production plan.

The consumption plan c is (Y_0, Y^I, Y^m) -efficient if there is no other (Y_0, Y^I, Y^m) -feasible consumption plan that is strictly Pareto-preferred, given the production plan a .

When the production plan is given, the allocation problem is a matter of sharing the risk, i.e., sharing the aggregate consumption possibilities among investors and managers. Investors' consumption plans are measurable with respect to public information. However, managers' consumption plans can vary with their private information. The following proposition states that this does not happen for efficient consumption plans.

Proposition 2.1.

For an (Y_0, Y^I, Y^m) -efficient consumption plan c , given the production plan, the managers' consumption plans are measurable with respect to public information.

Proof:

Assume that for some $y_0 \in Y_0$ and $y_j^I \in Y_j^I$ there are at least two signals y_j^m and $y_j^{m'}$ both in y_j^I for which

$$c_{j1}^m(y_0, y_1^I, \dots, y_j^m, \dots, y_J^I) \neq c_{j1}^m(y_0, y_1^I, \dots, y_j^{m'}, \dots, y_J^I)$$

That is, the consumption plan at $t=1$ for the manager of firm j is not measurable with respect to public information at $t=1$, y^I . If we offer the manager the conditional expected value of the original plan for all signals y_j^m in y_j^I :

$$\begin{aligned} & c_{j1}^{m*}(y_0, y_1^I, \dots, y_j^I, \dots, y_J^I) \\ &= c_{j1}^{m*}(y_0, y_1^I, \dots, y_j^m, \dots, y_J^I) \quad \forall y_j^m \in y_j^I \\ &= \sum_{y_j^m \in y_j^I} c_{j1}^m(y_0, y_1^I, \dots, y_j^m, \dots, y_J^I) \cdot f(y_j^m | y_0, y_j^I) \end{aligned}$$

then this plan is strictly preferred by the manager, since he is strictly risk averse.

The other investors and managers are no worse off, since the supply-demand restriction on feasible consumption plans is stated only in terms of expected values of the managers' consumption conditional on public information, and these are the same for both plans by construction.

Hence, the consumption plan c_{j1}^m cannot be part of an (Y_0, Y^I, Y^m) -efficient consumption plan c . A contradiction to the assumption that c_{j1}^m is not measurable with respect to public information is obtained. **Q.E.D.**

The proposition implies that insider trading based on managers' superior information about firm-specific events cannot occur as part of an efficient resource allocation. As it appears from the idea of the proof, the reason is that in order to obtain the gains from insider trading, the managers must take some unnecessary risks. Thus, insider trading is not just a matter of movements along the Pareto-frontier and "fair" allocations; it is a matter of Pareto-inefficiency. However, recall that the consumption plans are not required to be individually rational. Therefore, efficient allocations may not be obtainable in a market setting without restrictions on the managers' trading possibilities. We return to this question in Chapter 4.

The idea applied in the proof of Proposition 2.1 can be used to identify some additional characteristics of (Y_0, Y^I, Y^M) -efficient consumption plans.

Proposition 2.2.

If c is an (Y_0, Y^I, Y^M) -efficient consumption plan given the production plan, then c is measurable with respect to economy-wide events.

Proof:

The proof is basically the same as the proof of Proposition 1. Assume that the consumption plan for at least one economic agent is not measurable with respect to the economy-wide events. Then offer all investors and managers another plan, where each economic agent for each event gets the expected value of the original plan conditional on the economy-wide event. Then at least one economic agent is strictly better off, since all investors and managers are strictly risk averse. The other economic agents are at least as well off. The new plan is quasi-feasible, since the original plan does not for any public signal y^I distribute more than the expected aggregate consumption possibilities conditional on the economy-wide events. This leads to a contradiction of the initial assumption. **Q.E.D.**

Given risk averse investors and managers there is no reason why any economic agent should bear a risk, which by the strong law of large numbers for independent random variables average out when aggregated for all investors and managers. Thus, with efficient risk sharing no investor or manager is exposed to firm-specific risk. They only share the risk associated with the economy-wide events.

Firm-specific events are only those events that affect a single firm and which are conditionally independent. All other events are defined to be economy-wide events. Therefore, given the production plan, many economy-wide events can result in the same level of expected aggregate consumption possibilities conditional on the economy-wide event:

$$\bar{c}_{0t}(s_0, \mathbf{a}) = \begin{cases} \sum_i \hat{c}_{i1} + \sum_j \hat{c}_{j1}^m + \sum_j \bar{d}_j(s_0, \mathbf{a}_j) & \text{if } t=1 \\ \sum_i \hat{c}_{i2} + \sum_j \hat{c}_{j2}^m + \sum_j \bar{x}_j(s_0, \mathbf{a}_j) & \text{if } t=2 \end{cases}$$

The conditional expected aggregate consumption level can be viewed as defining a partition on the set of economy-wide events (or the set of economy-wide signals at $t=1$). That partition at time t , given production plan $\mathbf{a}$, is denoted $\Gamma_t(\mathbf{a}) = \{\gamma_t\}$, where γ_t denotes both the level of the conditional expected aggregate consumption and the set of economy-wide events that results in that level, i.e.,

$$\gamma_t \equiv \{ s_0 \in S_0 \mid \bar{c}_{0t}(s_0, \mathbf{a}) = \gamma_t \}$$

Observe that $\bar{c}_{0t}(s_0, \mathbf{a})$ depends on the production plan and, hence, the partition Γ_t is a function of the production plan. Note also that Y_0 is a subpartition of Γ_1 whereas there is no general relationship between Γ_2 and Y_0 .

Proposition 2.3.

The consumption plan $\mathbf{c}$ is (Y_0, Y^I, Y^M) -efficient given the production plan $\mathbf{a}$ if, and only if, implicit non-negative prices P_t and positive parameters $\lambda_1, \dots, \lambda_N, \delta_1, \dots, \delta_J$ exist, such that for all i

$$(2.1) \quad u_{i1}'(c_{i1}(y_0)) = \lambda_i \cdot P_1(y_0)/f(y_0) = \lambda_i \cdot P_1(\gamma_1)/f(\gamma_1), \quad \forall y_0 \subseteq \gamma_1$$

$$(2.2) \quad u_{i2}'(c_{i2}(s_0)) = \lambda_i \cdot P_2(s_0)/f(s_0) = \lambda_i \cdot P_2(\gamma_2)/f(\gamma_2), \quad \forall s_0 \in \gamma_2$$

and for all j

$$(2.3) \quad u_{j1}'(c_{j1}^m(y_0)) = \delta_j \cdot P_1(y_0)/f(y_0) = \delta_j \cdot P_1(\gamma_1)/f(\gamma_1), \quad \forall y_0 \subseteq \gamma_1$$

$$(2.4) \quad u_{j2}'(c_{j2}^m(s_0)) = \delta_j \cdot P_2(s_0)/f(s_0) = \delta_j \cdot P_2(\gamma_2)/f(\gamma_2), \quad \forall s_0 \in \gamma_2$$

and such that

$$(2.5) \quad \sum_i c_{it}(s_0) + \sum_j c_{jt}^m(s_0) = \bar{c}_{ot}(s_0, \mathbf{a}), \quad \forall s_0 \in S_0, \quad t = 1, 2$$

where

$$P_1(y_0) = P_1(\gamma_1) \cdot f(y_0|\gamma_1)$$

$$P_2(s_0) = P_2(\gamma_2) \cdot f(s_0|\gamma_2)$$

Furthermore, all (Y_0, Y^I, Y^M) -efficient consumption plans are $(\Gamma_1(\mathbf{a}), \Gamma_2(\mathbf{a}))$ -measurable and strictly increasing with γ_t .⁷

Proof:

Given the concavity of preferences, (A5), the convexity of the consumption possibility set, and given that efficient consumption plans are measurable with respect to economy-wide events (Proposition 2.2), any Pareto-efficient consumption plan can be represented as a solution to the following decision problem:

$$\text{maximize}_{\mathbf{c}} \quad \sum_i \lambda_i \cdot U_i(\mathbf{c}_i) + \sum_j \delta_j \cdot U_j^m(\mathbf{c}_j^m)$$

subject to

$$c_{it}(s_0) \geq 0 \quad \forall i, \forall s_0, \quad t=1, 2$$

$$c_{jt}^m(s_0) \geq 0 \quad \forall j, \forall s_0, \quad t=1, 2$$

$$\sum_i c_{it}(s_0) + \sum_j c_{jt}^m(s_0) \leq \bar{c}_{ot}(s_0, a) \quad [P_t(s_0)] \quad \forall s_0, t=1,2$$

$$c_{i1}(s_0) \text{ and } c_{j1}^m(s_0) \text{ is } Y_0\text{-measurable } \forall i, \forall j, \forall s_0$$

If we let P_t denote the multipliers for the third set of constraints, then (2.1), (2.2), (2.3) and (2.4) are the first-order conditions for this decision problem, since infinite marginal utility for zero consumption implies that the multipliers for the two first sets of constraints are zero. Condition (2.5) follows from non-satiation. Under the given assumptions, the first order conditions are also sufficient conditions.

$(\Gamma_1(a), \Gamma_2(a))$ -measurability of the consumption plan can be seen as follows. (2.1)-(2.4) implies that

$$\frac{u'_{i1}(c_{i1}(y_0))}{u'_{i1}(c_{i1}(y'_0))} = \frac{\lambda_1}{\lambda_i} \quad \forall i \quad \forall y_0 \in Y_0$$

$$\frac{u'_{j1}(c_{j1}(y_0))}{u'_{j1}(c_{j1}(y'_0))} = \frac{\lambda_1}{\delta_j} \quad \forall j \quad \forall y_0 \in y_0$$

That is, the ratios of marginal utilities of time $t=1$ consumption across investors and managers are independent of the signals $y_0 \in Y_0$. For any $\gamma_1 \in \Gamma_1(a)$ take two signals $y_0, y'_0 \subseteq \gamma_1$, i.e. the conditional expected aggregate consumption possibility level is the same for both signals. Suppose that $c_{i1}(y_0) > c_{i1}(y'_0)$. Then $c_{i1}(y_0) > c_{i1}(y'_0)$ for all i and $c_{j1}^m(y_0) > c_{j1}^m(y'_0)$ for all j by the equality of the ratio of marginal utilities of consumption for the two signals. This implies that

$$\sum_i c_{i1}(y_0) + \sum_j c_{j1}^m(y_0) > \sum_i c_{i1}(y'_0) + \sum_j c_{j1}^m(y'_0)$$

but this is inconsistent with non-satiation and equal conditional expected aggregate consumption possibilities for signal y_0 and signal y'_0 . Thus, $c_{i1}(y_0) = c_{i1}(y'_0)$. The same argument for time $t=2$ consumption gives that the consumption plan must be $(\Gamma_1(a), \Gamma_2(a))$ -measurable. The same argument gives also that c_{it} and c_{jt}^m are strictly increasing with γ_t . Q.E.D.

The propositions 2.1 to 2.3 characterize unconstrained Pareto-efficient risk sharing given a particular production plan. No economic agent takes any firm-specific risk or economy-wide risk, which does not result from variations in the conditional expected aggregated consumption level. Consequently, the implicit prices P_t pertain only to events that affect the level of conditional expected aggregate consumption. To appreciate this fully, these prices are such that for any γ_1 (or γ_2) any variation in contingent consumption due to events in γ_1 is a "fair" gamble, i.e.,

$$\begin{aligned} \sum_{y_0 \in \gamma_1} c_{i1}(y_0) \cdot P_1(y_0) &= \sum_{y_0 \in \gamma_1} c_{i1}(y_0) \cdot P_1(\gamma_1) \cdot f(y_0|\gamma_1) \\ &= P_1(\gamma_1) \cdot \sum_{y_0 \in \gamma_1} c_{i1}(y_0) \cdot f(y_0|\gamma_1) \end{aligned}$$

Since the investors and the managers are strictly risk averse, it is not optimal for them to take part in any such gamble. Hence, their consumption only varies with and is an increasing function of γ_t , i.e., the conditional expected aggregated consumption level associated with economy-wide events.

We now turn to the case of production. As the managers base their production decisions on their private information and as the resulting dividends are observable by investors, more of the managers' private firm-specific information than the information revealed by accounting reports can be revealed to investors. How much more, clearly depends on the production plan. Therefore, the firm-specific information available to investors depends on the production plan. However, we have already shown that efficient consumption plans do not depend on firm-specific events and, hence, for efficient consumption/production plans we can ignore the fact that the firm-specific information available to investors depends on the production plan. Moreover, in the definition of feasible consumption/production plans, dividends are not required to be measurable with respect to public information at $t=1$. Hence, although the information available to investors depends on the production plan, we can define efficient consumption/production plans as follows.

Definition 2.3: (Y_0, Y^I, Y^m) -efficient consumption/production plans.

The consumption/production plan (c, a) is (Y_0, Y^I, Y^m) -efficient if there is no other (Y_0, Y^I, Y^m) -feasible consumption/production plan (c', a') that is strictly Pareto-preferred.

The following proposition characterizes the optimal production plan for (Y_0, Y^I, Y^m) -efficient consumption/production plans.

Proposition 2.4.

(c^*, a^*) is an (Y_0, Y^I, Y^m) -efficient consumption/production plan if, and only if, implicit prices P_t exist such that (2.1) through (2.5) are satisfied and such that for all j the production plan of firm j , a_j^* , maximizes the value at $t=0$ of the dividends from that firm calculated at these prices,

$$(2.6) \quad W_{j0} = \sum_{y_0 \in Y_0} \bar{d}_j(y_0, a_j) \cdot P_1(y_0) + \sum_{s_0 \in S_0} \bar{x}_j(s_0, a_j) \cdot P_2(s_0).$$

Furthermore, the production plan of firm j also maximizes the intrinsic value at $t=1$ of the dividends, given the manager's information at $t=1$,

$$(2.7) \quad W_{j1}(y_0, y_j^m) = d_j(y_0, y_j^m, a_j(y_0, y_j^m)) \cdot P_1(y_0) \\ + \sum_{s_0 \in y_0} P_2(s_0) \cdot \sum_{s_j \in S_j} x_j(s_0, s_j, a_j(y_0, y_j^m)) \cdot f(s_j | s_0, y_j^m).$$

Proof:

An (Y_0, Y^I, Y^m) -efficient consumption/production plan is the solution to the following decision problem:

$$\underset{c, a}{\text{maximise}} \quad \sum_i \lambda_i \cdot U_i(c_i) + \sum_j \delta_j \cdot U_j^m(c_j^m)$$

subject to

$$c_{it}(s_0) \geq 0 \quad \forall i, \forall s_0, t=1,2$$

$$c_{jt}^m(s_0) \geq 0 \quad \forall j, \forall s_0, t=1,2$$

$$\sum_i c_{it}(s_0) + \sum_j c_{jt}^m(s_0) \leq \bar{c}_{0t}(s_0, a) \quad [P_t(s_0)] \quad \forall s_0, t=1,2$$

$$c_{i1}(s_0) \text{ and } c_{j1}^m(s_0) \text{ is } Y_0\text{-measurable } \forall i, \forall j, \forall s_0$$

$$a_j(s) \in A_j \text{ and } (Y_0 \times Y_j^m)\text{-measurable } \forall s, \forall j$$

Using the definition of the expected aggregate consumption conditional on the economy-wide event,

$$\bar{c}_{0t}(s_0, a) = \begin{cases} \sum_i \hat{c}_{i1}(s_0) + \sum_j \hat{c}_{j1}^m(s_0) + \sum_j \bar{d}_j(s_0, a_j) & \text{if } t=1 \\ \sum_i \hat{c}_{i2}(s_0) + \sum_j \hat{c}_{j2}^m(s_0) + \sum_j \bar{x}_j(s_0, a_j) & \text{if } t=2 \end{cases}$$

and the definition of conditional expected dividends,

$$\bar{d}_j(y_0, a_j) = \sum_{y_j^m \in Y_j^m} \{ x_{0j}(y_0, y_j^m) - q_j(y_0, y_j^m, a_j(y_0, y_j^m)) \} \cdot f(y_j^m | y_0)$$

$$\bar{x}_j(s_0, a_j) = \sum_{s_j \in S_j} x_j(s_0, s_j, a_j(y_0, y_j^m)) \cdot f(s_j | s_0),$$

the first order condition for an interior optimal solution of $a_j(y_0, y_j^m)$ is that

$$\frac{\partial \bar{d}_j}{\partial s_j}(y_0, y_j^m, a_j^*(y_0, y_j^m)) \cdot f(y_j^m | y_0) \cdot F_1(y_0)$$

$$\begin{aligned}
& + \sum_{s_0 \in y_0} P_2(s_0) \cdot \sum_{s_j \in y_j^m} \frac{\partial x_j(s_0, s_j, a_j^*(y_0, y_j^m))}{\partial a_j} \cdot f(s_j | s_0) \\
& = 0
\end{aligned}$$

Since $f(s_j | s_0) / f(y_j^m | y_0) = f(s_j | s_0, y_j^m)$ for $s_0 \in y_0$ and $s_j \in y_j^m$, the first order condition is equivalent to⁸

$$\begin{aligned}
& \frac{\partial q_j(y_0, y_j^m, a_j^*(y_0, y_j^m))}{\partial a_j} \cdot P_1(y_0) = \\
& = \sum_{s_0 \in y_0} P_2(s_0) \cdot \sum_{s_j \in S_j} \frac{\partial x_j(s_0, s_j, a_j^*(y_0, y_j^m))}{\partial a_j} \cdot f(s_j | s_0, y_j^m)
\end{aligned}$$

This is the first order condition for a maximum of the intrinsic value of the dividends at $t=1$. Since the value of the dividends at $t=0$ is given by

$$\begin{aligned}
W_{j0} &= \sum_{y_0 \in Y_0} \sum_{y_j^m \in Y_j^m} W_{j1}(y_0, y_j^m) \cdot f(y_j^m | y_0), \\
&= \sum_{y_0 \in Y_0} \bar{W}_{j1}(y_0)
\end{aligned}$$

the production plan also maximizes the value at $t=0$. **Q.E.D.**

Note that the production plan maximizes the value of the dividends. It does not necessarily maximize the value of the firms which is the value of the dividends minus the value of the compensations to the managers. Only for some compensation contracts is maximizing the value of the dividends equivalent to maximizing the value of the firm. This will be shown in Chapter 4. An efficient production plan is determined by an efficient allocation of total consumption possibilities over time, both to investors and to managers, and by an efficient allocation of investments among firms. How the total consumption possibilities are distributed among investors on one side

and managers on the other side is not of direct concern for an efficient production plan.

An efficient allocation of total investments among firms requires that production decisions are based on the managers' superior information at $t=1$ and not only on public available information. The implicit state-contingent prices, however, pertain only to economy-wide events about which investors and managers have the same information. These prices are used to evaluate the expected dividends conditional on the managers' superior information. Thus, the managers have only superior information about the production conditions of their own firms and not about the relevant prices to evaluate the stream of dividends for alternative production plans. The following proposition gives some initial characteristics of the consequences of changing the information structures.

Proposition 2.5

Let $C^* \times A^* = \{(c^*, a^*)\}$ be the set of (Y_0, Y^I, Y^m) -efficient consumption/production plans.

- (2.8) Any (Y_0, Y^I, Y^m) -efficient consumption/production plan is also an $(Y_0, Y^{I'}, Y^m)$ -efficient consumption/production plan even though $Y^{I'}$ may be more or less informative than Y^I .
- (2.9) For any consumption/production plan which is based only on publicly available information, there exists a (weakly) Pareto-preferred consumption/production plan $(c^*, a^*) \in C^* \times A^*$.
- (2.10) If $Y_0 \times Y_1^m \times \dots \times Y_J^m$ is at least as informative as $Y_0' \times Y_1^{m'} \times \dots \times Y_J^{m'}$, then for any (Y_0, Y^I, Y^m) -efficient consumption/production plan there exists a (weakly) Pareto-preferred consumption/production plan $(c^*, a^*) \in C^* \times A^*$.
- (2.11) There is strict Pareto-preference in the relations (2.9) and (2.10) if, and only if, for all $(c^*, a^*) \in C^* \times A^*$ which is weakly Pareto-preferred, the production plan a^* is not feasible under the less informative information structures.

Proof:

(2.8) follows from Proposition 2.2, since c^* is measurable with respect to the economy-wide events and a^* is not required to be measurable with respect to public information.

(2.9) holds, since a consumption/production plan based on public information is also an (Y_0, Y^I, Y^m) -feasible plan. A similar argument gives (2.10).

(2.11): A production plan is feasible if, and only if, $a_j(y_0, y_j^m) \in A_j$ and a_j is $(Y_0 \times Y_j^m)$ -measurable. Therefore a^* is not feasible under the less informative information structures if, and only if, a^* varies with the signals (y_0, y_j^m) that belongs to the same signal for the less informative information structures $y_0', y_j^{m'}$.

(Only if): Suppose a^* is feasible under the less informative information structures. Then a^* is also an efficient production plan for these information structures, since all feasible production plans under the less informative information structures are also feasible for the information structures (Y_0, Y^I, Y^m) . This implies that the partitions Γ_t and the elements γ_t are the same for both sets of information structures. Since efficient consumption plans are measurable with respect to the partitions Γ_t , the consumption/production plan (c^*, a^*) cannot be strictly Pareto-preferred in the relations (2.9) and (2.10). Hence, a^* is not feasible under the less informative information structures.

(If): An efficient production plan under the less informative information structures is also feasible for the information structures (Y_0, Y^I, Y^m) . If an efficient production plan for the information structures (Y_0, Y^I, Y^m) , which is feasible under the less informative information structures, does not exist, then the consumption/production plan (c^*, a^*) is strictly Pareto-preferred in the relations (2.9) and (2.10). This gives (2.11).

Q.E.D.

(2.8) states, that the consumption/production plan does not depend on how much information investors have about firm-specific events. The reasons are that efficient consumption plans only depend on economy-wide events and that efficient production plans are based on the information which is available to managers. Therefore, a Pareto-improvement of the consumption/production plan cannot be obtained by making investors better informed about firm-specific events. That is, reporting firm-specific information to investors is of no social value.

A production plan based on public information is weakly inferior to a production plan based on the managers' information, since measurability with respect to public information is at least as restrictive as measurability with respect the managers' information structures. What can be done with one information system can also be done with another information system, if that information system is more informative than the former, but a more informative information system is only valuable if it changes the optimal production plan,⁹ i.e., (2.11). That is, additional information is of no value in an exchange economy, but it can be valuable in a production economy due to a preferred allocation of consumption possibilities over time and due to a better allocation of investments among firms.

In the following section we give some examples that illustrate the results of the preceding analysis.

2.2.1 Examples.

The first example illustrates efficient risk sharing given a particular production plan such that there is no aggregate risk to share. In the second example, the production plan is determined as the optimal plan, given that it must be measurable with respect to public information. This corresponds to perceived market value maximization. In the third example we show how the aggregate investment determined in the second example can be re-allocated between firms based on the managers' superior information, such that all investors and all managers are strictly better off. Finally, we determine the efficient consumption/production plans in the fourth example.

The general characteristics of the examples below are as follows. The cash flows of the previous period are zero for all firms:

$$x_{j0} = 0 \quad \forall j$$

The set of production choices are $A_j = [0, \infty)$ for all firms, and the production decision is the amount to invest:

$$q(y_0, y_j^m, a_j) = a_j \quad \text{for all } y_0, y_j^m, a_j \text{ and for all firms } j.$$

The production functions at $t=2$ are

$$x_j(s_0, s_j, a_j) = \begin{cases} s_0 + s_j \cdot a_j & \text{if } j \text{ even} \\ 3 - s_0 + s_j \cdot a_j & \text{if } j \text{ odd} \end{cases}$$

The set of economy-wide events and firm-specific events are

$$S_0 = \{ 1, 2 \}$$

$$S_j = \{ 2, 4, 6, 8 \} \quad \text{for all } j$$

where all events are assumed to be independent and equally likely, i.e., $f(s_0) = 1/2$, $s_0 = 1, 2$ and $f(s_j) = 1/4$, $s_j = 2, 4, 6, 8$. Investors and managers get perfect information about the economy-wide event at $t=1$, i.e., $Y_0 = \{(1); (2)\}$. There are no accounting reports. Hence, if managers base their production decisions on public information, then investors get no informa-

tion about firm-specific events at $t=1$, i.e., $Y_j^I = \{ (2,4,6,8) \}$. However, the managers get information about firm-specific events at $t=1$, such that they are able to distinguish the two bad events from the two good events, i.e., $Y_j^m = \{ (2,4); (6,8) \}$.

Investors' consumption endowments in both periods are 6 units, i.e., $\hat{c}_{it} = 6$, while the managers' endowments are 4 units in each period, i.e., $\hat{c}_{jt}^m = 4$. Their respective utility functions are

$$u_{it}(c_{it}(s)) = \ln(c_{it}(s)) \quad \text{for all } i, t, \text{ and } s$$

$$u_{jt}^m(c_{jt}^m(s)) = \ln(c_{jt}^m(s)) \quad \text{for all } j, t \text{ and } s.$$

and there are J investors and J managers.

Example 2.1: Risk sharing given the production plan.

Let the production plan be

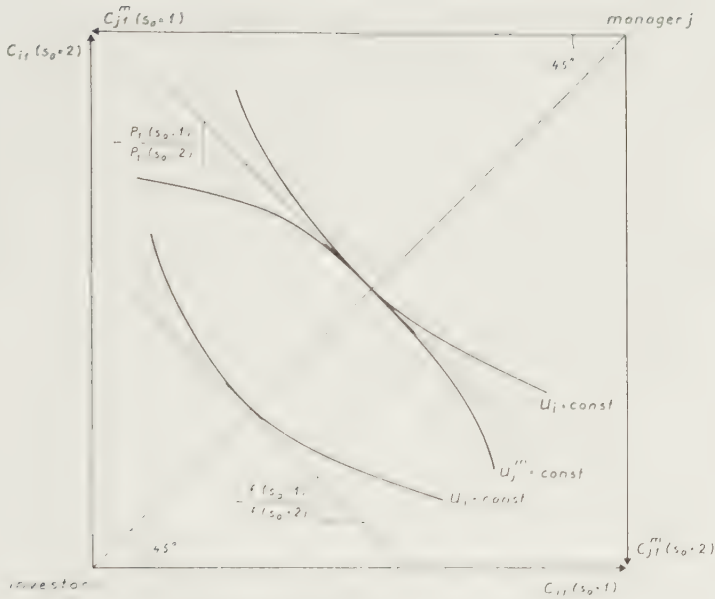
$$a_j(s_o, y_j^m) = 2 \quad \text{for all } j, s_o \text{ and } y_j^m$$

Then the conditional expected aggregate consumption levels are

$$\bar{c}_{o1}(s_o, a) = 6 \cdot J + 4 \cdot J - 2 \cdot J = 8J \quad s_o = 1, 2$$

$$\bar{c}_{o2}(s_o, a) = 10 \cdot J + 3 \cdot J + 10 \cdot J = 23J \quad s_o = 1, 2$$

There is no aggregate risk to share in either period 1 or period 2, and then no economic agent should take on any risk in either period. This can be illustrated by an Edgeworth box. Since all investors are identical and all managers are identical we reduce the allocation problem to the problem of allocating $\bar{c}_{ot}(s_o)/J$ units between one investor and one manager. That is, we choose the parameters $\lambda = \lambda_1 = \dots = \lambda_J$ and $\delta = \delta_1 = \dots = \delta_J$. Since preferences are time-additive, we can consider consumption at time $t=1$ and consumption at time $t=2$ separately. Therefore, consider the Edgeworth box for consumption at time $t=1$:



The slope of the indifference curves are

$$\frac{dc_{i1}(s_0=2)}{dc_{i1}(s_0=1)} = - \frac{u_{i1}'(c_{i1}(s_0=1))}{u_{i1}'(c_{i1}(s_0=2))} \cdot \frac{f(s_0=1)}{f(s_0=2)}$$

$$\frac{dc_{j1}^m(s_0=2)}{dc_{j1}^m(s_0=1)} = - \frac{u_{j1}^m(c_{j1}^m(s_0=1))}{u_{j1}^m(c_{j1}^m(s_0=2))} \cdot \frac{f(s_0=1)}{f(s_0=2)}$$

Optimal risk sharing is characterized by a tangency point between the indifference curves of the investor and the manager. In a no-risk position for the investor (i.e., along the 45° line) the slope of the indifference curve is

$$\frac{dc_{i1}(s_0=2)}{dc_{i1}(s_0=1)} = - \frac{f(s_0=1)}{f(s_0=2)} ; \quad c_{i1}(s_0=1) = c_{i1}(s_0=2)$$

since the utility function is state-independent. The same holds for the manager. Since the Edgeworth box is squared, both the investor's 45° line

and the manager's 45° line collapse into the single main 45° diagonal and, since they have homogeneous beliefs this line also becomes the contract curve of all efficient allocations. Therefore, neither the investor nor the manager gets any risk. The situation is similar for period 2 consumption.

Example 2.2: Efficient consumption plan given production plan based on public information.

The public information is perfect information about the economy-wide event s_0 and no information about firm-specific events. Hence, the feasible production plans are

$$a_j(s_0, y_j^m) = a_j(s_0) \quad \text{for all } j, s_0 \text{ and all } y_j^m \in Y_j^m$$

Suppose that the production plan does not vary with the economy-wide event either. Then $\bar{c}_{01}(s_0=1) = \bar{c}_{01}(s_0=2)$ and $\bar{c}_{02}(s_0=1) = \bar{c}_{02}(s_0=2)$, and no economic agent will be exposed to any risk for an efficient consumption allocation. The implicit prices will therefore be

$$P_1(s_0) = P_1 \cdot f(s_0) \quad s_0 = 1, 2$$

$$P_2(s_0) = P_2 \cdot f(s_0) \quad s_0 = 1, 2$$

where P_1 and P_2 are some constants independent of the economy-wide event. From the first order condition for optimal production,

$$P_1(s_0) = P_2(s_0) \cdot \sum_{s_j} f(s_j) \cdot s_j,$$

it follows that

$$P_1 = 5 \cdot P_2$$

Furthermore, given the form of the utility functions, it follows from the first order conditions for an optimal risk sharing that

$$\frac{u_{i2}'(c_{i2}(s_0))}{u_{i1}'(c_{i1}(s_0))} = \frac{c_{i1}(s_0)}{c_{i2}(s_0)} = \frac{P_2}{P_1} = 1/5$$

$$\frac{u_{j2}^m(c_{j2}^m(s_0))}{u_{j1}^m(c_{j1}^m(s_0))} = \frac{c_{j1}^m(s_0)}{c_{j2}^m(s_0)} = \frac{P_2}{P_1} = 1/5$$

Hence, for each investor and each manager the consumption in period 2 is five times higher than in period 1 and, therefore

$$\bar{c}_{02}(s_0, \mathbf{a}) = 5 \cdot \bar{c}_{01}(s_0, \mathbf{a})$$

This gives that the production plan is

$$a_j(s_0) = 3.7$$

and this plan is indeed optimal.

If we again choose $\lambda_1 = \dots = \lambda_J = \lambda$ and $\delta_1 = \dots = \delta_J = \delta$, and $\lambda = 1$, $\delta = 2$, then

$$\frac{u_{i1}'(c_{i1}(s_0))}{u_{j1}^m(c_{j1}^m(s_0))} = \frac{c_{j1}^m(s_0)}{c_{i1}(s_0)} = \lambda/\delta = 1/2$$

$$\frac{u_{i2}'(c_{i2}(s_0))}{u_{j2}^m(c_{j2}^m(s_0))} = \frac{c_{j2}^m(s_0)}{c_{i2}(s_0)} = \lambda/\delta = 1/2$$

Hence, investors consume twice as much as managers in both periods and for $a_j = 3.7$ this gives that

$$c_{i1}(s_0) = 4.2 \quad c_{i2}(s_0) = 21 \quad \text{for all } i, s_0$$

$$c_{j1}^m(s_0) = 2.1 \quad c_{j2}^m(s_0) = 10.5 \quad \text{for all } j, s_0$$

The expected utility of investors and managers are

$$U_i = \ln(4.2) + \ln(21) = 4.48 \quad \text{for all } i$$

$$U_j^m = \ln(2.1) + \ln(10.5) = 3.09 \quad \text{for all } j$$

Example 2.3: Re-allocation of investments based on the managers' information.

In example 2.2, where production decisions are based on public information, all firms invest 3.7 units for all events, firm-specific as well as economy-wide. The production plan does not vary with the economy-wide events, because it is not optimal to do so, but the production plan does not vary with the firm-specific events, because it is not possible, given the public information structures, Y_j^I . Here we will improve the allocation by making a re-allocation of the total investments based on the managers' information. Let the production plan for firms with the bad signal, $y_{j1}^m = (2,4)$, be zero investment,

$$a_j(s_o, y_{j1}^m) = 0,$$

and let the production plan for firms with the good signal, $y_{j2}^m = (6,8)$, be twice the investment in the case of production decisions based on public information,

$$a_j(s_o, y_{j2}^m) = 7.4.$$

Then the expected total investment is still equal to $3.7 \cdot J$ and the expected aggregate consumption level at $t=1$ is also still equal to $6.3 \cdot J$. The expected aggregate consumption level at $t=2$, however, is now

$$\bar{c}_{02}(s_o, a) = 10 \cdot J + 3 \cdot J + 7.4 \cdot J \cdot (6 + 8) / 4 = 38.9 \cdot J$$

What we have done is that, based on the managers' information, we have re-allocated the same total investments from the low productivity firms and to the high productivity firms, such that the total consumption possibilities at $t=1$ are the same. The total consumption possibilities at $t=2$, however, have increased from $31.5 \cdot J$ to $38.9 \cdot J$. These additional consumption possibi-

lities can then be distributed to investors and managers and make everyone strictly better off. If we share aggregate consumption possibilities as we did in Example 2.2, then the consumption plans are

$$c_{i1}(s_0) = 4.2 \quad c_{i2}(s_0) = 25.9 \quad \text{for all } i, s_0$$

$$c_{j1}^m(s_0) = 2.1 \quad c_{j2}^m(s_0) = 13 \quad \text{for all } j, s_0$$

and the expected utilities of investors and managers are

$$U_i = \ln(4.2) + \ln(25.9) = 4.690 \quad \text{for all } i$$

$$U_j^m = \ln(2.1) + \ln(13) = 3.307 \quad \text{for all } j$$

which, obviously, are higher than the expected utilities investors and managers obtain in Example 2.2.

Example 2.4: Efficient consumption/production plan.

Here we allow the manager to vary the production decision with his private information, i.e.,

$$a_j = a_j(s_0, y_j^m) \quad \text{for all } j, s_0, y_j^m \in Y_j^m$$

and we want to determine the optimal investments conditional on his private information.

The manager's information is either $y_{j1}^m = (2, 4)$ or $y_{j2}^m = (6, 8)$.

Suppose again that $\bar{c}_{ot} = \bar{c}_{ot}(s_0)$ for all t and s_0 . Then, given the production functions, the first order condition (with possible boundary solutions) for optimal production given y_{j1}^m is

$$\begin{aligned} P_1 &\geq P_2 \cdot \sum_{s_j} f(s_j | y_{j1}^m) \cdot s_j \\ &= 3 \cdot P_2 \end{aligned}$$

and for y_{j2}^m

$$\begin{aligned} P_1 &\geq P_2 \cdot \sum_{s_j} f(s_j | y_{j2}^m) \cdot s_j \\ &= 7 \cdot P_2 \end{aligned}$$

Since these inequalities cannot be fulfilled as equalities at the same time, we get that $a_j(y_0, y_{j1}^m) = 0$, i.e., the firms with bad signals do not invest while firms with good signals do (as in Example 2.3). Then we have that

$$P_1 = 7 \cdot P_2$$

Now investors and managers consume seven times as much in period 2 as in period 1 and, hence,

$$\bar{c}_{02}(s_0) = 7 \cdot \bar{c}_{01}(s_0).$$

We can now determine $a_j(y_0, y_{j2}^m)$ from¹⁰

$$7 \cdot (10 \cdot J - a_j \cdot J/2) = 10 \cdot J + 3 \cdot J + a_j \cdot J \cdot (6 + 8)/4$$

which is equivalent to

$$a_j(y_0, y_{j2}^m) = 8.14.$$

If we compare this production plan with the production plan based on public information, then the expected investment has increased from 3.7 to 4.07, and only firms with good signals invest, while firms with bad signals do not invest. Hence, due to a more efficient allocation of capital among firms, the trade-off between current consumption and future consumption has changed such that total investments are higher.

With $\lambda = 1$ and $\delta = 2$ the consumption plans of investors and managers are

$$c_{i1}(s_0) = 3.95$$

$$c_{i2}(s_0) = 27.65$$

$$c_{j1}^m(s_0) = 1.98$$

$$c_{j2}^m(s_0) = 13.86$$

and the expected utilities are

$$U_i = \ln(3.95) + \ln(27.65) = 4.694$$

$$U_j^m = \ln(1.98) + \ln(13.86) = 3.312$$

Thus, both the investors and the managers gain from delegating production responsibility to managers due to a more efficient allocation of investments among firms, and due to a more efficient allocation of consumption over time. Observe also that delegating production responsibility to managers implies that the investors are able to infer the managers' information from the dividend and their knowledge of the optimal production plans in this example.

In these examples the economy-wide events are only a matter of whether it is the "odd" firms that get the windfall-gain of 2 units instead of 1 unit or it is the "even" firms. The total windfall gain for all firms are always 3 units times the number of firms, and it is not optimal to vary the production decisions with the economy-wide events. Hence, for the optimal consumption/production plan the conditional expected aggregate consumption level does not vary with the economy-wide events either. However, this does not necessarily hold for non-optimal production plans, and this distinguishes the economy-wide events from the firm-specific events. Observe also that the optimal consumption/production plan is independent of the information structure for the economy-wide events, since neither optimal consumption plans nor optimal production plans vary with that information, i.e., information about economy-wide events is of no social value in these examples.

2.2.2 Improved allocation of resources among firms.

One of the gains to additional information in the examples above was a more efficient allocation of resources among firms (Example 2.3). In order to develop this idea more formally, define the set of expected aggregated production by

$$DX(Y_0, Y^m) = \{ \bar{d}_0, \bar{x}_0 \mid \bar{d}_0(y_0) = \sum_j \bar{d}_j(y_0, a_j), \quad \bar{x}_0(s_0) = \sum_j \bar{x}_j(s_0, a_j), \\ (d_j, x_j) \in DX_j(Y_0, Y_j^m) \}$$

One information structure is more informative than another information structure if it defines a finer partition on the set of events. We now define more productive information structures as follows.

Definition 2.4: Productive information structures.

In a large economy, information structure $Y'_0 \times Y^{m'}$ is more productive than information structure $Y_0 \times Y^m$ if, for every $(\bar{d}_0, \bar{x}_0) \in DX(Y_0, Y^m)$ there exist an $(\bar{d}'_0, \bar{x}'_0) \in DX(Y'_0, Y^{m'})$ such that $\bar{d}'_0(s_0) \geq \bar{d}_0(s_0)$ and $\bar{x}'_0(s_0) \geq \bar{x}_0(s_0)$ for all $s_0 \in S_0$ and strict equality for at least some $s_0 \in S_0$.

There are two reasons for expressing this definition in terms of conditional expected aggregate production instead of in terms of production sets for individual firms. First, only conditional expected aggregate consumption is of concern in our large economy, and secondly, in order to reflect that improved productivity may result from firms using additional information to allocate resources among projects within firms more efficiently as well as to allocate resources among firms more efficiently.¹¹ If $Y'_0 \times Y^{m'}$ is more productive than $Y_0 \times Y^m$, then $Y'_0 \times Y^{m'}$ provides a dominant vector of expected aggregate production levels and thereby a dominant vector of expected aggregate consumption possibilities. These additional consumption possibilities can then be distributed among investors and managers, and

make everyone better off with the information structure $Y'_0 \times Y^{m'}$ than with the information structure $Y_0 \times Y^m$.

Proposition 2.6.

If, in a large economy, information structure $Y'_0 \times Y^{m'}$ is more productive than $Y_0 \times Y^m$, then for any consumption/production plan (c, a) that is (Y_0, Y^I, Y^m) -feasible, there exists a strictly Pareto preferred consumption/production plan (c', a') which is $(Y'_0, Y^I, Y^{m'})$ -feasible.

Proof:

The proof follows from the arguments preceding the proposition and from the fact that investors and managers strictly prefer more rather than less (i.e non-satiation). **Q.E.D.**

More productive information structures provide a basis for Pareto preferred consumption/production plans due to the existence of dominant conditional expected aggregate consumption vectors (as in Example 2.3). However, the optimal consumption/production plan is not necessarily dominant over the optimal consumption/production plans under less productive information structures. This follows from the fact that a more efficient allocation of resources among firms also changes the trade-off between current and future consumption (as in Example 2.4).

2.2.3 Windfall and productivity information

Some general results on the use of information to improve resource allocations within an economy are presented above. In this section we impose some additional structure on the production functions in order to obtain further insight into the characteristics of valuable information. In particular, we distinguish between windfall events and productivity events. The impact of windfall events on future dividends is independent of the current production plan, whereas the impact of productivity events on future dividends depends on the production plan. To depict these two types of events assume that

$$s_0 = (\theta_0, \xi_0), \quad s_0 \in S_0 = \Theta_0 \times \Xi_0$$

and

$$s_j = (\theta_j, \xi_j), \quad s_j \in S_j = \Theta_j \times \Xi_j$$

and such that

$$x_j(s_0, s_j, a_j) = g_j(\theta_0, \theta_j) + h_j(\xi_0, \xi_j, a_j)$$

That is, while both types of events influence the cash flow at $t=2$, the windfall events (θ_0 and θ_j) do not influence the incremental effect of the managers' production decisions, whereas the productivity events (ξ_0 and ξ_j) do influence those incremental effects.

Definition 2.5: Windfall and Productivity events.

Let $S_0 = \Theta_0 \times \Xi_0$ and $S_j = \Theta_j \times \Xi_j$ for all j . Then the set $\Theta = \Theta_0 \times \Theta_1 \times \dots \times \Theta_J$ defines windfall events and the set $\Xi = \Xi_0 \times \Xi_1 \times \dots \times \Xi_J$ defines productivity events if

- (a) For each firm j , functions $g_j(\cdot)$ and $h_j(\cdot)$ exist, such that

$$x_j(s_0, s_j, a_j) = g_j(\theta_0, \theta_j) + h_j(\xi_0, \xi_j, a_j)$$

for all $s_0 \in \Theta_0 \times \Xi_0$ and $s_j \in \Theta_j \times \Xi_j$

where the marginal productivity of a_j is positive and decreasing, i.e., $h'_j(\cdot, a_j) > 0$, $h''_j(\cdot, a_j) < 0$.

(b) The capital investment depends on the production plan only,

$$q_j(y_0, y_j^m, a_j) = q_j(a_j) \quad \text{for all } y_0, y_j^m \text{ and all } j,$$

and $q'_j(a_j) > 0$, $q''_j(a_j) \geq 0$.

(c) The events are ordered such that for any feasible production plan a the following conditions hold:

(i) $g_0(\theta) \equiv \sum_j g_j(\theta_0, \theta_j)$ is strictly increasing in θ_0 .

(ii) $g_j(\theta_0, \theta_j)$ is strictly increasing in θ_j , for all j .

(iii) $h_0(\xi, a) \equiv \sum_j h_j(\xi_0, \xi_j, a_j)$ and

$$h'_0(\xi, a) \equiv \sum_j h'_j(\xi_0, \xi_j, a_j)$$

are both strictly increasing in ξ_0

(iv) $h_j(\xi_0, \xi_j, a_j)$ and $h'_j(\xi_0, \xi_j, a_j)$ are both strictly increasing in ξ_j .

(d) The economy-wide windfall and productivity events are independent

$$f(\theta_0, \xi_0) = f(\theta_0) \cdot f(\xi_0).$$

and the firm-specific windfall and productivity events are conditionally independent, such that

$$f(\theta_j, \xi_j | \theta_0, \xi_0) = f(\theta_j | \theta_0) \cdot f(\xi_j | \xi_0).$$

Condition (a) formalizes the requirement that the impact of windfall events on future cash flows is independent of a_j . Condition (b) simplifies the analysis, since the productivity of the actions then only depends on the production functions for future cash flows. We could perform the analysis without this requirement, and then the productivity would be measured as the ratio of marginal output to marginal input for the particular signals. Condition (c) facilitates the analysis by ensuring that both windfall events and productivity events have a well defined impact on future cash flows. Both types of events are ordered such that the cash flow at $t=2$ increases with the event and the productivity events are such that an increase in a productivity event increases the marginal productivity of the input. Condition (d) ensures that information on windfall events do not tell anything about productivity events and vice versa.

We define information about windfall and productivity events as partitions on the set of events as follows.

Economy-wide information:

$$y_0 = (y\theta_0, y\xi_0) \in Y\Theta_0 \times Y\Xi_0$$

$$Y_0 = Y\Theta_0 \times Y\Xi_0$$

Firm-specific information:

$$y_j^I = (y\theta_j^I, y\xi_j^I) \in Y\Theta_j^I \times Y\Xi_j^I$$

$$Y_j^I = Y\Theta_j^I \times Y\Xi_j^I$$

$$y_j^m = (y\theta_j^m, y\xi_j^m) \in Y\Theta_j^m \times Y\Xi_j^m$$

$$Y_j^m = Y\Theta_j^m \times Y\Xi_j^m$$

In order to secure a well defined impact of information we assume that for each partition higher signals correspond to subsets of events which are higher than those for lower signals. Taking the information structure for economy-wide windfall events as an example, that is, if $y\theta_0 < y\theta'_0$ for $y\theta_0$,

$y\theta'_0 \in Y\theta_0$ then $\theta_0 < \theta'_0$ for all $\theta_0 \in y\theta_0$ and all $\theta'_0 \in y\theta'_0$. Since the events are ordered such that the production and the productivity increase with the events, we get that the production and the productivity also increase with the signals. This assumption is made to ensure that finer information structures in fact also provide additional information on expected windfall gains or losses and on expected productivity. If we take the managers' information structures on firm-specific events in the examples above, $Y_j^m = \{ (2,4), (6,8) \}$, then they have this characteristic. For the low signal, y_{j1}^m , the expected marginal product of investments is 3, and for the high signal, y_{j2}^m , the expected marginal product of investments is 7, and that information is useful (see example 2.3). However, if the information structure is $Y_j^m = \{ (2,8), (4,6) \}$, then the expected marginal product of investments is 5 for both signals. This information structure is useless compared to no information, since it does not distinguish between low productivity firms and high productivity firms. The assumption on the type of information structures above is made in order to avoid this situation.

Information about windfall and productivity events will be compared to minimal information structures Y_0^O , Y_0^I and Y_0^m which are defined as follows:

$Y_0^O \times Y_0^m$: The coarsest partition of the sets of events which make the cash flows of the previous period, x_0 , measurable with respect to this system for all firms.

Y_0^I : The coarsest partition of the sets of firm-specific events which make the dividends at $t=1$, d , measurable with respect to this system given the optimal production plan for $Y_0^O \times Y_0^m$.

These information structures represent the minimal information available in the economy, given that the cash flows of the previous period depend on the events, and given (Y_0^O, Y_0^I, Y_0^m) -efficient production plans. Observe that if the cash flows of the previous periods are known with certainty at $t=0$, i.e., $x_{0j}(s) = x_{0j} \quad \forall s, j$, then the minimal information structures provide no information at all.

The following proposition establishes which of the information structures is of value relative to the minimal information structures.¹²

Proposition 2.7

For any consumption/production plan (c, a) based on the minimal information structures, Y_O^O , Y_O^I and Y_O^m , consumption/production plans exist based on the following structures that are strictly Pareto-preferred:

(2.12) Y_{Θ_O} - information on economy-wide windfall events.

(2.13) Y_{Ξ^m} - information on firm-specific productivity events to managers.

Similarly, consumption/production plans exist generically based on the following structure that are strictly Pareto-preferred:

(2.14) Y_{Ξ_O} - information on economy-wide productivity events.

If (c, a) is an (Y_O^O, Y_O^I, Y_O^m) -efficient consumption/production plan, then there is no feasible consumption/production plans based on the following information structures which are strictly Pareto-preferred to (c, a) :

(2.15) Y_{Θ^m} - information on firm-specific windfall events to managers.

(2.16) Y_{Θ^I} - information on firm-specific windfall events to investors.

(2.17) Y_{Ξ^I} - information on firm-specific productivity events to investors.

Proof:

(2.16) and (2.17) follow immediately from (2.8) in Proposition 2.5. In general, information is of strict positive value if, and only if, the production plan varies with that information (see (2.11) in Proposition 2.5). Therefore, we will focus on the optimal production decision criterion. From Proposition 2.4 we have that the optimal production plan maximizes the intrinsic value of the dividends,

$$W_{j1}(y_O, y_j^m, a_j) = d_j(y_O, y_j^m, a_j) \cdot P_1(y_O) \\ + \sum_{s_O \in y_O} P_2(s_O) \cdot \sum_{s_j \in S_j} x_j(s_O, s_j, a_j) \cdot f(s_j | s_O, y_j^m)$$

given the information available to the manager of firm j , $y_0 = (y_{\theta_0}, y_{\xi_0})$ and $y_j^m = (y_{\theta_j^m}, y_{\xi_j^m})$. Using the definition of windfall and productivity events and the fact that the cash flow of the previous period does not depend on the current production plan, we get that maximizing the intrinsic value of the dividends is equivalent to maximizing the following expression:

$$\begin{aligned}
 & - q_j(a_j) \cdot P_1(y_0) \\
 & + \sum_{\theta_0 \in y_{\theta_0}} \sum_{\xi_0 \in y_{\xi_0}} P_2(\theta_0, \xi_0) \cdot \sum_{\theta_j \in \Theta_j} \sum_{\xi_j \in \Xi_j} x_j(\theta_0, \xi_0, \theta_j, \xi_j, a_j) \\
 & \quad \cdot f(\theta_j, \xi_j | \theta_0, \xi_0, y_{\theta_j^m}^m, y_{\xi_j^m}^m) \\
 & = - q_j(a_j) \cdot P_1(y_0) \\
 & + \sum_{\theta_0 \in y_{\theta_0}} \sum_{\theta_j \in \Theta_j} g_j(\theta_0, \theta_j) \cdot f(\theta_j | \theta_0, y_{\theta_j^m}^m) \cdot \sum_{\xi_0 \in y_{\xi_0}} P_2(\theta_0, \xi_0) \\
 & + \sum_{\xi_0 \in y_{\xi_0}} \sum_{\xi_j \in \Xi_j} h_j(\xi_0, \xi_j, a_j) \cdot f(\xi_j | \xi_0, y_{\xi_j^m}^m) \cdot \sum_{\theta_0 \in y_{\theta_0}} P_2(\theta_0, \xi_0) \\
 & = - q_j(a_j) \cdot P_1(y_0) \\
 & + \sum_{\theta_0 \in y_{\theta_0}} E[g_j(\theta_0, \theta_j) | \theta_0, y_{\theta_j^m}^m] \cdot \sum_{\xi_0 \in y_{\xi_0}} P_2(\theta_0, \xi_0) \\
 & + \sum_{\xi_0 \in y_{\xi_0}} E[h_j(\xi_0, \xi_j, a_j) | \xi_0, y_{\xi_j^m}^m] \cdot \sum_{\theta_0 \in y_{\theta_0}} P_2(\theta_0, \xi_0)
 \end{aligned}$$

where $E[\cdot]$ is the expectation operator.

The implicit state-contingent prices $P_t(\cdot)$ do not depend on firm-specific signals, and therefore, the optimal production plan does not vary with firm-specific windfall signals, since

$$\frac{\partial E[g_j(\theta_0, \theta_j) | \theta_0, y_{\theta_j^m}^m]}{\partial a_j} = 0 \quad \forall \theta_0, y_{\theta_j^m}^m$$

This establishes (2.15).

For firm-specific productivity signals the marginal expected output,

$$\frac{\partial E[h_j(\xi_0, \xi_j, a_j) \mid \xi_0, y\xi_j^m]}{\partial a_j},$$

is strictly increasing in $y\xi_j^m$, since $h_j'(\xi_0, \xi_j, a_j)$ is strictly increasing in ξ_j and since higher signals $y\xi_j^m$ correspond to higher events ξ_j . Since, $h_j(\cdot)$ is increasing and concave in a_j and $q_j(\cdot)$ is increasing and convex in a_j , the optimal production plan and the optimal capital investment are both strictly increasing in $y\xi_j^m$ for given economy-wide signals. Therefore, the optimal production plan varies with firm-specific productivity signals and this gives (2.13).

The first order condition for an optimal production choice is

$$\begin{aligned} & \frac{dq_j}{da_j} (a_j^*(y\theta_0, y\xi_0, y\xi_j^m)) \\ &= \sum_{\xi_0 \in y\xi_0} \frac{\partial E[h_j(\xi_0, \xi_j, a_j) \mid \xi_0, y\xi_j^m]}{\partial a_j} \cdot \sum_{\theta_0 \in y\theta_0} \frac{P_2(\theta_0, \xi_0)}{P_1(y\theta_0, y\xi_0)} \Big|_{a_j = a_j^*} \end{aligned}$$

It appears from this optimality condition that the optimal production plan only varies with economy-wide windfall signals given $y\xi_0$ and $y\xi_j^m$ if

$$\sum_{\theta_0 \in y\theta_0} \frac{P_2(\theta_0, \xi_0)}{P_1(y\theta_0, y\xi_0)}$$

varies with $y\theta_0$ for some $\xi_0 \in y\xi_0$. Assume that the optimal production plan does not vary with the economy-wide windfall signals, $y\theta_0$. Then, within a given signal from the minimal information structure for economy-wide windfall events, $y\theta_0^0$, the conditional expected aggregate consumption level at $t=1$ does not vary with $y\theta_0$. From Proposition 2.3 we then have that individual consumption plans at $t=1$ do not vary with $y\theta_0$ either, and that

$$\sum_{\theta_0 \in y\theta_0} \frac{P_2(\theta_0, \xi_0)}{P_1(y\theta_0, y\xi_0)}$$

$$\begin{aligned}
&= \sum_{\theta_0 \in y\theta_0} \frac{u'_{i2}(c_{i2}(\theta_0, \xi_0))}{u'_{i1}(c_{i1}(y\theta_0, y\xi_0))} \cdot \frac{f(\theta_0, \xi_0)}{f(y\theta_0, y\xi_0)} \\
&= \frac{f(\xi_0|y\xi_0)}{u'_{i1}(c_{i1}(y\theta'_0, y\xi_0))} \cdot \sum_{\theta_0 \in y\theta_0} u'_{i2}(c_{i2}(\theta_0, \xi_0)) \cdot f(\theta_0|y\theta_0)
\end{aligned}$$

for $y\theta'_0, y\theta_0 \in y\theta_0^0$. However, from condition (c)-i) in Definition 2.5 we have that the conditional expected aggregate consumption level at $t=2$ is strictly increasing in the economy-wide windfall events when the production plan does not depend on these events. Using Proposition 2.3 again we then have that

$$c_{i2}(\theta_0, \xi_0) > c_{i2}(\theta'_0, \xi_0) \quad \forall i$$

for $\theta_0 \in y\theta_0$, $\theta'_0 \in y\theta_0$ and $y\theta_0 > y\theta'_0$. This implies, that

$$\sum_{\theta_0 \in y\theta_0} u'_{i2}(c_{i2}(\theta_0, \xi_0)) \cdot f(\theta_0|y\theta_0) < \sum_{\theta'_0 \in y\theta'_0} u'_{i2}(c_{i2}(\theta'_0, \xi_0)) \cdot f(\theta'_0|y\theta'_0)$$

or equivalently that

$$\sum_{\theta_0 \in y\theta_0} \frac{P_2(\theta_0, \xi_0)}{P_1(y\theta_0, \xi_0)} < \sum_{\theta'_0 \in y\theta'_0} \frac{P_2(\theta'_0, \xi_0)}{P_1(y\theta'_0, \xi_0)}$$

for $y\theta_0 > y\theta'_0$. From the first order condition for optimal production decisions, this means that $a_j^*(y\theta_0, y\xi_0, y\xi_j^m)$ is strictly decreasing in the economy-wide windfall signal $y\theta_0$. This contradicts the initial assumption that the optimal production plan does not vary with the economy-wide windfall signals, and that gives (2.12). (2.14) follows from similar arguments, since economy-wide productivity signals influence both the marginal expected output and the ratio of period 1 and period 2 state-contingent prices. However, "knife-edge" cases exist where for all signals the change in productivity precisely offsets the change in the state-contingent prices such that the production plan does not depend on economy-wide productivity information.

Q.E.D.

Information about economy-wide or firm-specific productivity events is of value, because it allows a better allocation of the resources at $t=1$ between aggregate current consumption and aggregate investment in production, as well as a better allocation of the aggregate investment among firms and among projects within firms. Firm-specific productivity information derives its main benefit from allowing a higher aggregate output at $t=2$ for the same aggregate investment at $t=1$ by allocating more investment to the more productive firms and less to the less productive firms. Higher aggregate output at $t=2$ reduces the implicit state-contingent prices, and a more efficient allocation of investments at $t=1$ gives a higher average productivity of investments. Hence, the effect on aggregate investment can go either way relative to what would occur with the minimal information structure.

The effect of economy-wide productivity information on aggregate investment can also be positive as well as negative. When the productivity is high, the expected output is also high and, thus, the state-contingent prices are low. Increased productivity has a positive impact on optimal aggregate investment, while reduced state-contingent prices has a negative impact on optimal aggregate investment. The net-effect on aggregate investment relative to the minimal information structure depends on the relative size of the variations in productivity, the variations in the expected output, and on the resulting changes in the investors' ratio of marginal utilities of future and current consumption. "Knife-edge" cases exist where economy-wide productivity has no impact on the production plan.¹³

The proposition establishes that prior knowledge of windfall gains and losses is of value only if it pertains to economy-wide events. Recall that if the production plan does not change, then additional information is of no value. The value of having information about economy-wide windfall events derives from the fact that it influences production activity. For economy-wide windfall information it is optimal to let the aggregate amount invested in production be inversely related to the signal. For example, if we know at $t=1$ that there will be large economy-wide windfall gains at $t=2$, then it will be optimal to increase consumption at $t=1$ and reduce investment in production relative to the case with the minimal information structure. Hence, due to these changes in the aggregate production, the impact

of economy-wide windfall information is a smoothing of the aggregate consumption possibilities over the two time periods. Therefore, economy-wide windfall information permits a more efficient smoothing of individual consumption levels. This follows from the fact that individual consumption plans are increasing functions of the level of conditional expected aggregate consumption in each period.

On the other hand, knowledge at $t=1$ of firm-specific windfall gains and losses is of no value. This information merely tells which firms will realize the gains and which will realize the losses. In a large economy, these gains and losses will offset each other, and the beliefs about the aggregate output at $t=2$ will not vary with that information. Hence, the managers can do nothing to take advantage of firm-specific windfall information.

Reporting firm-specific windfall as well as productivity information to the investors is of no value because their consumption plans do not depend on their knowledge of firm-specific events, and investors are not responsible for making production decisions. Therefore, there is nothing investors can do to gain from that information.

2.3 Conclusion.

In this chapter we have described the economy in section 2.1 and characterized efficient consumption/production plans in section 2.2.

The uncertainty in the economy is depicted by a set of states which consist of economy-wide events as well as of firm-specific events. Firm-specific events are those events which only affect the dividends of individual firms, and which are independent conditional of the economy-wide events. Information structures are defined as partitions on the sets of economy-wide and firm-specific events. The information structure for the economy-wide events is common to all investors and managers, while the managers' information structures for the firm-specific events of their own firms are at least as informative as that of investors and other managers. Furthermore, the information is such that, given the economy-wide information, the firm-specific information for one firm does not convey information about the firm-specific events of other firms. Hence, the manager of any single firm may have superior information about the firm-specific events of his own firm, but not about the economy-wide events and the firm-specific events of other firms.

The economy is assumed to be large with many investors and firms, and it is assumed that no firm is large relative to the total economy, such that the firm-specific events only have a negligible impact on "per capita supply" for a given production plan. Using the strong law of large numbers for independent random variables, a concept of quasi-feasible consumption/production plan is defined in which the aggregate consumption possibilities are equal to the expected aggregate consumption possibilities conditional on the economy-wide events. This concept ensures the possibility of choosing market clearing consumption plans which are measurable with respect to economy-wide events. Hence, the definition of quasi-feasible consumption/production plans facilitates diversification in a large economy.

Pareto efficient consumption plans, given the production plan, are characterized in Propositions 2.1, 2.2, and 2.3. Proposition 2.3 states that efficient consumption plans for investors as well as for managers are measurable with respect to the conditional expected aggregate consumption

level at each date, and that individual consumption is strictly increasing with that level. Furthermore, implicit state-contingent prices exist such that variations in consumption plans not due to variations in the level of conditional expected aggregate consumption are fair gambles with respect to these prices. Hence, the basic state-contingent prices pertain only to the level of conditional expected aggregate consumption for a given production plan.

Pareto efficient production plans are characterized in Proposition 2.4. It is shown that, prior to the release of information, efficient production plans maximize the value of the dividends of firms. The value of the dividends is calculated as the expected dividends conditional on the economy-wide events evaluated with the implicit state-contingent prices pertaining to these events. After the release of information, production plans maximize the intrinsic value of the dividends, given the manager's superior information about the firm-specific events of the firm.

As an immediate consequence of Proposition 2.3, it is shown in Proposition 2.5 that additional information is only valuable if it changes the production plan. Otherwise, the levels of conditional expected aggregate consumption are identical, and thus, individual consumption plans are also identical. Feasible production plans are only restricted by the manager's information. Hence, reporting firm-specific information to the investors does not change the production plan and, therefore, it is of no social value to report that information. On the other hand, it is of social value that the production plan is based on the managers' information, if their superior information is useful.

The critical assumptions for these results are time-additive utility functions and homogeneous beliefs. If these assumptions are not fulfilled, then additional information may be of social value even if the production plan does not change. This is due to the fact that individual consumption plans are not measurable with respect to aggregate consumption possibilities in that case. If the utility functions are not time-additive, then additional information may facilitate a more efficient intertemporal allocation of individual consumption. In the case of heterogeneous beliefs, additional information may provide additional opportunities for betting on the events.

In section 2.2.1 a number of examples are given which illustrate that the managers' superior information about firm-specific events can be useful to allocate resources more efficiently among firms. In section 2.2.2 this is formalized in the definition of productive information structures. An information structure is more productive than another if it provides a dominant vector of expected aggregate production levels conditional on the economy-wide events. These additional consumption possibilities can then be distributed among investors and managers, and make everyone better off. Hence, more productive information structures are valuable as long as investors and managers have strictly increasing utility functions and the concept of quasi-feasible consumption/production plans is well-defined.

In order to obtain further insight into the characteristics of valuable information, additional structure is imposed on the production functions in section 2.2.3. We distinguish between windfall events and productivity events. The impact of windfall events on future dividends is independent of the current production plan, whereas the impact of productivity events on future dividends depends on the production plan. It is shown in Proposition 2.7 that Pareto-preferred consumption/production plans can be obtained with more information about productivity, whether or not the variations in productivity are due to economy-wide or firm-specific events, whereas information about windfall gains and losses is only valuable if it pertains to economy-wide events. Information about firm-specific productivity events to the managers achieves its main gains from permitting a more efficient allocation of resources among firms. Information about economy-wide windfall events permits a more efficient allocation of resources over time. It is optimal that the aggregate investment in production is negatively related to the expected future windfall gains. Hence, the aggregate risk in the economy can be distributed more evenly between time periods based on this information. Information about economy-wide productivity events permits a more efficient allocation of resources among firms as well as over time.

It is of no value that managers receive information about firm-specific windfall events. This information does not affect the beliefs about the future level of expected aggregate consumption conditional on the economy-wide events. Hence, the implicit state-contingent prices pertaining to the economy-wide events does not depend on this information. Since the impact

of windfall events on future dividends do not depend on the current production plan, the managers can do nothing to take advantage of firm-specific windfall information.

The analysis in this chapter has considered the role of information for efficient consumption/production plans. Efficient production plans depend on all the information in the economy which either affects the productivity of firms or the beliefs about the future expected aggregate consumption possibilities conditional on the economy-wide events. On the other hand, efficient consumption plans depend only on the level of expected aggregate consumption conditional on the economy-wide events at each date. Hence, only information which makes the level of conditional expected aggregate consumption measurable with respect to that information is of value to the investors. In the next chapter, we consider the implementation of efficient consumption plans in a market setting. In that respect, additional information to investors is generally required. The implementation of efficient production plans is considered in Chapter 4.

Footnotes for Chapter 2.

1. Let $y = \{y_0, y_1, \dots, y_J\}$ be the investors' information or the information of the manager of firm j , i.e.,

$$y \in Y_0 \times Y_1^I \times \dots \times Y_J^I \quad \text{or} \quad y \in Y_0 \times Y_1^I \times \dots \times Y_j^m \times \dots \times Y_J^I$$

We must show that

$$f(s_j|y) = f(s_j|y_0, y_j).$$

The probability of s_j given the information y can be written as

$$\begin{aligned} f(s_j|y) &= f(s_j, y) / f(y) \\ &= \{ \sum_{s_0 \in y_0} f(s_j, s_0, y_1, \dots, y_J) \} / f(y) \\ &= \{ \sum_{s_0 \in y_0} f(s_j|s_0, y_1, \dots, y_J) \cdot f(s_0, y_1, \dots, y_J) \} / f(y) \end{aligned}$$

From (A1) and $s_0 \in y_0$ we get that

$$\begin{aligned} f(s_j|y) &= \{ \sum_{s_0 \in y_0} f(s_j|s_0, y_j) \cdot f(s_0, y_1, \dots, y_J) \} / f(y) \\ &= \{ \sum_{s_0 \in y_0} f(s_j|s_0, y_j) \cdot f(s_0, y_0, y_1, \dots, y_J) \} / f(y) \\ &= \{ \sum_{s_0 \in y_0} f(s_j|s_0, y_j) \cdot f(s_0|y_0, y_1, \dots, y_J) \cdot f(y) \} / f(y) \\ &= \sum_{s_0 \in y_0} f(s_j|s_0, y_j) \cdot f(s_0|y) \end{aligned}$$

From (A3) it follows that

$$\begin{aligned} f(s_j|y) &= \sum_{s_0 \in y_0} f(s_j|s_0, y_j) \cdot f(s_0|y_0) \\ &= f(s_j|y_0, y_j) \end{aligned}$$

which is what we had to show.

2. Let y be the investors' information or the information of the manager of firm j as in footnote 1. We must show that the signal beliefs are given by

$$f(y) = f(y_1|y_0) \cdot \dots \cdot f(y_J|y_0) \cdot f(y_0)$$

The signal beliefs can be written as

$$\begin{aligned} f(y) &= f(y_0, y_1, \dots, y_J) \\ &= f(y_0, y_2, \dots, y_J) \cdot f(y_1|y_0, y_2, \dots, y_J) \\ &= f(y_0, y_2, \dots, y_J) \cdot \sum_{s_0 \in y_0} \{ f(y_1|s_0, y_2, \dots, y_J) \cdot f(s_0|y_0) \} \\ &= f(y_0, y_2, \dots, y_J) \cdot \sum_{s_0 \in y_0} \sum_{s_1 \in y_1} \{ f(s_1|s_0, y_2, \dots, y_J) \cdot f(s_0|y_0) \} \end{aligned}$$

From (A1) we get that

$$\begin{aligned} f(y) &= f(y_0, y_2, \dots, y_J) \cdot \sum_{s_0 \in y_0} \sum_{s_1 \in y_1} \{ f(s_1|s_0) \cdot f(s_0|y_0) \} \\ &= f(y_0, y_2, \dots, y_J) \cdot \sum_{s_0 \in y_0} \{ f(y_1|s_0) \cdot f(s_0|y_0) \} \\ &= f(y_0, y_2, \dots, y_J) \cdot f(y_1|y_0) \end{aligned}$$

If this procedure is repeated for all firm-specific signals y_j we get the result we had to show.

3. It is assumed that firms are all-equity firms and that they are not able to trade in securities. This is a critical assumption, because borrowing and lending by firms may partly reveal the managers' private information (see Feltham and Christensen (1988) for a discussion of this).

4. The production sets are defined only in terms of the information received by managers, since only this information limits the production choices.
5. The "null" vector is the vector of dividends given no changes in the present production at $t=1$.
6. Berninghaus (1977) considers economy-wide and personal risks. However, whether the independent risks arise from risks about personal endowments or firm-specific risks makes no difference, since only the supply in the consumption market matters for the definition of feasible consumption/production plans. In our analysis of the implementation of efficient consumption plans in Chapter 3 it is a critical assumption that there are no personal risks about endowments. In that case, investors might not be able to implement those consumption plans through dynamic trading in well-diversified portfolios. Berninghaus extends a similar analysis by Malinvaud (1972), who only considered personal risks. Other papers that examine the impact of personal and firm-specific risks include Malinvaud (1973), Caspi (1974, 1978) and Ross (1976, 1977).
7. Given the result in Proposition 2.2, namely that efficient consumption plans are measurable with respect to economy-wide events, this Proposition is similar to Proposition 3 in Feltham and Christensen (1988). Prior analyses have identified the crucial role of aggregate consumption for efficient consumption plans and implicit state-contingent prices when beliefs are homogeneous and preferences are time additive. These analyses include Breeden and Litzenberger (1978), Amershi (1981, 1982, 1985, 1988), and Feltham (1985). With our definition of quasi-feasibility, the expected aggregate consumption conditional on the economy-wide event has the same role in our model as the aggregate consumption in those analyses.
8. Let $s_0 \in y_0$ and $s_j \in y_j^m$. We must then show that

$$f(s_j | s_0) / f(y_j^m | y_0) = f(s_j | s_0, y_j^m)$$

The ratio of conditional probabilities can be written as

$$\begin{aligned}
 f(s_j | s_o) / f(y_j^m | y_o) &= \{ f(s_j, s_o) \cdot f(y_o) \} / \{ f(s_o) \cdot f(y_j^m, y_o) \} \\
 &= \{ f(s_j, y_j^m, s_o) \cdot f(y_o) \} / \{ f(s_o, y_o) \cdot f(y_j^m, y_o) \} \\
 &= \{ f(s_j | s_o, y_j^m) \cdot f(y_j^m, s_o) \cdot f(y_o) \} \\
 &\quad / \{ f(s_o | y_o) \cdot f(y_o) \cdot f(y_j^m, y_o) \} \\
 &= \{ f(s_j | s_o, y_j^m) \cdot f(s_o, y_o, y_j^m) \} \\
 &\quad / \{ f(s_o | y_o) \cdot f(y_j^m, y_o) \} \\
 &= \{ f(s_j | s_o, y_j^m) \cdot f(s_o | y_o, y_j^m) \cdot f(y_o, y_j^m) \} \\
 &\quad / \{ f(s_o | y_o) \cdot f(y_j^m, y_o) \}
 \end{aligned}$$

From (A3) we have that

$$f(s_o | y_o, y_j^m) = f(s_o | y_o).$$

Substituting this into the equation above gives the result we had to show.

9. In similar models, Feltham (1985) and Feltham and Christensen (1988) show that a more informative information structure is only valuable if it results in a change in production plans. Time-additive preferences and homogeneous beliefs are critical assumptions for this result. If utility functions are not time-additive, then additional information may facilitate a more efficient intertemporal allocation of individual consumption. In the case of heterogeneous beliefs, additional information may provide additional opportunities for "betting" on the occurrence of events. See, for example, Ohlson and Buckman (1981), Hakansson, Kunkel, and Ohlson (1982), and Feltham (1985).

10. Note that there are constant stochastic returns to scale in this example. Hence, only the aggregate investment for the firms with the good signal can be determined, whereas it cannot be determined how this investment is allocated among those firms. We assume without loss of generality that all firms with the good signal invest the same amount.
11. Note that a more informative information structure is also more productive if the additional information is useful to allocate aggregate investments more efficiently, while a more productive information structure is not necessarily more informative. Hence, information structures that cannot be ranked according to informativeness might be ranked according to productiveness.
12. Feltham (1983) and Feltham and Christensen (1988) have similar results for the symmetric information case.
13. Feltham and Christensen (1988) do not consider these knife-edge cases. Also, their interpretation of the impact of economy-wide productivity information, namely that the aggregate amount invested in production is inversely related to productivity, is not valid in general.

CHAPTER 3.

Efficient Allocations as Equilibrium Allocations.

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In Chapter 2 we have identified some characteristics of Pareto efficient consumption/production plans for given information structures as well as given characteristics of valuable and non-valuable information. However, no consideration is given to the implementation of those plans. In particular, the distribution of initial endowments is ignored, and the resulting state-contingent plans are not required to be individually rational, given the economic agents' endowments and information at $t=0$ and at $t=1$.

We will now consider a decentralized and competitive market setting, where each economic agent acts solely in his self-interest given his current endowment and information. Investors take the market prices of firms as given and the managers take implicit state-contingent prices as given. Given this behavior, the solution concept is that of non-cooperative game theory. This means that a state-contingent plan can only be a market equilibrium, if it is a sequentially rational Nash-equilibrium. In section 3.1 we define sequential equilibria for a large economy, and in section 3.2 we give sufficient conditions for the implementation of an unconstrained Pareto efficient consumption plan for investors as a sequential equilibrium given the production plan.

In a market economy, exchange of consumption possibilities must be performed through trading in ownership claims to firms and financial claims in zero net-supply. Production decisions are determined by managers in accordance with their self-interest given their superior information about firm-specific events. However, owners of firms can influence the managers' preferences with respect to alternative production decisions through the design of the management compensation schemes offered to the managers.

In this chapter, we consider sequential equilibria for given compensation schemes to managers and given the production plan. In Chapter 4, we return to the design of compensation schemes that motivate the managers to implement efficient production plans in their self-interest.

3.1 Sequential equilibria.

Taking the market prices of firms as given, the investors' decision problem can be expressed as follows:

Investor i's decision problem:

$$\begin{aligned} &\text{maximize } U_i(c_i) \\ &\quad c_i, z_i \end{aligned}$$

subject to

$$\sum_k z_{iko} \cdot V_{ko} \leq \sum_k \hat{z}_{ik} \cdot V_{ko}$$

$$c_{i1}(y^I) + \sum_k z_{ik1}(y^I) \cdot v_{k1}(y^I) \leq \hat{c}_{i1} + \sum_k z_{iko} \cdot V_{k1}(y^I) \quad \text{all } y^I \in Y_0 \times Y^I$$

$$c_{i2}(s) \leq \hat{c}_{i2} + \sum_k z_{ik1}(y^I) \cdot v_{k2}(s) \quad \text{all } s \in y^I, y^I \in Y_0 \times Y^I$$

The investor faces a sequence of budget constraints. The first states that the market value of the portfolio he acquires at $t=0$ must not exceed the market value of his endowed portfolio. The second states that, given the signal y^I , his consumption plus the ex net-dividend market value of the portfolio he acquires at $t=1$ must not exceed his endowed consumption at that date plus the pre net-dividend market value of the portfolio he acquired at $t=0$. The third states that, given s , his consumption at $t=2$ cannot exceed his endowed consumption at that date plus the net-dividends paid by the portfolio he acquired at $t=1$.

In a market equilibrium investors have no incentive to deviate from the consumption/portfolio plan specified by the equilibrium given the market values of firms. Therefore, a necessary condition for a market equilibrium is that there are no arbitrage opportunities for investors at either $t=0$ or $t=1$. This implies that there exist (not necessarily unique) implicit non-negative prices

$$P_1 = \{ P_1(y^I) \}_{y^I \in Y_0 \times Y^I}$$

$$P_2(y^I) = \{ P_2(s|y^I) \}_{s \in y^I} \quad \text{for all } y^I \in Y_0 \times Y^I$$

such that

$$(3.1) \quad V_{j0} = \sum_{y^I \in Y_0 \times Y^I} V_{j1}(y^I) \cdot P_1(y^I)$$

and such that

$$(3.2) \quad v_{j1}(y^I) = \sum_{s \in Y^I} V_{j2}(s) \cdot P_2(s|y^I) \quad \text{for all } y^I \in Y_0 \times Y^I$$

The existence of such prices follows from using Farkas' lemma at $t=0$ and for each $y^I \in Y_0 \times Y^I$ at $t=1$ (see, e.g., Garman and Ohlson (1980)). Alternatively, we can express the market value of firm j as a linear function of its state-contingent net-dividends at $t=1$ and $t=2$, i.e.,

$$(3.3) \quad v_{j0} = \sum_{y^I \in Y_0 \times Y^I} \{ d_j(y^I) - M_{j1}(y^I) \} \cdot P_1(y^I) \\ + \sum_{s \in S} \{ x_j(s) - M_{j2}(s) \} \cdot P_2(s)$$

where

$$(3.4) \quad P_2(s) = P_1(y^I) \cdot P_2(s|y^I) \quad \text{for all } s \in y^I$$

As a matter of normalization we assume that the market values of the firms are priced relative to the cost at $t=0$ of getting one unit of the consumption good at $t=1$ with certainty, i.e.,

$$\sum_{y^I \in Y_0 \times Y^I} P_1(y^I) = 1$$

The market values of firms and the implicit state-contingent prices depend on the managers' compensation schemes and the production plan. Managers act in their self-interest like the investors do. Therefore, the managers will only choose a production plan which is in their self-interest, given the compensation schemes and their private information. That is, the production plan must be incentive compatible.

The production plan of firm j influences the dividends of firm j , and it influences the information about the firm-specific event that can be inferred from the dividends, c.f. the examples. There are two effects of this on the implicit state-contingent prices, and thereby, on the market values of firms. In incomplete markets, a change in the production plan of a single firm may change the set of consumption plans that can be achieved by the investors. This might change the structure of the implicit state-contingent

prices (see, e.g., Baron (1979)). The other effect is due to the fact that the investors' information structures for firm-specific events depend both on the information reported directly to them and on the information that can be inferred from the dividends given the production plan. When managers evaluate different production plans they must consider both of these effects on the implicit state-contingent prices.

Let $Y_0 \times Y^I$ be the public information structure for an equilibrium production plan, such that the dividends are measurable with respect to that information structure. We assume that the managers realize that no matter what production plan they are going to choose for their own firm, the market values of firms are going to be such that no arbitrage opportunities are available. This implies that there exist implicit state-contingent prices which satisfy (3.1) - (3.4) for the resulting public information structure. We also assume that the managers are in fact able to calculate such prices taking the production plans of other firms as given, i.e.,¹

$$\{ P_1(y^I(a_j)) \}_{y^I(a_j) \in Y_0 \times Y^I(a_j)}$$

$$\{ P_2(s|y^I(a_j)) \}_{s \in Y^I(a_j)} \text{ for all } y^I(a_j) \in Y_0 \times Y_1^I \times \dots \times Y_j^I(a_j) \times \dots \times Y_J^I$$

Of course, these prices are, in general, subjective prices for the manager of firm j . However, it will be shown in Chapter 4 how the prices can be calculated in efficient markets.

Given the portfolio acquired at $t=0$, z_{j0}^m , and given the information to the manager of firm j , y^{mj} , we can now formulate the manager's decision problem at $t=1$ as follows.

Manager j's decision problem at t=1:

$$\text{maximize}_{c_j^m, z_{j1}^m, a_j} u_{j1}^m(c_{j1}^m(y^{mj})) + \sum_{s \in S} u_{j2}^m(c_{j2}^m(s)) \cdot f(s|y^{mj})$$

subject to

$$c_{j1}^m(y^{mj}) + \sum_k z_{jk1}^m(y^{mj}) \cdot v_{k1}(y^I(a_j)) \leq \hat{c}_{j1}^m + M_{j1}(y^I(a_j), a) + \sum_k z_{jk0}^m \cdot v_{k1}(y^I(a_j))$$

$$c_{j2}^m(s) \leq \hat{c}_{j2}^m + M_{j2}(s, a) + \sum_k z_{jk1}^m(y^{mj}) \cdot v_{k2}(s) \quad \text{all } s \in y^{mj}$$

$$v_{k1}(y^I(a_j)) = d_k(y^I(a_j), a_k) - M_{k1}(y^I(a_j), a) + v_{k1}(y^I(a_j)) \quad \text{all } y^I(a_j) \subseteq y^{mj}$$

$$v_{k1}(y^I(a_j)) = \sum_{s \in S} \{ x_k(s, a_k) - M_{k2}(s, a) \} \cdot P_2(s|y^I(a_j)) \quad \text{all } y^I(a_j) \subseteq y^{mj}$$

$$a_j(y^{mj}) \in A_j \quad \text{and } (Y_0 \times Y_j^m) \text{ - measurable}$$

This formulation of the managers decision problem at t=1 recognizes that the production plan of firm j influences the dividends of firm j. The implicit state-contingent prices $\{ P_1(y^I(a_j)), P_2(s|y^I(a_j)) \}$ are used by the manager to evaluate these dividends in order to calculate his perception of the market values of firms for alternative production plans. These evaluations are made as if market values of firms are based on all the information that can be inferred from the dividends given the production plan being considered. The production plans of all firms may affect any manager's compensations at t=1 and t=2. However, each manager acts as if the production plans of all other firms are given. He then calculates how alternative production plans for his own firm influence the dividends of his firm and the compensation he gets.

An incentive compatible production plan and public information structure can now be defined as follows.

Definition 3.1: Incentive compatible production plans and public information structure Y^I given the information structure (Y_0, Y^m) and given the compensation functions.

Given the information structure (Y_0, Y^m) and given the compensation functions to managers, the production plan a and the information structure Y^I are incentive compatible, if for all j , $d_j(y^I(a_j), a_j)$ is $(Y_0 \times Y^I)$ -measurable, and (c_j^m, z_j^m, a_j) is a solution to the following decision problem for the manager of firm j at $t=0$:

$$\begin{aligned} & \text{maximize } U_j^m(c_j^m) \\ & c_j^m, z_j^m, a_j \end{aligned}$$

subject to

$$\sum_k z_{jko}^m \cdot v_{ko} \leq \sum_k \hat{z}_{jk}^m \cdot v_{ko}$$

$$\begin{aligned} c_{j1}^m(y^{mj}) + \sum_k z_{jk1}^m(y^{mj}) \cdot v_{k1}(y^I(a_j)) & \leq \hat{c}_{j1}^m + M_{j1}(y^I(a_j), a) \\ & + \sum_k z_{jko}^m \cdot v_{k1}(y^I(a_j)) \quad \text{all } y^{mj} \in Y_0 \times Y^{mj} \end{aligned}$$

$$\begin{aligned} c_{j2}^m(s) & \leq \hat{c}_{j2}^m + M_{j2}(s, a) + \sum_k z_{jk1}^m(y^{mj}) \cdot v_{k2}(s) \\ & \quad \text{all } s \in y^{mj}, y^{mj} \in Y_0 \times Y^{mj} \end{aligned}$$

$$\begin{aligned} v_{ko} & = \sum_{y^I(a_j) \in Y_0 \times Y^I(a)} \{ d_k(y^I(a_j), a_k) - M_{k1}(y^I(a_j), a) \} \cdot P_1(y^I(a_j)) \\ & \quad + \sum_{s \in S} \{ x_k(s, a_k) - M_{k2}(s, a) \} \cdot P_2(s) \end{aligned}$$

$$\begin{aligned} v_{k1}(y^I(a_j)) & = \sum_{s \in y^I(a_j)} \{ x_k(s, a_k) - M_{k2}(s, a) \} \cdot P_2(s | y^I(a_j)) \\ & \quad \text{for all } y^I(a_j) \in Y_0 \times Y^I(a) \end{aligned}$$

$\{ z_{j1}^m(y^{mj}), a_j(y^{mj}) \}$ is an optimal solution to the manager's decision problem at $t=1$, given z_{j0}^m , for all $y^{mj} \in Y_0 \times Y^{mj}$

The definition recognizes that the production plan and the resulting public information structure must be part of a sequential Nash-equilibrium such that, for all dates and for all signals, no single manager has incentives to deviate from the production plan of his own firm, given the production plans of other firms and his compensation scheme.

The last constraint in the manager's decision problem at $t=0$, states that a feasible production plan at $t=0$ must be such that the manager has no incentives to deviate from that plan at $t=1$ given his portfolio choice at $t=0$. Otherwise, the production plan will not be credible to the investors, and hence, not reflected in the market values of firms at $t=0$. The production plan, which maximizes the manager's expected utility at $t=0$, is partly determined by his endowed portfolio of firms. At $t=1$, the endowed portfolio is irrelevant to his production decision. Only the portfolio acquired at $t=0$ is of concern at $t=1$. This is the reason, why we have to impose the sequential rationality constraint in the manager's decision problem.

For the production plan to be compatible with the public information structure for firm-specific events, the dividend at $t=1$ is required to be measurable with respect to public information. Observe that this requirement is not formulated as a restriction in the managers' decision problem but only as a restriction on the dividend function for incentive compatible production plans. It reflects the fact that the public information structure is partly endogenously determined. This requirement was not made in the definition of efficient consumption/production plans. It is made here, because the consumption/production plans are now required to be individually rational, given the information available to each of the economic agents.

For given compensation schemes, the managers' decision problems are similar to that of investors with respect to the consumption/portfolio decisions. The differences are that managers get additional consumption possibilities from the management compensation at $t=1$ and $t=2$, and that managers' portfolio choices are measurable with respect to their private information, and not necessarily with respect to public information. That is, investors are not trying to draw inference about the managers' private information from the managers portfolio choices at $t=1$, either because these are not obser-

vable to the investors or because investors are "unsophisticated" in that respect.

The description of the market is in many respects similar to that described in Radner (1982), and the equilibrium concept for given information structures and compensation schemes defined below is also similar.

Definition 3.2: Sequential quasi-equilibria for given information structures (Y_O, Y^m) and given compensation schemes.

$(Y_O, Y^I, Y^m, c, z, a, M, V)$ constitute a sequential quasi-equilibrium given (Y_O, Y^m) and M if:

- (a) For each investor i , (c_i, z_i) solves the investor's decision problem given a , M and V .
- (b) For each manager j , (c_j^m, z_j^m) solves the manager's decision problem given a , M and V .
- (c) The production plan, a , and the public information structure, Y^I , are incentive compatible given the compensation schemes M and given the information structures (Y_O, Y^m) .
- (d) All markets clear, i.e.

$$\sum_i c_{i1}(y^I) + \sum_j \bar{c}_{j1}^m(y^I) = \sum_i \hat{c}_{i1} + \sum_j \hat{c}_{j1}^m + \sum_j \bar{d}_j(y_O, a_j) \text{ for all } y^I \in Y_O \times Y^I$$

$$\sum_i c_{i2}(s) + \sum_j \bar{c}_{j2}^m(s) = \sum_i \hat{c}_{i2} + \sum_j \hat{c}_{j2}^m + \sum_j \bar{x}_j(s_O, a_j) \text{ for all } s \in S$$

$$\sum_i z_{iko} + \sum_j z_{jko}^m = 1 \quad k = 1, \dots, J$$

$$\sum_i z_{ikl}(y^I) + \sum_j z_{jkl}^m(y^{mj}) = 1 \quad \forall y^I, y^{mj} \text{ and } k = 1, \dots, J$$

The definition of a sequential quasi-equilibrium states that all plans, consumption, portfolio and production plans, are individually rational, given the information structures and given the compensation schemes for the managers. The security market is required to clear at $t=0$ and to clear for each signal at $t=1$. In our large economy, the supply in the consumption market is taken to be the expected aggregate consumption possibilities conditional on the economy-wide signal at $t=1$ and conditional on the economy-wide event at $t=2$.

A main difference between our market and the market described by Radner (1982) is that in our model managers are individual economic agents who are consumers, investors in the capital market as well as agents responsible for making production decisions. In particular, the ultimate concern of managers in our model is their consumption plans, whereas in the Radner-model the managers' utility functions are defined as functions of the dividends of their respective firms. In our model, the managers' preferences over different production plans are created indirectly by offering them management compensation schemes.

A sequential quasi-equilibrium may not be an unconstrained Pareto efficient allocation, either because of an inefficient risk sharing or because managers do not select efficient production plans. We are interested in giving sufficient conditions for a sequential equilibrium being an unconstrained Pareto efficient allocation. That is, we will identify conditions on the structure of market values of firms and conditions on the compensation schemes, such that if investors and managers act individually rational, then the resulting consumption/production plan is (Y_0, Y^I, Y^m) -efficient. We consider the market values of firms in section 3.2 and the behavior of managers in Chapter 4.

3.2. Efficient sequential markets.

In this section we identify conditions on the structure of equilibrium market values of firms leading to an efficient risk sharing among investors. The compensation schemes and an incentive compatible production plan are taken as given. The following proposition identifies necessary conditions on the structure of equilibrium market values of firms.

Proposition 3.1.

If the sequential quasi-equilibrium $(Y_0, Y^I, Y^m, c, z, a, M, V)$ is (Y_0, Y^I, Y^m) -efficient given the production plan, then there exist non-negative prices, $P = \{P_1, P_2\}$, which satisfy (3.1) through (3.4) and the following two conditions

$$(3.5) \quad P_1(y^I) = P_1(\gamma_1) \cdot f(y_1^I, \dots, y_J^I | y_0) \cdot f(y_0 | \gamma_1) \quad \text{all } y^I \in Y_0 \times Y^I, y_0 \in \gamma_1$$

$$(3.6) \quad P_2(s) = P_2(\gamma_2) \cdot f(s_1, \dots, s_J | s_0) \cdot f(s_0 | \gamma_2) \quad \text{all } s \in S, s_0 \in \gamma_2$$

where $P = \{P_1(\gamma_1), P_2(\gamma_2)\}$ satisfies (2.1) through (2.4) for some positive parameters $\lambda_1, \dots, \lambda_N, \delta_1, \dots, \delta_J$.

Proof:

Let the production plan be given. Without loss of generality, we can consider the first order conditions for investor i 's decision problem. These conditions are

$$(a) \quad u'_{i1}(c_{i1}(y^I)) = P_{i1}(y^I)/f(y^I) \quad \text{for all } y^I \in Y_0 \times Y^I$$

$$(b) \quad u'_{i2}(c_{i2}(s)) = P_{i2}(s)/f(s) \quad \text{for all } s \in S$$

$$(c) \quad v_{k0} = \sum_{y^I \in Y_0 \times Y^I} \{ P_{i1}(y^I)/\lambda_i \} \cdot v_{k1}(y^I)$$

$$(d) \quad v_{k1}(y^I) = \sum_{s \in Y^I} \{ P_{i2}(s)/P_{i1}(y^I) \} \cdot v_{k2}(s) \quad \text{for all } y^I \in Y_0 \times Y^I$$

where λ_i , P_{i1} and P_{i2} are the multipliers for the three constraints respectively. Since the sequential equilibrium is (Y_0, Y^I, Y^M) -efficient we have from Proposition 3.3 that there exist prices P_t and a parameter λ_i such that

$$u'_{i1}(c_{i1}(y^I)) = \lambda_i \cdot P_1(\gamma_1)/f(\gamma_1) \text{ for all } y^I \in Y_0 \times Y^I, y_0 \subseteq \gamma_1$$

$$u'_{i2}(c_{i2}(s)) = \lambda_i \cdot P_2(\gamma_2)/f(\gamma_2) \text{ for all } s \in S, s_0 \in \gamma_2$$

Hence, if we define the state-contingent prices by

$$P_1(y^I) \equiv P_{i1}(y^I)/\lambda_i$$

$$P_2(s) \equiv P_{i2}(s)/\lambda_i$$

then those prices satisfy (3.1) through (3.6). **Q.E.D.**

Suppose the compensation schemes are of the form

$$(3.7) \quad M_{k1}(y^I) = M_{k1}(y_0, y_k^I) \text{ for all firms and } y^I \in Y_0 \times Y^I$$

$$(3.8) \quad M_{k2}(s) = M_{k2}(s_0, s_k) \text{ for all firms and } s \in S.$$

That is, given the production plan, the compensations to any single manager only depend on the economy-wide event and the firm-specific event for his own firm. Since the firm-specific events are conditionally independent the proposition implies that the market values of firms for an (Y_0, Y^I, Y^M) -efficient consumption plan can be written as

$$(3.9) \quad V_{k0} = \sum_{y_0 \in Y_0} E[d_k(y_0, y_k^I) - M_{k1}(y_0, y_k^I) | y_0] \cdot P_1(y_0) \\ + \sum_{s_0 \in S_0} E[x_k(s_0, s_k) - M_{k2}(s_0, s_k) | s_0] \cdot P_2(s_0)$$

$$\begin{aligned}
(3.10) \quad V_{k1}(y_0, y_k^I) &= d_k(y_0, y_k^I) - M_{k1}(y_0, y_k^I) \\
&\quad + \sum_{s_0 \in y_0} E[x_k(s_0, s_k) - M_{k2}(s_0, s_k) | s_0, y_k^I] \cdot P_2(s_0 | y_0) \\
&= d_k(y_0, y_k^I) - M_{k1}(y_0, y_k^I) \\
&\quad + v_{k1}(y_0, y_k^I)
\end{aligned}$$

$$(3.11) \quad V_{k2}(s_0, s_k) = x_k(s_0, s_k) - M_{k2}(s_0, s_k)$$

where

$$P_1(y_0) = P_1(\gamma_1) \cdot f(y_0 | \gamma_1)$$

$$P_2(s_0) = P_2(\gamma_2) \cdot f(s_0 | \gamma_2)$$

$$P_2(s_0 | y_0) = P_2(s_0) / P_1(y_0) \quad \text{for } s_0 \in y_0.$$

Hence, the market values of firms at $t=0$ are determined as the discounted sum of the expected net-dividends to investors conditional on the economy-wide signals and events. The discounting factors are the implicit state-contingent prices pertaining to the economy-wide signals and events. The market values at $t=1$ are the discounted sum of the expected net-dividends conditional on the economy-wide events and the public firm-specific signal. At $t=2$ the market values are simply the net-dividends at $t=2$.

To show that efficient risk sharing can be obtained as a sequential quasi-equilibrium we must propose a structure of equilibrium market values of firms and a set of compensation schemes, and then show that given these the investors choose to implement an efficient consumption plan in their self-interest. Therefore, let state-contingent prices satisfying (3.5) and (3.6) and the derived market values of firms determined by (3.9) through (3.11) be given. Let also compensation schemes satisfying (3.7) and (3.8) be given.

Not all efficient consumption plans can be obtained as sequential quasi-

equilibria. The investors (and managers) have the option not to trade in the capital market and then consume their consumption endowments and the dividends obtained from firms in which they have an endowed ownership. Hence, only those efficient consumption plans which give each investor at least the same expected utility as he would get without trading can be candidates for a sequential equilibrium, i.e., the core of the economy. Given the endowments, only those plans in the core of the economy for which prices exist, that clear all markets when the investors acts individually rational given those prices, can be achieved as an equilibrium. In general, redistributions of initial endowments are required in order to be able to obtain any efficient consumption plan as an equilibrium.

The market value of firm k at $t=1$ depends on the economy-wide signal y_0 and the public firm-specific signal for that firm y_k^I . The expected market value conditional on the economy-wide signal is given by

$$\bar{V}_{k1}(y_0) = \sum_{y_k^I \in Y_k^I} V_{k1}(y_0, y_k^I) \cdot f(y_k^I | y_0)$$

since the firm-specific signals of the other firms do not convey information about the firm-specific event of firm k . Similarly, at $t=2$ the market value of firm k only depends on the economy-wide event and the firm-specific event for that firm, and the expected market value of the firm conditional on the economy-wide event is given by

$$\bar{V}_{k2}(s_0) = \sum_{s_k \in S_k} V_{k2}(s_0, s_k) \cdot f(s_k | s_0)$$

Divide the firms into M portfolios (e.g., mutual funds). For each $m = 1, \dots, M$ let J_m denote the set of firms in portfolio m and let m_j be the number of firms in portfolio m , i.e. $m_j = |J_m|$. The strong law of large numbers for independent random variables then implies that for all $y_0 \in Y_0$:

$$\text{Prob} \left[\left\{ y^I \in Y_0 \times Y^I \mid \lim_{m_j \rightarrow \infty} \left| \frac{1}{m_j} \sum_{k \in J_m} (V_{k1}(y_0, y_k^I) - \bar{V}_{k1}(y_0)) \right| = 0 \right\} \mid y_0 \right] = 1$$

and for all $s_0 \in S_0$:

$$\text{Prob} [\{ s \in S \mid \lim_{m_j \rightarrow \infty} |(1/m_j) \cdot \sum_{k \in J_m} (V_{k2}(s_0, s_k) - V_{k2}(s_0))| = 0 \} \mid s_0] = 1$$

That is, if a portfolio is large, then, if signal y_0 is received at $t=1$, the average market value of the firms in the portfolio will, with probability one, be equal to their average expected value conditional on y_0 . A similar relationship holds for the market values at $t=2$ conditional on the economy-wide event s_0 . An investor's portfolio is said to be well-diversified, if he holds a small percentage of one or more large portfolios. The total expected market values of portfolio m at $t=0$, at $t=1$ and at $t=2$ are denoted

$$V_{m0} = \sum_{k \in J_m} V_{k0}$$

$$V_{m1}(y_0) = \sum_{k \in J_m} V_{k1}(y_0)$$

$$\bar{V}_{m1}(y_0) = \sum_{k \in J_m} \bar{V}_{k1}(y_0)$$

$$V_{m2}(s_0) = \sum_{k \in J_m} V_{k2}(s_0)$$

and z_{imt} denotes the fraction of the portfolio m the investor i acquires at time t .

Market values of firms at $t=1$ are measurable with respect to public information at $t=1$ and may therefore be over- or undervalued relative to the managers' superior information about firm-specific events. That is, the market value of a firm may be higher or lower than the intrinsic value of the firm given the manager's information as defined by

$$(3.12) \quad V_{k1}(y_0, y_k^m) = d_k(y_0, y_k^I) - M_{k1}(y_0, y_k^I) \\ + \sum_{s_0 \in y_0} E[x_k(s_0, s_k) - M_{k2}(s_0, s_k) \mid s_0, y_k^m] \cdot P_2(s_0 \mid y_0) \quad \text{for } y_k^m \subseteq y_k^I$$

$$= d_k(y_0, y_k^I) - M_{k1}(y_0, y_k^I) \\ + v_{k1}(y_0, y_k^m) \quad \text{for } y_k^m \subseteq y_k^I$$

However, as demonstrated by the following proposition, if an investor acquires a well-diversified portfolio, then the market value of that portfolio has the correct value relative to the managers' information.

Proposition 3.2.

Given equilibrium market values of firms in our large economy determined by (3.9) through (3.11), the market value of any well-diversified portfolio is equal to its intrinsic value, and it is measurable with respect to economy-wide information, i.e.,

$$(3.13) \quad z_{imo} \cdot \sum_{k \in J_m} V_{k1}(y_0, y_k^I) = z_{imo} \cdot \sum_{k \in J_m} V_{k1}(y_0, y_k^m) = z_{imo} \cdot \bar{V}_{m1}(y_0)$$

$$(3.14) \quad z_{im1}(y^I) \cdot \sum_{k \in J_m} v_{k1}(y_0, y_k^I) \\ = z_{im1}(y^I) \cdot \sum_{k \in J_m} v_{k1}(y_0, y_k^m) = z_{im1}(y^I) \cdot \bar{v}_{m1}(y_0)$$

Proof:

If an investor acquires a well-diversified portfolio, then he only holds a small percentage of a large portfolio. Therefore, let the fraction of the portfolio m the investor acquires at time t be proportional to $1/m_j$:

$$z_{imo} = z'_{imo}/m_j \quad z_{im1}(y^I) = z'_{im1}(y^I)/m_j.$$

Considering a sequence of economies in which m_j is increasing, the strong law of large numbers for independent random variables implies (3.13) and (3.14).

Q.E.D.

If investor i restricts his trading to well-diversified portfolios, then he is not only able to insure himself against firm-specific risk, but he is also able to insure himself against the deviations between market values and intrinsic values of individual firms due to limited public information about firm-specific events relative to the managers' information about those events. If he does this and, moreover, restricts his portfolio plan at $t=1$ to be measurable with respect to the economy-wide signal at that date, then his decision problem can be stated as

Investor i 's decision problem for well-diversified portfolios:

$$\begin{aligned} &\text{maximize } U_i(c_i) \\ &\quad c_i, z_i \end{aligned}$$

subject to

$$\sum_m z_{imo} \cdot V_{mo} \leq \sum_k \hat{z}_{ik} \cdot V_{ko}$$

$$c_{i1}(y_o) + \sum_m z_{im1}(y_o) \cdot \bar{v}_{m1}(y_o) \leq \hat{c}_{i1} + \sum_m z_{imo} \cdot v_{m1}(y_o) \quad \text{all } y_o \in Y_o$$

$$c_{i2}(s_o) \leq \hat{c}_{i2} + \sum_m z_{im1}(y_o) \cdot v_{m2}(s_o) \quad \text{all } s_o \in y_o, y_o \in Y_o$$

In the following two subsections we give conditions on the structure of equilibrium market values of firms, which ensure that the investors can restrict their portfolio plans to trading in well-diversified portfolios.

3.2.1 Dynamically quasi-complete markets.

In our large economy, the investors can obtain consumption plans which are measurable with respect to the economy-wide signals and events by only acquiring well-diversified portfolios at $t=0$ and at $t=1$. If the market values of firms are such that the investors can obtain any economy-wide state-contingent consumption plan by only acquiring well-diversified portfolios, then they do not want to be undiversified. This holds, because the market values of firms satisfying (3.9) through (3.11) are such that any variation in consumption plans due to firm-specific events is a fair gamble and no risk averse investor wants to take part in such a gamble. Therefore, the issue for efficient risk sharing in a sequential equilibrium is a question of sufficient flexibility within the set of available well-diversified portfolios. That flexibility is available if the given market prices provide a dynamically quasi-complete market as defined below.

Let

$$\mathbf{V}_1 \equiv [\mathbf{V}_{m1}(y_0)]_{|Y_0| \times M}$$

and

$$\mathbf{V}_2(y_0) \equiv [\mathbf{V}_{m2}(s_0)]_{|Y_0| \times M}$$

Definition 3.3: Dynamically quasi-complete market.

The equilibrium market values of firms provide a dynamically quasi-complete market if the following two conditions are satisfied:

- (a) There exist implicit state-contingent prices which satisfy conditions (3.5) and (3.6).
- (b) There exist large portfolios $J_1, \dots, J_M$ such that the matrix $\mathbf{V}_1$ has rank $|Y_0|$ and, for each $y_0 \in Y_0$, the matrix $\mathbf{V}_2(y_0)$ has rank $|y_0|$.

Proposition 3.3.

If $(Y_0, Y^I, Y^m, c, z, a, M, V)$ is a sequential quasi-equilibrium and the equilibrium market values of firms provide a dynamically quasi-complete market then the consumption plan for investors c^I is (Y_0, Y^I, Y^m) -efficient given the production plan.

Proof:

Consider an otherwise identical economy where in addition to ownership claims to firms it is possible to trade in a complete set of primitive securities contingent on public information at $t=1$ and contingent on the state at $t=2$. These primitive securities are in zero net-supply, i.e. financial securities. Let the prices of these securities be implicit state-contingent prices, $P_1(y^I)$ and $P_2(s)$, satisfying (3.5) and (3.6) and consistent with the equilibrium market values of firms V . In this economy all trading occurs at $t=0$ (see Rubinstein (1975)). We will first show that the equilibrium consumption plan in this economy is the (Y_0, Y^I, Y^m) -efficient consumption plan corresponding to the state-contingent prices $P_1(y^I)$ and $P_2(s)$. Secondly, it will be shown that the equilibrium consumption plan in the original economy is the same as in the extended economy.

First, we show that the equilibrium consumption plan in the extended economy is measurable with respect to economy-wide information at $t=1$ and with respect to the economy-wide event at $t=2$.

The wealth equivalent at $t=0$ of any equilibrium consumption plan in the extended economy is given by

$$\begin{aligned} W_{i0}(c) = & \sum_{y_0 \in Y_0} E[c_{i1}(y^I) \mid y_0] \cdot P_1(y_0) \\ & + \sum_{s_0 \in S_0} E[c_{i2}(s) \mid s_0] \cdot P_2(s_0) \end{aligned}$$

This implies that any variation in consumption plans due to firm-specific events is a fair gamble. Since any consumption plan with the same wealth

equivalent can be achieved in the extended economy and since the investors are risk averse, the equilibrium consumption plans must be measurable with respect to the economy-wide information and events.

From the first order conditions for optimal holdings of primitive securities in the investors' decision problem it follows that (see the proof of Proposition 3.1)

$$P_1(y^I) = u'_{i1}(c_{i1}(y^I)) \cdot f(y^I) / \lambda_i$$

and

$$P_2(s) = u'_{i2}(c_{i2}(s)) \cdot f(s) / \lambda_i$$

where λ_i is the multiplier for the $t=0$ budget constraint. Since the equilibrium consumption plans do not vary with firm-specific events, this implies that

$$u'_{i1}(c_{i1}(y_0)) = \lambda_i \cdot P_1(y^I) / f(y^I) = \lambda_i \cdot P_1(y_0) / f(y_0)$$

and

$$u'_{i2}(c_{i2}(s_0)) = \lambda_i \cdot P_2(s) / f(s) = \lambda_i \cdot P_2(s_0) / f(s_0)$$

Since the consumption plans are equilibrium plans, all markets clear, and these two conditions are necessary and sufficient conditions for efficient risk sharing among investors (see the proof of Proposition 2.3). This establishes that the investors' equilibrium consumption plan in the economy with a complete set of primitive securities is the (Y_0, Y^I, Y^m) -efficient consumption plan corresponding to the state-contingent prices $P_1(y^I)$ and $P_2(s)$.

The next step is to show that the equilibrium consumption plan in the original economy is the same as in the economy with a complete set of primitive securities.

Let $c_i^* = (c_{i1}^*(y_0), c_{i2}^*(s_0))$ be investor i 's equilibrium consumption plan in the extended economy. One way to implement this consumption plan is to sell his endowed portfolio of ownership claims at $t=0$ and buy a portfolio of primitive securities with the following pay-offs at $t=1$ and at $t=2$, respectively:

$$\delta_{i1}(y_0) = c_{i1}^*(y_0) - \hat{c}_{i1}$$

$$\delta_{i2}(s_0) = c_{i2}^*(s_0) - \hat{c}_{i2}$$

The price at $t=0$ of that portfolio is

$$w_{i0} = \sum_{y_0 \in Y_0} \delta_{i1}(y_0) \cdot P_1(y_0) + \sum_{s_0 \in S_0} \delta_{i2}(s_0) \cdot P_2(s_0) = \sum_k \hat{z}_{ik} \cdot v_{k0},$$

and its market value at $t=1$ is

$$w_{i1}(y_0) = \delta_{i1}(y_0) + \sum_{s_0 \in y_0} \delta_{i2}(s_0) \cdot P_2(s_0|y_0).$$

In the original economy it is possible to acquire a well-diversified portfolio of ownership claims at $t=1$, $\{z_{im1}(y_0)\}_{m=1, \dots, M}$, with a pay-off vector $\{\delta_{i2}(s_0)\}_{s_0 \in y_0}$ at $t=2$, since the matrix $\mathbf{V}_2(y_0)$ has rank $|y_0|$ for every y_0 . The given equilibrium market value at $t=1$ of that portfolio must be

$$\sum_m z_{im1}(y_0) \cdot \bar{v}_{m1}(y_0) = \sum_{s_0 \in y_0} \delta_{i2}(s_0) \cdot P_2(s_0|y_0)$$

since, otherwise, arbitrage opportunities would exist at $t=1$ in the extended economy.

Similarly, since the matrix $\mathbf{V}_1$ has rank $|Y_0|$ it is possible to acquire a well-diversified portfolio at $t=0$, $\{z_{imo}\}_{m=1, \dots, M}$, with a pay-off vector $\{w_{i1}(y_0)\}_{y_0 \in Y_0}$ at $t=1$, and to avoid arbitrage opportunities in the extended economy, the market value of that portfolio at $t=0$ must be w_{i0} .

This establishes that in the original economy it is possible for any investor to implement his equilibrium consumption plan in the extended economy through trading in well-diversified portfolios. Since all feasible consumption plans in the original economy are also feasible in the extended economy, the equilibrium consumption plans in the original economy must be the equilibrium consumption plans in the extended economy, and this proves the proposition. Q.E.D.

The key characteristic of a dynamically quasi-complete market is that investors can insure themselves against firm-specific risk without paying a premium (i.e., condition (a)) by restricting their portfolio plan to well-diversified portfolios. At the same time, they still have sufficient flexibility through dynamic trading to implement any financially feasible consumption plan contingent on economy-wide information (i.e. condition (b)).² Hence, diversification and sequentially trading are complementary methods for achieving an unconstrained Pareto efficient risk sharing among investors in a seemingly incomplete and large market economy.

3.2.2 Dynamically investment wealth quasi-complete markets.

When the market values of firms are such that the implicit state-contingent prices satisfy (3.5) and (3.6) then the desired consumption plans are only those which are measurable with respect to the levels of expected aggregate consumption at $t=1$ and at $t=2$, i.e. measurable with respect to $(\Gamma_1(a), \Gamma_2(a))$. This follows from the fact that any variation in consumption plans due to events not influencing γ_t is a fair gamble. Therefore, the set of large portfolios do not have to span the hole space of plans contingent on y_0 at $t=1$ and contingent on s_0 at $t=2$.

As an example consider the case where some economy-wide signals at $t=1$ only determine which firms get the good economy-wide windfall event and which firms get the bad economy-wide event within a group of firms. Investors do not want to gamble on this. Hence, if these firms are aggregated into a single portfolio, the pay-off of the portfolio is independent of the particular signal, and investors can avoid taking that risk. If this aggregation of firms into a portfolio does not limit the space of desired consumption plans obtainable through dynamic trading, then investors will restrict their portfolio plan such that these firms only enter the portfolio plan as a portfolio. It is important, however, that the aggregation does not restrict the obtainable space of desired consumption plans. Clearly, all risk not related to the level of expected aggregate consumption can be avoided by restricting the portfolio plan to, e.g., a riskless firm and the market portfolio. However, a riskless firm and the market portfolio does not in general span (through dynamic trading) the space of desired consumption plans. This occurs only when the probability distributions or the utility functions are such that two fund separation is obtained (see, e.g., Cass and Stiglitz (1970)).

Without making further assumptions about preferences and probability distributions we refine the concept of dynamically quasi-complete markets by recognizing that only the space of desired consumption plans needs to be spanned by sequentially trading in well-diversified portfolios.

Definition 3.4: Minimal investment wealth statistics.

The partitions Ψ_1 of Y_0 and Ψ_2 of S_0 are minimal investment wealth statistics with respect to Y_0 and S_0 , if

- (a) $\Psi_1 = \{ \psi_1 \}$ is the coarsest partition of Y_0 such that γ_1 and $f(\gamma_2|y_0)$ are Ψ_1 -measurable.
- (b) $\Psi_2 = \{ \psi_2 \}$ is the coarsest partition of S_0 such that it is at least as fine as Y_0 and such that γ_2 is Ψ_2 -measurable. Let

$$\Psi_2(y_0) = \{ \psi_2(y_0) \} \equiv \{ \psi_2 \in \Psi_2 \mid \psi_2 \subseteq y_0 \}$$

Definition 3.5: Dynamically investment wealth quasi-complete market.

The equilibrium market values of firms provide a dynamically investment wealth quasi-complete market if the following three conditions are satisfied:

- (c) There exist implicit state-contingent prices which satisfy conditions (3.5) and (3.6).
- (d) There exists a partition of firms into large portfolios $J_1, \dots, J_M$ such that

$$DW_1 \subseteq \text{span}\{ \nabla_{11}, \dots, \nabla_{M1} \}$$

$$DW_1 \equiv \{ w_1 \in R^{|Y_0|} \mid w_1(y_0) = w_1(y'_0) \}$$

for all $y_0, y'_0 \subseteq \psi_1$, and for all $\psi_1 \in \Psi_1$ }

- (e) For all $y_0 \in Y_0$ there exists a partition of firms into large portfolios $J_1(y_0), \dots, J_{M(y_0)}(y_0)$ such that

$$DW_2(y_0) \subseteq \text{span}\{ \nabla_{12}(y_0), \dots, \nabla_{M(y_0)2}(y_0) \}$$

$$DW_2(y_0) \equiv \{ w_2 \in R^{|Y_0|} \mid w_2(s_0) = w_2(s'_0) \text{ all } s_0, s'_0 \in \psi_2(y_0),$$

$\psi_2(y_0) \in \Psi_2(y_0) \}$

Proposition 3.4.

If $(Y_0, Y^I, Y^m, c, z, a, M, V)$ is a sequential quasi-equilibrium and the equilibrium market values of firms provide a dynamically investment wealth quasi-complete market, then the consumption plan for investors c^I is (Y_0, Y^I, Y^m) -efficient given the production plan.

Proof:

The proof is similar to the proof of Proposition 3.3. First, in an economy with a complete set of primitive securities the investors' equilibrium consumption plans are measurable with respect to the level of expected aggregate consumption at $t=1$ and at $t=2$, since any variation in these is a fair gamble given that the implicit state-contingent prices satisfy (3.5) and (3.6). Again, it follows from the first order conditions for the investors' decision problems that the equilibrium consumption plans are (Y_0, Y^I, Y^m) -efficient given the production plan.

The final step is to show that the equilibrium consumption plans in the extended economy can be obtained through dynamic trading in the original economy if the market values of firms provide a dynamically investment wealth quasi-complete market.

Let again

$$\delta_{i1}(y_0) = c_{i1}^*(y_0) - \hat{c}_{i1}$$

$$\delta_{i2}(s_0) = c_{i2}^*(s_0) - \hat{c}_{i2}$$

denote the equilibrium net consumption requirements from the portfolio plan in the extended economy. Since the equilibrium consumption plan in the extended economy is measurable with respect to Γ_t , the net consumption requirements are also measurable with respect to Γ_t , i.e.,

$$\delta_{i1}(\gamma_1) = c_{i1}^*(\gamma_1) - \hat{c}_{i1}$$

$$\delta_{i2}(\gamma_2) = c_{i2}^*(\gamma_2) - \hat{c}_{i2}$$

The market value at $t=0$ of a portfolio with these pay-offs are

$$w_{i0} = \sum_{\gamma_1 \in \Gamma_1} \delta_{i1}(\gamma_1) \cdot P_1(\gamma_1) + \sum_{\gamma_2 \in \Gamma_2} \delta_{i2}(\gamma_2) \cdot P_2(\gamma_2) = \sum_k \hat{z}_{i0} \cdot V_{k0},$$

and its market value at $t=1$ contingent on the information y_0 is

$$w_{i1}(y_0) = \delta_{i1}(\gamma_1) + \sum_{\gamma_2 \in \Gamma_2} \delta_{i2}(\gamma_2) \cdot P_2(\gamma_2 | y_0) \quad \text{for } y_0 \subseteq \gamma_1.$$

where

$$\begin{aligned} P_2(\gamma_2 | y_0) &= \sum_{s_0 \in \gamma_2 \cap y_0} P_2(s_0 | y_0) \\ &= \sum_{s_0 \in \gamma_2 \cap y_0} P_2(s_0) / P_1(y_0) \\ &= \{ \sum_{s_0 \in \gamma_2 \cap y_0} P_2(\gamma_2) \cdot f(s_0 | \gamma_2) \} / P_1(y_0) \\ &= \{ P_2(\gamma_2) \cdot f(y_0 | \gamma_2) \} / \{ P_1(\gamma_1) \cdot f(y_0 | \gamma_1) \} \\ &= \{ P_2(\gamma_2) \cdot f(\gamma_1) \cdot f(\gamma_2 | y_0) \} / \{ P_1(\gamma_1) \cdot f(\gamma_2) \} \end{aligned}$$

This implies that, for all investors, the equilibrium investment wealth in the extended economy at $t=1$, $w_{i1}(y_0)$, is measurable with respect to the minimal investment wealth statistic at $t=1$, i.e., Ψ_1 -measurable. This follows from the fact that the level of expected aggregate consumption γ_1 and the probability distribution of the level of future expected aggregate consumption is measurable with respect to that statistic. That is,

$$\{ w_{i1}(y_0) \}_{y_0 \in Y_0} \in DW_1 \quad \text{for all investors } i.$$

Similarly, for each $y_0 \in Y_0$ the equilibrium investment wealth at $t=2$, $w_{i2}(s_0) = \delta_{i2}(\gamma_2)$, is $\Psi_2(y_0)$ -measurable, and this implies that

$$\{ w_{i2}(s_0) \}_{s_0 \in y_0} \in DW_2(y_0)$$

The definition of a dynamically investment wealth quasi-complete market is precisely such that any of these equilibrium investment wealth plans in the extended economy can be implemented in the original economy through sequentially trading in well-diversified portfolios, and this proves the proposition. Q.E.D.

The key characteristics of a dynamically investment wealth quasi-complete market are that investors can insure themselves against diversifiable risk without paying a premium by restricting their portfolio plan to well-diversified portfolios and still have enough flexibility through sequentially trading to implement any desirable and financially feasible consumption plan as opposed to any financially feasible consumption plan in the definition of a dynamically quasi-complete market. Hence, the market may not be dynamically quasi-complete, but still, an efficient risk sharing can be obtained if the market is dynamically investment wealth quasi-complete.

The desirable consumption plans are only those which are measurable with respect to the level of expected aggregate consumption at $t=1$ and at $t=2$. Therefore, for any economy-wide signal, $y_0 \in Y_0$, received at $t=1$ it must be possible to choose a portfolio of well-diversified portfolios with any payoff at $t=2$ which is measurable with respect to the level of expected aggregate consumption at that date. This is ensured by the conditions (b) and (e). Observe, that the composition and the number of these portfolios may depend on the economy-wide signal received at $t=1$.

The portfolio problem at $t=0$ is somewhat more complicated, since the value of the selected portfolio, i.e., the investment wealth, at $t=1$ must fulfil two purposes. Some of the investment wealth is used for consumption at $t=1$, and the remaining investment wealth is used to invest in a new portfolio to generate additional consumption possibilities at $t=2$. Hence, even if the level of expected aggregate consumption at $t=1$ (i.e., γ_1) is the same for two different economy-wide signals at $t=1$, the desired investment wealth may be different, since the desired amount for new investments may be different for the two signals. Therefore, it may not be enough that the well-diversified portfolios span the linear subspace of investment wealths mea-

surable with respect to the level of expected aggregate consumption at $t=1$. In other words, the investors' portfolio choice at $t=0$ is not only determined by their desired consumption at $t=1$, but also by a demand for insurance against the future consumption prospects.

If the level of current expected aggregate consumption as well as the conditional probability distribution of future expected aggregate consumption are the same for two economy-wide signals at $t=1$, then the desired current consumption as well as the desired amount for new investments are also the same for the two signals. This is true, because, in terms of individual and aggregate consumption possibilities, the future looks exactly the same for the two signals. That is, the conditional probability distribution of individual consumption at $t=2$ and the set of relevant implicit state-contingent prices are the same for the two signals. Therefore, the space of desired investment wealths at $t=1$ is the linear subspace of investment wealths measurable with respect to the minimal investment wealth statistic Ψ_1 , and well-diversified portfolios exist at $t=0$ which span that linear subspace (condition (d)).

The definition of a dynamically quasi-complete market is expressed in terms of rank conditions, whereas the definition of a dynamically investment wealth quasi-complete market is expressed in terms of spanning conditions. If the matrices of market values on well-diversified portfolios have full row-rank, as in the definition of dynamically quasi-complete markets, then the rank conditions are equivalent to spanning conditions. However, in the definition of dynamically investment wealth quasi-complete markets those price matrices do not necessarily have full row-rank, and then, rank conditions are not equivalent to spanning conditions. Here we have to ensure that the linear dependencies are such that the portfolios span the desired linear subspace.³

3.3 Conclusion.

Sequential equilibria for our large economy has been defined in section 3.1 and sufficient conditions for an equilibrium consumption plan for investors being Pareto efficient has been given in section 3.2.

A sequential quasi-equilibrium is defined by the market clearing conditions for the security market and the consumption market, and by all plans, consumption, portfolio and production plans, being individually and sequentially rational.

The investors' decision problem is that of choosing a consumption plan and a portfolio plan which implements the consumption plan. In doing that, they take the market values of firms and the net dividends as given. The investors' decision problem is a sequential decision problem. Before the release of information, they choose an optimal portfolio, given the endowments. After the release of public information, they make a consumption decision and select a new portfolio which generates future consumption possibilities.

The managers' decision problem is that of choosing a consumption plan, a portfolio plan, and a production plan for their own firms given their compensation schemes. Given the production plan, the managers' decision problem for consumption and portfolio plans is similar to that of investors, except that these plans are only measurable with respect to their private information. In choosing the production plan, the managers realize that both the dividends and the firm-specific information, which can be inferred from the dividends, depend on the production plan. It is assumed that, whatever production plan they are going to choose for their own firms, the market values of firms are going to be such that no arbitrage opportunities are available. This implies, that there exist implicit state-contingent prices which can be used to calculate the managers' perceptions of the market values of firms for alternative production plans. On this basis, incentive compatible production plans and public information structures were defined.

Sufficient conditions on the structure of equilibrium market values of

firms leading to an efficient risk sharing among investors are given in section 3.2.

First, a necessary condition is given in Proposition 3.1. This condition is that the equilibrium market values of firms are such that there exist implicit state-contingent prices which are equal to implicit prices pertaining to the level of conditional expected aggregate consumption times the probability of the signal or the event conditional on that level. It is assumed that the compensation schemes are such that the compensations to any manager depend only on the economy-wide event and on the firm-specific event of his own firm. Given these conditions, it is shown in Proposition 3.2 that the market value of any well-diversified portfolio is equal to its intrinsic value, and that it is measurable with respect to the economy-wide information. This allows investors to restrict their portfolio plans to trading in well-diversified portfolios if there exist well-diversified portfolios with enough flexibility to implement an efficient consumption plan. This is the issue in sections 3.2.1 and 3.2.2.

In section 3.2.1 a dynamically quasi-complete market is defined as a market in which there exist implicit state-contingent prices that satisfy the conditions in Proposition 3.1, and in which any financially feasible consumption plan measurable with respect to the economy-wide information can be implemented through dynamic trading in well-diversified portfolios. This concept is refined in section 3.2.2 to a dynamically investment wealth quasi-complete market in which any financially feasible and desired consumption plan can be implemented through dynamic trading in well-diversified portfolios. This latter market only requires that there exist well-diversified portfolios with pay-off vectors which span the linear subspace determined by the level of current conditional expected aggregate consumption and the probability distribution of the future level of conditional expected aggregate consumption.

In both types of markets, additional information about firm-specific events plays no part in the implementation of efficient consumption plans. However, the role of additional information about the economy-wide events are different in the two markets. In a dynamically quasi-complete market with a limited set of well-diversified portfolios, the number of economy-wide

signals at $t=1$ must be large enough in order to have a complete market at $t=1$. However, the number of economy-wide signals must not be too large, such that the market at $t=0$ becomes incomplete. In a dynamically investment wealth quasi-complete market the number of economy-wide signals at $t=1$ must also be large enough in order to have a complete market at $t=1$. The difference is that additional economy-wide signals at $t=1$ do not destroy the completeness of the market at $t=0$ in a dynamically investment wealth quasi-complete market, if the pay-off vectors of the well-diversified portfolios which span the desired linear subspace of investment wealths, remain unchanged. This will be the case for economy-wide signals that merely tell which firms will realize the gains and which firms will realize the losses within given well-diversified portfolios.

Footnotes for Chapter 3.

1. In general, the managers have monopolistic power in determining the implicit state-contingent prices, both because they might be able to change the set of obtainable consumption plans and because the public information about firm-specific events depends on the production plan. That is, the managers are required to predict the equilibrium prices for alternative production plans. However, note that if managers are not allowed to trade in the shares of firms, then they only have to predict these prices in so far as those prices affect their management compensations. Hence, if the management compensation schemes depend on the dividends only, then they do not have to make any predictions about equilibrium prices in order to evaluate alternative production plans.
2. Ohlson and Buckman (1981), Feltham (1985), Duffie and Huang (1985), Duffie (1986) and Ohlson (1987) have shown similar results for dynamic trading. However, only Feltham (1983) and Feltham and Christensen (1988) consider dynamic trading and the possibility of diversification. Our Proposition 3.3 is identical to Proposition 8 in Feltham and Christensen (1988).
3. Feltham (1985) and Feltham and Christensen (1988) consider refinements of the dynamic complete market requirement by excluding information and events that have no impact on current and future market prices of firms (or well-diversified portfolios). Feltham and Christensen (1988) shows that if there exist well-diversified portfolios with a matrix of pre-dividend market values which has rank equal to the number of price relevant signals in the next period then the equilibrium consumption plan is generically efficient. That is, "knife-edge" cases exist where this is not the case. This is due to the fact that even if two economy-wide signals at $t=1$ are irrelevant for the pre-dividend market values of well-diversified portfolios, they might be relevant for the aggregate dividend and, thus, for the level of conditional expected aggregate consumption at that date. Our definition of a dynamically investment quasi-complete market avoids these cases, since it is defined explicitly in terms of the level of conditional expected aggregate

consumption. Moreover, the required number of well-diversified portfolios are, in general, lower, because, even if two economy-wide signals are relevant for the market values of well-diversified portfolios, they may not be relevant for the level of conditional expected aggregate consumption.

CHAPTER 4.

Efficient Behavior of Managers.

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In the previous chapter we have identified conditions on the structure of market values of firms such that, given those conditions, the equilibrium consumption plans of investors are (Y_0, Y^I, Y^M) -efficient given the production plan. In this chapter we impose additional conditions on the behavior of managers which ensure that the equilibrium consumption as well as the equilibrium production plan are (Y_0, Y^I, Y^M) -efficient. Two issues have to be addressed. First, we must ensure that the managers choose to implement their part of an efficient risk sharing, and secondly, we must ensure that managers select efficient production plans.

4.1 Insider trading.

It follows from Proposition 2.3 that an efficient consumption plan for investors and managers has the characteristic that both the investors' and the managers' consumption plans are measurable with respect to the level of expected aggregate consumption. Hence, efficient consumption plans of managers do not vary with their private information about the firm-specific events.

Market values of firms only depend on public information. Hence, managers have private information about the intrinsic values of their respective firms. If the managers act individually rational at the time they get their private information, then they will use this information to obtain abnormal gains from speculative positions in their firms (see, e.g., Hirshleifer (1971)). The speculative positions depend on the managers' risk aversion and on the difference between the market values and the intrinsic values of the firms given some particular private signals. Therefore, without restrictions on the managers' portfolio plans their equilibrium consumption plans may vary with their private information about firm-specific events and, hence, it may be impossible to obtain an efficient risk sharing as an equilibrium where each economic agent acts individually rational.

Managers taking speculative positions in their respective firms may not only influence the sharing of risks. It may also influence the production choices. The managers choose the production plan that maximizes their own expected utilities and, hence, the production choices are influenced by the

compensation schemes as well as by the return managers can obtain from investing in firms. Therefore, if we do not impose restrictions on the managers' portfolio plans, then the production plan may partly be determined so as to create large differences between the market values and the intrinsic values of firms, such that large abnormal gains can be obtained from speculative positions based on private information.

To obtain efficient plans as equilibrium plans we impose the following institutional restriction on insider trading:

- (A7) At $t=1$ managers are not allowed to take speculative positions in the shares of their own firms. That is, managers are only allowed to trade in the shares of their own firms to the extent that they are part of a well-diversified portfolio.¹

Managers may have superior information about their own firms at $t=1$ but they do not have superior information about the true values of well-diversified portfolios (Proposition 3.2).² Hence, managers are not able to obtain abnormal gains based on their private information, when they are allowed to trade only in well-diversified portfolios. Provided that a sufficiently varied set of well-diversified portfolios exist, the restriction on insider trading still allows an efficient risk sharing, since this only requires trading in well-diversified portfolios (Proposition 3.3 and Proposition 3.4).

The institutional restriction on insider trading does not represent a market failure in the sense that it requires a governmental intervention in the market. In fact, at the time of contracting between investors and managers, i.e. at $t=0$, it is in the investors' and the managers' mutual interest that the managers precommit themselves to that rule. To see this, suppose the market for managers are such that a given manager requires a minimum expected utility to accept employment in a firm (i.e., he requires his reservation utility). There are two ways in which the manager can obtain that expected utility, one without the restriction on insider trading and one with that restriction. Without the restriction on insider trading,

the investors realize that the manager through his operation of the firm obtains superior information about the firm-specific events of the firm and that this enables him to obtain abnormal gains from insider trading. Hence, the investors will offer him a compensation scheme such that he can only obtain his reservation utility if he uses his private information optimally to obtain the gains from insider trading. However, the manager must take some firm-specific risk to obtain these gains, and thus, he requires a risk premium to take that risk. Investors can diversify all firm-specific risk in the capital market, and consequently, it is not optimal for them to pay the manager a risk premium for taking such risk. Therefore, it is cheaper for the investors to provide the manager with his reservation utility if the institutional restriction on insider trading is imposed. The manager is indifferent, because he only obtains his reservation utility in both cases. Of course, to enforce the institutional restriction on insider trading, the managers' positions in their own firms must be observable to the investors.

4.2 Management compensation.

We do not want managers to speculate on the basis of their private information, but we want managers to use their private information in the production decisions so as to allocate resources more efficiently among firms. This last objective can be achieved by offering the managers suitable compensation functions.

As the model has been formulated up until now, the economy-wide event as well as the firm-specific events are revealed to both managers and investors at $t=2$. The revelation of the events, the observed cash flows at $t=2$, and the investors' knowledge of the production functions enable the investors to infer the production decisions implemented by the managers at $t=1$. Moreover, the investors' knowledge of the managers' private information structures and the revealed firm-specific events enable them at $t=2$ to infer the private information received by the managers at $t=1$. This means that the investors can enforce any production decision rule contingent on both public and private information by offering the managers compensation schemes which at $t=2$ impose high penalties on the managers if they have not followed these rules. The penalties are so high that all utility maximizing

managers are deterred from not following the prescribed production decision rules. To avoid this rather uninteresting solution we make a small change in the assumptions of the model. We now assume that the economy-wide event and the cash flows is the only new information received by the investors at $t=2$. In that case, investors have only incomplete information about the production decisions implemented and the private information received by the managers at $t=1$. A nontrivial incentive problem is now present in the model. The results of the prior analysis are unaffected, since efficient consumption plans do not depend on firm-specific events.

Only compensation schemes contingent on public information are enforceable. Therefore, the feasible compensation schemes can be written as

$$M_{j1} = M_{j1}(y^I, d) \quad \text{for all firms } j \text{ and } y^I \in Y_0 \times Y^I$$

$$M_{j2} = M_{j2}(y^I, s_0, d, x) \quad \text{for all firms } j \text{ and } s_0 \in y_0, y^I \in Y_0 \times Y^I$$

where d and x are the vectors of dividends for all firms at $t=1$ and $t=2$, respectively. The dividends depend on the production plans but only the public information about the events and the observed dividends can be contracted upon.

We have to show that there exist a particular form of these compensation schemes such that the equilibrium consumption/production plan is an (Y_0, Y^I, Y^m) -efficient consumption/production plan. First, the compensations to the managers must be measurable with respect to the economy-wide events, since efficient consumption plans are measurable with respect to those events, and since managers are not able to undo firm-specific risk through trading in the capital market given the restriction on insider trading. Secondly, managers are responsible for production decisions as well as being investors in the capital market, although they are not allowed to speculate on the basis of their private information. Therefore, the compensation schemes must be such that the managers' production incentives both as receivers of management compensation and as investors in the capital market are such that they choose to implement an efficient production plan in their self-interest. Since the managers are investing in the capital market, we must specify how managers are making predictions of the return

they can get from investments for alternative production plans both because the dividends and the information that can be inferred from the dividends depend on the production plan.

Definition 4.1: Price taking behavior by managers.

Let the equilibrium market values of firms provide a dynamically investment wealth quasi-complete market. The managers are price takers if they act as if

- (a) the implicit state-contingent prices pertaining to the economy-wide information and events, $\{ P_1(y_0), P_2(s_0) \}$, are independent of the production plan of their own firms.
- (b) the market value of well-diversified portfolios are equal to the intrinsic value of those portfolios for any production plan.

The first condition corresponds to the usual price taking assumption in standard analyses (see, e.g., Baron (1979)). It implies that the manager of a firm is not able to change the market values of other firms by varying the production plan of his own firm. The second condition has its basis in Proposition 3.2. If managers act according to this condition, and if they are only allowed to trade in well-diversified portfolios, then they are not concerned about the information that can be inferred from the dividends for alternative production plans. The two conditions taken together imply that the managers act as if the change in the market value of a large portfolio for a given change in the production plan for any single firm is equal to the change in the intrinsic value of that firm alone taking the state-contingent prices pertaining to the economy-wide signals and events as given.

The following proposition identifies compensation schemes which ensure that the risk sharing between both investors and managers is efficient and that the managers implement efficient production plans in their self-interest, given they are price takers according to Definition 4.1.

Proposition 4.1.

Let $(Y_0, Y^I, Y^m, c, z, a, M, V)$ be a sequential quasi-equilibrium. If the managers are price takers and the following four conditions are satisfied then the equilibrium consumption/production plan is an (Y_0, Y^I, Y^m) -efficient consumption/production plan:

- (a) The equilibrium market values of firms provide a dynamically investment wealth quasi-complete market.
- (b) The compensation schemes are of the form

$$M_{j1}(\psi_1, \bar{d}) = \tau_{j1}(\psi_1) + \pi_j \cdot \bar{d}, \quad \pi_j > 0, \quad j = 1, \dots, J, \quad \psi_1 \in \Psi_1$$

$$M_{j2}(\psi_2, \bar{x}) = \tau_{j2}(\psi_2) + \pi_j \cdot \bar{x}, \quad \pi_j > 0, \quad j = 1, \dots, J, \quad \psi_2 \in \Psi_2$$

where $\bar{d}$ and $\bar{x}$ are the average dividends for all firms at $t=1$ and $t=2$ respectively:

$$\bar{d}(y_0, a) \equiv 1/J \cdot \sum_k d_k(y_0, y_k^I, a_k)$$

$$\bar{x}(s_0, a) \equiv 1/J \cdot \sum_k x_k(s_0, s_k, a_k)$$

- (c) $\pi_j/J > -\hat{z}_{jj}^m / (1 - \hat{z}_{jj}^m)$ for all firms and managers j .
- (d) $\pi_j/J > -z_{jjo}^m / (1 - z_{jjo}^m)$ for all firms and managers j .

Proof:

The proof for the equilibrium consumption plan being (Y_0, Y^I, Y^m) -efficient is similar to the proof of Proposition 3.4. Suppose it is possible in addition to ownership claims to trade in a complete set of primitive securities contingent on public information, and let the prices of these securities be the implicit state-contingent prices satisfying (3.5) and (3.6).

The institutional restriction on insider trading implies that the managers'

equilibrium consumption plans are measurable with respect to public information, since their consumption endowments, management compensations, and the returns from investments are all measurable with respect to that information. Given the structure of the implicit state-contingent prices any variation in consumption plans not due to the level of expected aggregate consumption is a fair gamble for investors as well as for managers. Risk averse investors and managers do not want to take part in such gambles. Their equilibrium consumption plans in the extended economy are therefore measurable with respect to the level of expected aggregate consumption.

The first order conditions for holdings of primitive securities in the extended economy implies that positive parameters $\lambda_1, \dots, \lambda_N$, and $\delta_1, \dots, \delta_J$ exist, such that for all investors i

$$u'_{i1}(c_{i1}(y_0)) = \lambda_i \cdot P_1(y_0)/f(y_0) = \lambda_i \cdot P_1(\gamma_1)/f(\gamma_1), \quad \forall y_0 \subseteq \gamma_1$$

$$u'_{i2}(c_{i2}(s_0)) = \lambda_i \cdot P_2(s_0)/f(s_0) = \lambda_i \cdot P_2(\gamma_2)/f(\gamma_2), \quad \forall s_0 \in \gamma_2$$

and for all managers k

$$u'^{m}_{k1}(c^m_{k1}(y_0)) = \delta_k \cdot P_1(y_0)/f(y_0) = \delta_k \cdot P_1(\gamma_1)/f(\gamma_1), \quad \forall y_0 \subseteq \gamma_1$$

$$u'^{m}_{k2}(c^m_{k2}(s_0)) = \delta_k \cdot P_2(s_0)/f(s_0) = \delta_k \cdot P_2(\gamma_2)/f(\gamma_2), \quad \forall s_0 \in \gamma_2$$

Since all markets clear for equilibrium consumption plans these conditions are both necessary and sufficient conditions for the equilibrium consumption plans in the extended economy being (Y_0, Y^I, Y^m) -efficient given the equilibrium production plan.

The final step to show that the equilibrium consumption plan in the original economy is (Y_0, Y^I, Y^m) -efficient is to show that the equilibrium consumption plan in the extended economy can be implemented through dynamic trading in the original economy. For the investors, the proof of this is like the proof of Proposition 3.4. The managers' net consumption requirements from the portfolio plan in the extended economy are given by

$$\delta_{k1}(\psi_1) = c_{k1}^{m*}(\gamma_1) - \hat{c}_{k1}^m - \tau_{k1}(\psi_1) - \pi_k \cdot \bar{d}(\gamma_1)$$

$$\delta_{k2}(\psi_2) = c_{k2}^{m*}(\gamma_2) - \hat{c}_{k2}^m - \tau_{k2}(\psi_2) - \pi_k \cdot \bar{x}(\gamma_2)$$

These net consumption requirements are measurable with respect to the minimal investment wealth statistics, and from the same argument as in the proof of Proposition 3.4, the investment wealths of managers are also measurable with respect to those statistics. Given that the equilibrium market values of firms provide a dynamically investment wealth quasi-complete market, the managers can implement their equilibrium consumption plans in the extended economy through dynamic trading in well-diversified portfolios in the original economy, and this completes the proof that the equilibrium consumption plan in the original economy is (Y_0, Y^I, Y^m) -efficient given the equilibrium production plan.

The last part of the proof shows that the equilibrium production plan is (Y_0, Y^I, Y^m) -efficient.

The managers are responsible for making production decisions, and their preferences for alternative production plans are determined by the obtainable return from investments in the shares of firms and by the resulting management compensations. Since the managers are only allowed to trade in well-diversified portfolios, and since they are price takers, they can evaluate the return from their investments by only evaluating the intrinsic value of their own firms. Hence, from their perspective as investors they are indifferent as to the information about the firm-specific events that will be conveyed by the dividends for alternative production plans. From the managers' perspective as receivers of management compensation, they do not care about that information either, since their compensation consists of terms that only depend on economy-wide events.

If the consumption plan is efficient and the managers' compensations only depend on economy-wide events, then the consumption plans and the portfolio plans as well depend only on economy-wide events. Hence, only the production plan can vary with firm-specific events. Given the structure of the implicit state-contingent prices, (3.5) and (3.6), and the definition of intrinsic values, (3.12), we can now formulate the decision problem of the

manager of firm j as follows

$$\text{maximize } U_j^m(c_j^m)$$

$$c_j^m, z_j^m, a_j$$

subject to

$$\sum_k z_{jko}^m \cdot v_{ko} \leq \sum_k \hat{z}_{jk}^m \cdot v_{ko} \quad [\delta_j]$$

$$c_{j1}^m(y_o) + \sum_k z_{jk1}^m(y_o) \cdot v_{k1}(y^{mj}) \leq \hat{c}_{j1}^m + M_{j1}(y_o, a) \\ + \sum_k z_{jko}^m \cdot v_{k1}(y^{mj}) \quad [P_{j1}^m(y^{mj})] \text{ all } y^{mj} \in Y_o \times Y^{mj}$$

$$c_{j2}^m(s_o) \leq \hat{c}_{j2}^m + M_{j2}(s_o, a) + \sum_k z_{jk1}^m(y_o) \cdot v_{k2}(s) \quad [P_{j2}^m(s)] \\ \text{all } s \in y^{mj}, y^{mj} \in Y_o \times Y^{mj}$$

$$v_{jo} = \sum_{y_o \in Y_o} E[d_j(y_o, y_j^m, a_j) - M_{j1}(y_o, a) \mid y_o] \cdot P_1(y_o) \\ + \sum_{s_o \in S_o} E[x_j(s_o, s_j, a_j) - M_{j2}(s_o, a) \mid s_o] \cdot P_2(s_o)$$

$$v_{j1}(y^{mj}) = \sum_{s_o \in Y_o} E[x_j(s_o, s_j, a_j) - M_{j2}(s_o, a) \mid y_j^m, s_o] \cdot P_2(s_o \mid y_o) \\ \text{for all } y^{mj} \in Y_o \times Y^{mj}$$

$$a_j(y^{mj}) \in A_j \text{ for all } y^{mj} \in Y_o \times Y^{mj}$$

In this decision problem, we have ignored the sequential rationality constraint, but it will be shown below that this constraint is not binding. Also, we have ignored that the production plan of firm j can not vary with the public firm-specific information for the other firms. It is shown below that the production plan does not vary with that information. Hence, the managers' planned production decisions at t=0 are determined by the following first order condition for an interior optimal production decision for

the manager of firm j given optimal consumption and portfolio rules, and given the signal y^{mj} :

$$\begin{aligned}
0 = & (\hat{z}_{jj}^m - z_{jjo}^m) \cdot \frac{\partial V_{jo}}{\partial a_j(y^{mj})} \cdot \delta_j \\
& + z_{jjo}^m \cdot \frac{\partial V_{j1}(y^{mj})}{\partial a_j(y^{mj})} \cdot P_{j1}^m(y^{mj}) \\
& + \frac{\partial M_{j1}(y_o, a)}{\partial a_j(y^{mj})} \cdot P_{j1}^m(y^{mj}) \\
& - z_{jj1}^m(y_o) \cdot \frac{\partial v_{j1}(y^{mj})}{\partial a_j(y^{mj})} \cdot P_{j1}^m(y^{mj}) \\
& + \sum_{s \in y^{mj}} z_{jj1}^m(y_o) \cdot \frac{\partial V_{j2}(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_{j2}^m(s) \\
& + \sum_{s \in y^{mj}} \frac{\partial M_{j2}(s_o, a)}{\partial a_j(y^{mj})} \cdot P_{j2}^m(s)
\end{aligned}$$

where $\{ \delta_j, P_{j1}^m(y^{mj}), P_{j2}^m(s) \}$ are the optimal Lagrange-multipliers. Since the managers' equilibrium consumption plans are efficient consumption plans, we have the following relations between the optimal Lagrange-multipliers and the state-contingent prices (see the proof of Proposition 3.1):

$$P_{j1}^m(y^{mj}) = \delta_j \cdot P_1(y_o) \cdot f(y^{mj}|y_o) \quad \text{for all } y^{mj} \in Y_o \times Y^{mj}$$

$$P_{j2}^m(s) = \delta_j \cdot P_2(s_o) \cdot f(s|s_o) \quad \text{for all } s \in S$$

Using these relations and the expressions for the value of firm j at $t=0$ and at $t=1$, the first order condition reduces to

$$0 = \hat{z}_{jj}^m \cdot \frac{\partial V_{jo}}{\partial a_j(y^{mj})} \cdot \delta_j$$

$$\begin{aligned}
& + \frac{\partial M_{j1}(y_0, \mathbf{a})}{\partial a_j(y^{mj})} \cdot \delta_j \cdot P_1(y_0) \cdot f(y^{mj}|y_0) \\
& + \sum_{s \in y^{mj}} \frac{\partial M_{j2}(s_0, \mathbf{a})}{\partial a_j(y^{mj})} \cdot \delta_j \cdot P_2(s_0) \cdot f(s|s_0) \\
= & \hat{z}_{jj}^m \cdot \frac{\partial \{ x_{jo}(y^{mj}) - q(y^{mj}, a_j(y^{mj})) - M_{j1}(y_0, \mathbf{a}) \}}{\partial a_j(y^{mj})} \cdot P_1(y_0) \cdot f(y^{mj}|y_0) \cdot \delta_j \\
& + \hat{z}_{jj}^m \cdot \sum_{s \in y^{mj}} \frac{\partial \{ x_j(s, a_j(y^{mj})) - M_{j2}(y_0, \mathbf{a}) \}}{\partial a_j(y^{mj})} \cdot P_2(s_0) \cdot f(s|s_0) \cdot \delta_j \\
& + \frac{\partial M_{j1}(y_0, \mathbf{a})}{\partial a_j(y^{mj})} \cdot \delta_j \cdot P_1(y_0) \cdot f(y^{mj}|y_0) \\
& + \sum_{s \in y^{mj}} \frac{\partial M_{j2}(s_0, \mathbf{a})}{\partial a_j(y^m)} \cdot \delta_j \cdot P_2(s_0) \cdot f(s|s_0)
\end{aligned}$$

Using the definitions of the compensation schemes,

$$M_{j1}(\psi_1, \bar{d}) = \tau_{j1}(\psi_1) + \pi_j \cdot \bar{d}, \quad y_0 \in \psi_1$$

$$M_{j2}(\psi_2, \bar{x}) = \tau_{j2}(\psi_2) + \pi_j \cdot \bar{x}, \quad s_0 \in \psi_2$$

and rearranging terms give that the first order condition is equivalent to the following condition:

$$\begin{aligned}
0 = & - \{ \pi_j/J + \hat{z}_{jj}^m \cdot (1 - \pi_j/J) \} \cdot \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_1(y_0) \cdot f(y^{mj}|y_0) \\
& + \{ \pi_j/J + \hat{z}_{jj}^m \cdot (1 - \pi_j/J) \} \cdot \sum_{s \in y^{mj}} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0) \cdot f(s|s_0)
\end{aligned}$$

From condition (c) this equilibrium condition is satisfied if and only if the first order condition for maximizing the market value of the dividends

of firm j at $t=0$ is satisfied:

$$\begin{aligned}
 & \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_1(y_0) \\
 &= \sum_{s \in y^{mj}} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0) \cdot f(s|y^{mj}, s_0) \\
 &= \sum_{s_0 \in y_0} \frac{\partial E[x_j(s_0, s_j, a_j) | y_j^m, s_0]}{\partial a_j(y^{mj})} \cdot P_2(s_0)
 \end{aligned}$$

Furthermore, if we divide through with $P_1(y_0)$, then the first order condition for maximizing the intrinsic value of dividends of firm j at $t=1$ is obtained:

$$\begin{aligned}
 & \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} = \sum_{s \in y^m} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0|y_0) \cdot f(s|y^{mj}, s_0) \\
 &= \sum_{s_0 \in y_0} P_2(s_0|y_0) \cdot \frac{\partial E[x_j(s_0, s_j, a_j) | y_j^m, s_0]}{\partial a_j(y^{mj})}
 \end{aligned}$$

Since $q(y^{mj}, a_j)$ is $(Y_0 \times Y_j^m)$ -measurable, $a_j(y^{mj})$ is also $(Y_0 \times Y_j^m)$ -measurable. From Proposition 2.4 it now follows that the planned production is an efficient production plan. The planned production is only an equilibrium production plan if the managers have no incentives to deviate from that plan at $t=1$ given the information at $t=1$ and given the portfolio acquired at $t=0$. The first order condition for an optimal production decision at $t=1$ given a signal y^{mj} and given the portfolio z_{j0}^m acquired at $t=0$ is

$$\begin{aligned}
 0 &= z_{jj0}^m \cdot \frac{\partial v_{j1}(y^{mj})}{\partial a_j(y^{mj})} \cdot p_{j1}^m(y^{mj}) \\
 &+ \frac{\partial M_{j1}(y_0, a)}{\partial a_j(y^{mj})} \cdot p_{j1}^m(y^{mj})
 \end{aligned}$$

$$\begin{aligned}
& - z_{jj1}^m(y_0) \cdot \frac{\partial v_{j1}(y^{mj})}{\partial a_j(y^{mj})} \cdot P_{j1}^m(y^{mj}) \\
& + \sum_{s \in y^{mj}} z_{jj1}^m(y_0) \cdot \frac{\partial v_{j2}(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_{j2}^m(s) \\
& + \sum_{s \in y^{mj}} \frac{\partial M_{j2}(s_0, a)}{\partial a_j(y^{mj})} \cdot P_{j2}^m(s)
\end{aligned}$$

which by the same arguments as above can be reduced to

$$\begin{aligned}
0 = & - \{ \pi_j/J + z_{jj0}^m \cdot (1 - \pi_j/J) \} \cdot \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_1(y_0) \cdot f(y^{mj}|y_0) \\
& + \{ \pi_j/J + z_{jj0}^m \cdot (1 - \pi_j/J) \} \cdot \sum_{s \in y^{mj}} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0) \cdot f(s|s_0)
\end{aligned}$$

This condition is equivalent to the first order condition for maximizing the market value of the dividends of firm j at $t=0$, and for maximizing the intrinsic value of those dividends at $t=1$ if and only if

$$\pi_j/J + z_{jj0}^m \cdot (1 - \pi_j/J) > 0$$

which is equivalent to condition (d) in the proposition. Hence, the planned production is also an equilibrium production plan. This proves the proposition, since the planned production plan is an efficient production plan.

Q.E.D.

The institutional restriction on insider trading limits the managers' trading opportunities to investments in shares or portfolios of shares about which they have no superior information. Hence, given the structure of the implicit state-contingent prices, i.e., (3.5) and (3.6), the managers' as well as the investors' desired consumption plans are those which are measurable with respect to the level of expected aggregate consumption at $t=1$ and at $t=2$. For the equilibrium production plan, a dynamically investment

wealth quasi-complete market (i.e., condition (a)) ensures that there exist well-diversified portfolios with enough flexibility to implement any desired and financially feasible consumption plan through dynamic trading. The market clearing conditions then implies a Pareto efficient risk sharing.

The conditions (b), (c) and (d) ensure that the managers have no incentives not to implement an (Y_0, Y^I, Y^M) -efficient production plan. Condition (b) gives the form of the compensation schemes and condition (c) and (d) are restrictions on the managers' equilibrium portfolio plans.

The compensation schemes consist of two terms. The first term is contingent on the elements in the partitions of the set of economy-wide events given by the minimal investment wealth statistics for the equilibrium production plan. If a manager considers a production plan other than the equilibrium plan, then his compensation is not affected by this term. Hence, the managers' preferences for alternative production plans are only affected by the second term in the compensation scheme.

The second term gives each manager a proportional share of the average dividends, which in our large economy is the same as a small fraction of the expected aggregate dividends. Since the managers' equilibrium consumption plans are efficient, their marginal rates of substitution across events and over time are in equilibrium equated with the ratios of the implicit state-contingent prices. The managers can therefore be characterized as evaluating, conditional on their superior information, the change in the average dividends due to a change in their individual production plans taking the implicit state-contingent prices as given. Given that there are no externalities in production (i.e. **(A4)**), this decision rule is equivalent to maximizing the intrinsic values of the dividends at $t=1$ and maximizing the market values of the dividends at $t=0$, which are necessary and sufficient conditions for an (Y_0, Y^I, Y^M) -efficient production plan (Proposition 2.4). In the limit, as the number of firms goes to infinity, no individual manager is able to change the average dividends by changing his own production plan, but then, he is indifferent between alternative production plans. Hence, from the perspective as receivers of management compensation, the managers have strict positive incentives to implement an efficient production plan for a large but finite number of firms, and in

the limit, they have no incentives to make individual deviations from such production plans.

It is therefore a Nash equilibrium that managers implement efficient production plans. Any production plan may represent a Nash equilibrium in the limit as the number of firms goes to infinity. But even then, all managers and all investors strictly prefer that the managers collectively choose to implement the efficient production plan corresponding to the implicit state-contingent prices. It is therefore not unreasonable to assume that the efficient production plan is the Nash equilibrium that will be played.

The managers' preferences for alternative production plans are not determined by the compensation schemes only. The production plan implemented also affects the return the managers can get from their portfolio plans. Conditions (c) and (d) ensure that the managers' incentives as investors with respect to alternative production plans are the same as their incentives as receivers of management compensation. Conditions (c) and (d) state that all managers' endowed and equilibrium ownership at $t=0$ in their own firms are not too negative.³ That is, the conditions ensure that all managers' equilibrium consumption possibilities determined both by their compensations and by the return from their portfolio plans are positively related to the dividends of their own firm. Of course, if this was not the case, we could not hope for an equilibrium production plan which maximizes the value of the dividends of firms.

The managers' endowed ownership in their own firms are exogenously given, while the managers' equilibrium ownership acquired at $t=0$, z_{jj0}^m , is determined as a part of the equilibrium. This equilibrium ownership may be negative, since the managers might want to hedge against the uncertainty given by the minimal investment wealth statistics. However, the managers' hedging behavior is influenced by the first term in the compensation schemes, and this is the reason why this term is included. Suppose that the managers are not endowed with firm ownership at all, i.e. $\hat{z}^m = 0$. We know that an efficient consumption plan is a strictly increasing function of the level of expected aggregate consumption (Proposition 2.3), but we do not know that individual consumption plans are linear in the level of expected aggregate consumption. Hence, if the first term in the compensation schemes were not

included, then the managers might for a zero net-investment at $t=0$ have to take short positions in some well-diversified portfolios and long positions in other well-diversified portfolios in order to obtain the efficient individual state-contingent investment wealth at $t=1$. If it happens that some managers are sufficiently short in portfolios, of which their own firm is a part, then they will not have incentives to implement the efficient production plan at $t=1$. On the other hand, the first term in the compensation function can be constructed such that all managers get their desired state-contingent investment wealth at $t=1$ without trading at all.⁴ It is therefore possible to offer the managers compensation schemes in which the first term is such that condition (d) will be satisfied.

If some managers' endowed ownership in their own firms are sufficiently negative, then they do not like the efficient production plan that maximizes the market values of the dividends of their firms. On the contrary, they would like to plan to implement the production plan that minimizes those market values. For this plan to be sequentially rational, i.e., be credible to investors and reflected in the market values of firms at $t=0$, the equilibrium ownership in their own firms acquired at $t=0$ must also be sufficiently negative. Otherwise, they have incentives at $t=1$ to switch from the planned production at $t=0$ to the efficient production plan that maximizes the intrinsic values of the dividends of firms at $t=1$. If the first term in the compensation schemes are determined such that condition (d) is satisfied and the number of firms are large but finite, then the only production plan which is sequentially rational is the efficient production plan. In this case, the restriction on the managers' endowed ownership in their own firms is not required to ensure that the equilibrium production plan is an efficient production plan. However, in the limit as the number of firms goes to infinity, the managers are indifferent at $t=1$ between alternative production plans for their own firms, since they are well-diversified at that date. Thus, they have no incentives at $t=1$ to change the production plan which was planned at $t=0$. Managers are not necessarily well-diversified at $t=0$, and then, they are not indifferent between alternative production plans for their own firms at $t=0$. Hence, in the limit, as the number of firms goes to infinity, the restriction on the managers' endowed ownership in their own firms are required to ensure that the equilibrium production plan is an efficient production plan.⁵

The type of compensation schemes offered to managers do not look like compensation schemes usually advocated (e.g., stock options), where the managers' compensations are more directly related to the performance of the firm in which they are employed. There are two basic reasons for that type of compensation scheme not being optimal in our model.

Managers are, in our model, individual economic agents with preferences defined on consumption plans. To obtain an efficient sharing of the aggregate risk in the economy, managers must take their part of that risk, but they should not take unnecessary risk. Since an efficient production plan can be obtained without imposing firm-specific risk on managers it is not optimal to impose such risk on managers. No economic agent has to take firm-specific risk. If managers were offered compensation schemes directly related to the market values of their own firms, then they would be exposed to firm-specific risk, since the market values of individual firms depend on firm-specific events. Hence, such compensation schemes destroy efficient risk sharing without creating preferred production incentives for managers.

We want managers to use their private information optimally, when they make production decisions, such that resources are allocated most efficiently among firms. Therefore, managers should not only be concerned about the performance of their own firm but about the performance of the market as a whole. In the standard perfect and complete market analysis where all investors and managers have the same information, there is no conflict between maximizing the market values of firms and an efficient allocation of resources among firms (see, e.g., Radner (1982)). However, when managers have superior information at the time where production decisions have to be made, such conflicts do exist. Hence, the managers' compensations must be related to aggregate dividends and not to dividends or market values of individual firms, if we want to obtain an efficient resource allocation.

An efficient production plan maximizes the market values of the dividends at $t=0$, and the intrinsic values of the dividends at $t=1$. The following proposition states that an efficient production plan also maximizes the market values of firms at $t=0$, and the intrinsic values of firms at $t=1$, given the compensation schemes in Proposition 4.1.

Proposition 4.2.

Let $(Y_0, Y^I, Y^m, c, z, a, M, V)$ be a sequential quasi-equilibrium. If the equilibrium consumption/production plan is an (Y_0, Y^I, Y^m) -efficient consumption/production plan, and if the compensation schemes are as given in Proposition 4.1, then the equilibrium production plan has the following properties

- (a) The equilibrium production plan maximizes the market values of firms at $t=0$.
- (b) The equilibrium production plan maximizes the intrinsic values of firms at $t=1$.

Proof:

If the equilibrium production plan is (Y_0, Y^I, Y^m) -efficient, then it maximizes the intrinsic values of the dividends at $t=1$:

$$0 = - \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} + \sum_{s \in y^{mj}} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0 | y_0) \cdot f(s | y^{mj}, s_0)$$

This implies that

$$0 = - \pi_j / J \cdot \frac{\partial q(y^{mj}, a_j(y^{mj}))}{\partial a_j(y^{mj})} + \pi_j / J \cdot \sum_{s \in y^{mj}} \frac{\partial x_j(s, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot P_2(s_0 | y_0) \cdot f(s | y^{mj}, s_0)$$

Subtracting these two equations on both sides, and using the definition of the compensation schemes, give that

$$0 = \frac{\partial}{\partial a_j(y^{mj})} [- \{d_j(y^{mj}, a_j(y^{mj})) - M_{j1}(y_0, a)\} + \sum_{s \in y^{mj}} \{x_j(s, a_j(y^{mj})) - M_{j2}(s_0, a)\} \cdot P_2(s_0 | y_0) \cdot f(s | y^{mj}, s_0)]$$

and this is the first order condition for maximizing the intrinsic value of the firm at $t=1$. Multiplying both sides with $P_1(y_0)$ and taking the expectation with respect to the manager's information, y^{mj} , give the first order condition for maximizing the market value of the firm at $t=0$. Q.E.D.

Investors choose in equilibrium to restrict their portfolio plans to well-diversified portfolios to insure themselves against firm-specific risk and against the deviations between market values and intrinsic values of individual firms. As a consequence of this equilibrium portfolio rule, investors are concerned about the market values of well-diversified portfolios and not about the market values of individual firms, as long as well-diversified portfolios are correctly priced. Hence, the compensation functions offered to managers are precisely such that the managers' preferences over alternative production plans are the same as the investors' preferences over those plans in equilibrium.

Proposition 4.3.

Let $(Y_0, Y^I, Y^m, c, z, a, M, V)$ be a sequential quasi-equilibrium. If the equilibrium consumption/production plan is an (Y_0, Y^I, Y^m) -efficient consumption/production plan, and if the compensation schemes are as given in Proposition 4.1, then the equilibrium has the following properties:

- (a) The equilibrium production plan maximizes the market value of well-diversified portfolios at $t=0$ and at $t=1$.
- (b) All investors with non-negative endowed ownership and equilibrium ownership in firm j at $t=0$ unanimously support the equilibrium production plan of firm j .

Proof:

From Proposition 3.2 we get that the market value of any well-diversified portfolio at $t=1$ is equal to its intrinsic value. Since the equilibrium

production plan is an efficient production plan, and the compensation schemes are as given in Proposition 4.1, we get from Proposition 4.2 that the equilibrium production plan maximizes the intrinsic values of individual firms at $t=1$ and the market values of individual firms at $t=0$. Because there are no externalities in production, the production plan also maximizes the intrinsic values of well-diversified portfolios at $t=1$ and, hence, the production plan maximizes the market values of well-diversified portfolios at $t=1$ and at $t=0$. This is (a).

The property (b) means that in equilibrium no investor with non-negative endowed ownership and equilibrium ownership in firm j at $t=0$ would like the manager of firm j to change the equilibrium production plan of the firm. That is, given the production plans of all other firms, the equilibrium production of firm j maximizes all such investors' expected utilities at $t=0$ and their conditional expected utilities at $t=1$.

In equilibrium, the conditional expected utility of investor i at $t=1$ can be written as

$$\begin{aligned} U_i(c_i)|y^I = & u_{i1}(\hat{c}_{i1} + \sum_m z_{imo} \cdot \bar{v}_{m1}(y_0) - \sum_m z_{im1}(y_0) \cdot \bar{v}_{m1}(y_0)) \\ & + \sum_{s_0 \in y_0} u_{i2}(\hat{c}_{i2} + \sum_m z_{im1}(y_0) \cdot \bar{v}_{m2}(s_0)) \cdot f(s_0) \end{aligned}$$

From Proposition 3.2 the intrinsic values of well-diversified portfolios are equal to their average expected values. Hence, we can also write the conditional expected utility of investor i at $t=1$ as

$$\begin{aligned} U_i(c_i)|y^I = & u_{i1}(\hat{c}_{i1} + \sum_m z_{imo} \sum_{j \in J_m} v_{j1}(y_0, y_j^m) - \sum_m z_{im1}(y_0) \sum_{j \in J_m} v_{j1}(y_0, y_j^m)) \\ & + \sum_{s_0 \in y_0} u_{i2}(\hat{c}_{i2} + \sum_m z_{im1}(y_0) \sum_{j \in J_m} \sum_{s_j \in y_j^m} v_{j2}(s_0, s_j) \cdot f(s_j|s_0, y_j^m)) \cdot f(s_0) \end{aligned}$$

We now have to show that this conditional expected utility for each signal to the manager of firm j attains its maximum in the equilibrium production decision given that signal. If we take the derivative of the conditional expected utility with respect to the production decision of firm j for a given signal to the manager of firm j we get

$$\begin{aligned}
\frac{\partial U_i(c_i) | y^I}{\partial a_j(y^{mj})} &= z_{imo} \cdot \frac{\partial v_{j1}(y^{mj})}{\partial a_j(y^{mj})} \cdot u'_{i1}(c_{i1}(y_o)) \\
&- z_{im1}(y_o) \cdot u'_{i1}(c_{i1}(y_o)) \cdot \left\{ \frac{\partial v_{j1}(y^{mj})}{\partial a_j(y^{mj})} \right. \\
&- \sum_{s_o \in y_o} \frac{u'_{i2}(c_{i2}(s_o))}{u'_{i1}(c_{i1}(y_o))} \sum_{s_j \in y_j^m} \frac{\partial v_{j2}(s_o, s_j, a_j(y^{mj}))}{\partial a_j(y^{mj})} \cdot f(s_j | s_o, y_j^m) \left. \right\}
\end{aligned}$$

Since the ratio of the marginal utilities for state-contingent consumption at $t=2$ and at $t=1$ is equal to the ratio of the corresponding equilibrium implicit state-contingent prices for efficient consumption plans, the term in the $\{\cdot\}$ - parenthesis is equal to zero by the definition of the intrinsic value of firm j at $t=1$. The equilibrium production plan maximizes the intrinsic value (cum net-dividend) of the firm at $t=1$. The first term is therefore also equal to zero. Hence, the marginal conditional expected utility is equal to zero for the equilibrium production plan, and thus, the equilibrium production plan represents a maximum for the conditional expected utility if z_{imo} is non-negative. This proves that investors with non-negative equilibrium ownership in firm j , acquired at $t=0$, unanimously support the equilibrium production plan implemented by the manager of the firm.

That investors with non-negative endowed ownership in firm j also support that production plan can be shown in a similar way. The expected utility of investor i at $t=0$ is the expected value of the conditional expected utility at $t=1$. Write the investor's equilibrium consumption at $t=1$ as

$$\begin{aligned}
c_{i1}(y_o) &= \hat{c}_{i1} + \sum_j (\hat{z}_{ij} - z_{ijo}) \cdot v_{jo} \\
&+ \sum_m z_{imo} \cdot \bar{v}_{m1}(y_o) - \sum_m z_{im1}(y_o) \cdot \bar{v}_{m1}(y_o)
\end{aligned}$$

then the proof is exactly as above. Q.E.D.

The efficient production decision rule looks like the usual production decision rule in perfect and complete markets. This rule is to maximize the market values of firms taking the implicit state-contingent prices pertaining to all events as given. The difference is that in our model we should maximize the market values of well-diversified portfolios and not of individual firms. This, on the other hand, can be achieved by maximizing the intrinsic values of individual firms taking the implicit state-contingent prices pertaining only to the economy-wide events as given. As in the perfect and complete market analysis all investors with positive ownership in firms unanimously support this decision rule.

Since investors with positive ownership in firms agree on the desirable production plan in equilibrium they also agree on what compensation schemes to offer managers. Of course, investors with positive ownership in a firm would like to pay the manager of that firm as small a compensation as possible. But for a given level of the managers' expected utility at $t=0$, investors cannot gain anything by offering the manager another compensation scheme. They do not want to change the production plan, and the risk sharing between investors and managers is unconstrained Pareto-optimal. Hence, the compensation schemes can also be considered as endogenous decision variables, and we can state the following proposition without proof.

Proposition 4.4.

Let $(Y_0, Y^I, Y^m, c, z, a, M, V)$ be a sequential quasi-equilibrium. If the equilibrium consumption/production plan is an (Y_0, Y^I, Y^m) -efficient consumption/production plan, and if the compensation schemes are as given in Proposition 4.1, then the equilibrium has the property:

Given the equilibrium expected utility for the manager of firm j , no investor with a non-negative endowed ownership in firm j has any incentive to offer the manager another compensation scheme.

4.3 Conclusion.

In this chapter, we have considered the managers' implementation of their part of an efficient risk sharing, and the implementation of an efficient production plan.

It is shown in section 4.1 that, before the release of information to managers and investors, it is unanimously preferred that the managers precommit themselves to trade only in well-diversified portfolios. This follows from inefficient risk sharing and adverse production incentives due to insider trading. It is therefore assumed that this precommitment is made, and that it is enforceable.

In section 4.2, conditions on the managers price taking behavior and on the structure of compensation schemes are identified which ensure that the managers implement an efficient production plan in their self-interest. The managers are price takers with respect to the implicit state-contingent prices pertaining to the economy-wide events. Because the dividends convey information to the investors about the managers' private information it is also assumed that the managers act as if the market value of any well-diversified portfolio is equal to the intrinsic value of that portfolio for any production plan. These two assumptions ensure that the managers' production incentives are not influenced by the fact that alternative production plans may not convey the same information to the investors. However, note that the price taking assumption would not be necessary, if the managers were not allowed to invest in well-diversified portfolios either.

The compensation schemes consist at each date of a fixed state-contingent term and a proportional share of the average dividend in the economy. The fixed state-contingent term is included to influence the managers' trading behavior, such that the managers do not take short positions in well-diversified portfolios in which their own firms are included. This implies that the managers' return from investments as well as their compensations are positively related to the dividends of their own firms. The proportional share of the average dividend is measurable with respect to the level of conditional expected aggregate consumption at each date. Hence, this term does not impose unnecessary risks on the managers, and it gives the mana-

gers incentives to implement an efficient production plan in their self-interest.

An alternative set of conditions exists, which would ensure that risk sharing is efficient and that managers have no incentive not to implement an efficient production plan. The first condition is that the managers have non-negative endowed ownership in their own firms; the second is that they are only allowed to trade in a riskless security, and the third is that the compensation schemes consist of a fixed state-contingent term only. The managers would then be forced to liquidate their endowed portfolio and invest the proceeds in the riskless asset. The state-contingent term must be determined such that the managers take their part of the economy-wide risks, given the managers' endowed consumption and the return from the investment in the riskless asset. In that case, the managers are only required to be price takers with respect to the riskless interest rate, and the managers' consumption plans are not related to either the dividends of their own firms or to the aggregate dividends in the economy. Therefore, given that the managers have no nonpecuniary returns from production decisions, they are indifferent between all feasible production plans. In particular, they have no incentive not to implement the efficient production plan corresponding to the state-contingent prices which pertain to the economy-wide signals and events, and this is the only requirement for the efficient production plan being part of a Nash equilibrium. However, for a large but finite number of firms, the conditions on the compensation schemes given in Proposition 4.1 ensure that the managers have strict positive incentives (although they are small) to implement the efficient production plan.

Efficient production plans maximize the value of the dividends at $t=0$ and the intrinsic value of the dividends at $t=1$. The market value of a firm at $t=0$ is equal to the value of the dividends minus the value of the compensations to the manager of the firm. Similarly, the intrinsic value of a firm at $t=1$ is equal to the intrinsic value of the dividends minus the compensations to the manager of the firm. It is shown in Proposition 4.2 that the compensation schemes are such that efficient production plans also maximize the intrinsic values of firms at $t=1$ as well as the market values of firms at $t=0$, before any information is released. Since there are no externa-

lities in production, it follows that efficient production plans maximize the market values of well-diversified portfolios before the release of information as well as after the release of information at $t=1$. Investors trade only in well-diversified portfolios in an efficient equilibrium. Hence, it is shown in Proposition 4.3 that any investor, who has a non-negative endowed ownership in firm j at $t=0$, and who selects a portfolio at $t=0$ in which he has a non-negative position in firm j , unanimously supports the equilibrium production plan of firm j . Therefore, these investors have no incentives to offer the manager of firm j another compensation scheme at $t=0$, given the level of expected utility to manager. That is, the compensation schemes given in Proposition 4.1 can be sustained as part of the equilibrium.

This chapter concludes the analysis of our basic model. In Chapter 2, we have characterized efficient consumption/production plans and we have given characteristics of valuable and non-valuable information. In Chapter 3 and Chapter 4, we have considered the implementation of efficient consumption/production plans in a sequential market setting. In the next chapter, we consider some extensions of the basic model, and we give some conclusions about the social and private value of firm-specific information in market economies.

Footnotes for Chapter 4.

1. The assumption states that managers are not allowed to take speculative positions in the shares of their own firms. This also include the case where a manager buys (sells) a small percentage of a large portfolio including his own firm and at the same time sells (buys) the same portfolio without his own firm. In this way, he can indirectly take a small speculative position in his own firm based on his private information.
2. For a finite number of firms, the managers have superior information about the value of a well-diversified portfolio which includes their own firm in the sense that the value of the portfolio is equal to the intrinsic value of their own firm plus the market value of the other firms in the portfolio. However, any single manager does not know the firm-specific signal received by the other managers. Hence, he does not know the portfolio's intrinsic value given all the managers' private information. Therefore, even for a large but finite number of firms, it does not seem unreasonable to assume that the managers do not take speculative positions in well-diversified portfolios based on their private information given the risk about the intrinsic values of the other firms in the portfolio. That is, the managers behave as if they do not have superior information about the true value of well-diversified portfolios. Alternatively, we might preclude the managers from trading in their own firms at all.
3. Note that conditions (c) and (d) are always satisfied if all managers' endowed and equilibrium ownership at $t=0$ in their own firms are positive. Hence, the term in the compensation schemes consisting of the proportional share of the average dividend for all firms is not necessary in this case to ensure that the managers implement an efficient production plan. That is, the proportional share of the average dividend for all firms is only required in those cases, where either the endowed ownership or the equilibrium ownership is negative.
4. In this case, the term in the compensation schemes consisting of the proportional share of the average dividend for all firms is not re-

quired, because the managers have no incentives not to implement an efficient production plan.

5. In the limit, condition (c) states that the managers' endowed ownership in their own firms must be positive. If the managers are not endowed with ownership in their own firms, then they have no incentives not to implement an efficient production plan.

CHAPTER 5.

Extensions and Conclusions.

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5.2	The private and social value of firm-specific information.	149

In Chapter 3 and Chapter 4 we have shown that an unconstrained Pareto-optimal allocation can be achieved as a sequential quasi-equilibrium given conditions on the structure of security prices, the managers' trading opportunities, and the compensation schemes. The assumption of the asymmetric information between managers and investors has been limited to the firm-specific events. Events are characterized as firm-specific if they only affect the dividends of individual firms and if they are conditionally independent. Therefore, many economy-wide events might be irrelevant to the implementation of the investors' consumption and portfolio plans. As long as the managers have the information about those events and use it optimally, there is no need to report that information to investors. In section 5.1 we examine the implementation of efficient consumption/production plans in an economy, where managers have superior information about firm-specific events as well as about the economy-wide events which are irrelevant to the investors' implementation of efficient consumption and portfolio plans.

In the preceding analysis the consumption, production and portfolio plans as well as the compensation schemes for managers have all been determined as part of the equilibrium in the economy. However, the information structures was exogenously given. In section 5.2 we give an informal analysis of an economy where the economic agents also have the option to choose individually among a set of information structures for firm-specific events.

5.1. Sustainable efficient equilibria under general asymmetric information.

The information about economy-wide events, which is relevant to the investors' consumption plans, is the partitions given by the level of expected aggregate consumption at $t=1$ and $t=2$. In order to implement an efficient consumption plan investors may have to trade dynamically in well-diversified portfolios. The state-contingent payoffs from efficient portfolio plans restricted to well-diversified portfolios are measurable with respect to the minimal investment wealth statistics (see the proof Proposition 3.4). If the composition of the portfolios at $t=1$ and their state-contingent payoffs at $t=2$ are measurable with respect to those statistics as well, then the information, which is relevant to the investors' decisions at $t=1$, is the level of current expected aggregate consumption and the

probability distribution of the future level of expected aggregate consumption. Any finer partition on the set of economy-wide events is of no value to investors, if managers have that information and use it optimally in their production decisions.

Managers are responsible for making the production decisions of firms. To implement efficient production plans managers should act as if they maximize the intrinsic values of firms at $t=1$ given the state-contingent prices,

$$V_{j1}(y_0, y_j^m) = d_j(y_0, y_j^I, a_j) - M_{j1}(\psi_1, \bar{d}) \\ + \sum_{s_0 \in y_0} E[x_j(s_0, s_j, a_j) - M_{j2}(\psi_2, \bar{x}) | s_0, y_j^m] \cdot P_2(s_0 | y_0) \text{ for } y_j^m \subseteq y_j^I$$

For efficient consumption plans the implicit state-contingent prices are given by

$$P_2(s_0 | y_0) = \frac{P_2(s_0)}{P_1(y_0)} = \frac{P_2(\gamma_2) \cdot f(s_0 | \gamma_2)}{P_1(\gamma_1) \cdot f(y_0 | \gamma_1)} \text{ for } s_0 \in y_0, s_0 \in \gamma_2 \text{ and } y_0 \in \gamma_1 \\ = \frac{P_2(\gamma_2) \cdot f(\gamma_1)}{P_1(\gamma_1) \cdot f(\gamma_2)} \cdot f(s_0 | y_0) \text{ for } s_0 \in y_0, s_0 \in \gamma_2 \text{ and } y_0 \in \gamma_1 \\ \equiv P_2(\gamma_2 | \gamma_1) \cdot f(s_0 | y_0) \text{ for } s_0 \in y_0, s_0 \in \gamma_2 \text{ and } y_0 \in \gamma_1$$

The intrinsic value of firm j at $t=1$ can therefore also be written as

$$V_{j1}(y_0, y_j^m) = d_j(y_0, y_j^I, a_j) - M_{j1}(\psi_1, \bar{d}) \\ + \sum_{\gamma_2 \in \Gamma_2} E[x_j(s_0, s_j, a_j) - M_{j2}(\psi_2, \bar{x}) | \gamma_2, y_0, y_j^m] \cdot P_2(\gamma_2 | \gamma_1) \text{ for } y_j^m \subseteq y_j^I$$

Hence, the relevant implicit state-contingent prices to discount payments at $t=2$ depend only on the level of expected aggregate consumption. The relevant payments to discount are the expected net-dividend of the firm at

$t=2$ conditional on the level of expected aggregate consumption at $t=2$, γ_2 , the information about the economy-wide event at $t=1$, y_0 , and conditional on the managers' private information about the firm-specific event at $t=1$, y_j^m . Thus, even though the implicit state-contingent prices only pertain to the level of expected aggregate consumption at $t=2$, the managers need all the information about the economy-wide event at $t=1$ in order to be able to implement efficient production plans. Hence, more economy-wide information is, in general, needed to implement efficient production plans than to implement efficient consumption plans.

Managers implement the production plan which maximizes their own expected utilities. In order to achieve an efficient production plan managers must be given economic incentives such that maximizing their own expected utilities becomes equivalent to maximizing the intrinsic values of firms at $t=1$, given the implicit state-contingent prices. We examine this issue in the extreme case. The only information available to investors at $t=1$ is the signal from the minimal investment statistic at $t=1$ for a given efficient production plan, i.e., some signal $\psi_1 \in \Psi_1(a)$, and the information that can be inferred from the dividends at $t=1$. The minimal investment wealth statistic at $t=1$ is called the pre-specified information structure for investors. This reflects that it is assumed that this information will be available, no matter what production plan managers are going to implement. Managers, on the other hand, still have information according to $Y_0 \times Y^m$.

Proposition 5.1.

Let $(Y_0, Y^I, Y^m, c, z, a, M, V)$ be a sequential quasi-equilibrium and let the equilibrium consumption/production plan be an (Y_0, Y^I, Y^m) -efficient consumption/production plan, such that

- (a) The compensation schemes are such that the conditions (b) through (d) in Proposition 4.1 are all satisfied.

- (b) There exist a partition of firms into large portfolios $J_1, \dots, J_M$ at $t=0$ such that

$$DW_1 = \text{span}\{ \mathbf{V}_{11}, \dots, \mathbf{V}_{M1} \}.$$

For each $y_0 \in Y_0$, there exist a partition of firms into large portfolios $J_1(\psi_1), \dots, J_{M(\psi_1)}(\psi_1)$ at $t=1$ such that

$$DW_2(y_0) = \text{span}\{ \mathbf{V}_{12}(\psi_1), \dots, \mathbf{V}_{M(\psi_1)2}(\psi_1) \} \quad \text{for } y_0 \subseteq \psi_1$$

i.e. the portfolio compositions and the state-contingent pay-offs vectors for well-diversified portfolios are measurable with respect to the minimal investment wealth statistic at $t=1$.

Then the equilibrium consumption/production plan can be sustained as an equilibrium in an otherwise identical economy in which the pre-specified information structure for investors at $t=1$ is $\Psi_1(a)$, the information structures for managers are $Y_0 \times Y^m$, and in which managers only are allowed to trade in the portfolios given by (b).

Proof:

In order to show that the equilibrium consumption/production plan in the original economy can be sustained as an equilibrium in the economy with less information to investors, we must show that it is feasible in that economy and that there exist a set of market values of firms such that no economic agent has any incentives to deviate from the plan.

The equilibrium consumption/production plan in the original economy is an efficient plan. It is a feasible plan in the original economy, and it is measurable with respect to the information in the economy with less information to investors. Hence, it is a feasible plan in the economy with less information to investors.

For the other part of the proof we must propose a set of market values of firms which is measurable with respect to public information in the economy with less information to investors. The public information at $t=1$ in that

economy is given by the minimal investment wealth statistic at $t=1$ for the production plan in the original economy, and by the information that can be inferred from the dividends at $t=1$. The public information structure for economy-wide events is therefore given by

Y_0^I : The coarsest subpartition of $\Psi_1(a)$ such that the dividends at $t=1$ are measurable with respect to that partition given the public firm-specific information.

Without loss of generality we assume that the public information structure for firm-specific events is the same as in the original economy, Y^I . The proposed market values of firms is the same as in the original economy at $t=0$ and at $t=2$, since the public information is the same at these two dates for both economies. As the market values of firms at $t=1$ we take the expected values of the market values in the original economy conditional on public information in the economy with less information to investors:

$$v_{j1}(y_0^I, y_j^I) = \sum_{y_0 \subseteq y_0^I} v_{j1}(y_0, y_j^I) \cdot f(y_0 | y_0^I) \quad \text{for all } y_0^I \in Y_0^I, y_j^I \in Y_j^I$$

Since there exist well-diversified portfolios in the original economy which precisely span the sets of desired investment wealths (i.e. condition (b)), those portfolios have the same market values in the two economies given that the production plans are the same. Furthermore, since the composition of the portfolios acquired at $t=1$ and their vectors of state-contingent pay-offs at $t=2$ are measurable with respect to the minimal investment wealth statistic at $t=1$, the portfolio plans at $t=1$ in the original economy are also measurable with respect to that statistic. Hence, the equilibrium portfolio plan in the original economy is feasible, and it implements the same consumption plan in the economy with less information to investors as in the original economy. The implicit state-contingent prices pertaining to the level of expected aggregate consumption are also the same in the two economies, and given the structure of market values of well-diversified portfolios, investors have no incentives to deviate from the consumption plan in the original economy. Similarly, given the production plan, managers have no incentives to deviate from the consumption plan in the original economy, since they are only allowed to trade in well-diversified

portfolios which are of the same market values in both economies. That is, managers are only allowed to trade in portfolios about which they have no superior information.

Finally, we must show that the managers have no incentive to deviate from the production plan in the original economy given efficient risk sharing between investors and managers. All we have to do is to use the corresponding proof of Proposition 4.1, assuming that the managers are price takers with respect to the implicit state-contingent prices in the original economy pertaining to the economy-wide events:

$$P_2(s_0|y_0) = P_2(\gamma_2|\gamma_1) \cdot f(s_0|y_0) \quad \text{for } s_0 \in y_0, s_0 \in \gamma_2 \text{ and } y_0 \in \gamma_1$$

Then, the managers' preferences for alternative production plans as receivers of management compensation are the same as in the original economy. Furthermore, taking these prices as given, managers act as if they maximize the market values of the well-diversified portfolios given by (b), and as if those market values do not depend on the information about economy-wide events that can be inferred from the dividends at $t=1$. Hence, managers have no incentives to deviate from the production plan in the original economy just to change the information that can be inferred from the dividends. Since the production in the original economy maximizes the market value of well-diversified portfolios given the implicit state-contingent prices, this proves the proposition. Q.E.D.

Under the assumed conditions the proposition states that only information relevant to the level of current expected aggregate consumption and for the probability distribution of the level of future expected aggregate consumption has to be available to the investors in order to obtain a resource allocation which is efficient with respect all available information in the economy. That is, managers reporting additional information to investors is of no social value. Of course, the reasons are that the investors do not need more information to make efficient consumption and portfolio decisions, and that managers can be motivated to implement a production plan which is efficient with respect to their superior information.

The assumed conditions are sufficient conditions for the equilibrium in the full-information economy being efficient. However, condition (b) is more restrictive than the condition that the market in the full-information economy is dynamically investment wealth quasi-complete. It requires that the composition of the well-diversified portfolios acquired at $t=1$ and the vectors of their state-contingent pay-offs at $t=2$ are measurable with respect to the minimal investment wealth statistic at $t=1$. This is to ensure that investors have all the information they need to implement the portfolio plan in the full-information economy. Furthermore, condition (b) requires that the equilibrium market values at $t=1$ of all well-diversified portfolios acquired and sold at $t=1$ in the full-information economy are measurable with respect to the minimal investment wealth statistic at $t=1$. If this was not the case, then managers might be able to trade in well-diversified portfolios with intrinsic values not equal to their market values. Hence, there would be an insider trading problem.

Thus, the economy-wide events, about which we allow the managers to have superior information, are only those events which determine for which firms the event is a good event and for which firms it is a bad event. Those events can be considered as generalized firm-specific events. The distinction between firm-specific and economy-wide events is whether or not the events influence the composition and the market values of well-diversified portfolios. However, note that whether events are firm-specific or economy-wide according to this definition depends, in general, on the production plan, while this is not the case with the original distinction between firm-specific and economy-wide events.

5.2. The private and social value of firm-specific information.

The results of the analysis in Part I have shown that the managers' information about firm-specific productivity events is of social value, but it is of no social value that managers report any firm-specific information to the investors. The analysis in the previous section shows that many economy-wide events might also be irrelevant to the investors. Hence, from a social perspective, it is the information available to managers that matters and not the information available to investors.

In the preceding analysis we have considered Pareto-efficient equilibrium allocations of resources among investors, among firms and over time for exogenously given information structures. The consumption, production and portfolio plans as well as the compensation schemes for managers have all been determined as part of the equilibrium in the economy. In this section, we conclude Part I with an informal analysis of an economy, in which the economic agents also have the option to choose individually among a set of information structures for firm-specific events. The question we ask is, whether the information structures for firm-specific events available to the managers and those available to the investors can be sustained as part of an equilibrium. That is, do managers and investors have any incentives to change their information structures privately. First, we consider the case, where the managers choose among alternative information structures for firm-specific events.

The managers' information structures can be viewed as determined by the internal accounting systems and other decision support systems used, and as such, the information structures are part of the managers' decision problem. Of course, if these systems were costless to implement in terms of real resources, it would be optimal for both managers and investors that managers choose the most informative information structures. However, if they are costly to implement, then the question is whether the managers make socially optimal comparisons between the cost and the benefit to the economy as a whole of implementing alternative information structures.

The social benefit of more firm-specific information to managers is that it allows managers to make more efficient production decisions. That is, a

more efficient allocation of resources among firms and among projects within firms can be achieved, such that preferred vectors of expected aggregate consumption levels can be obtained (see Proposition 2.6 and Proposition 2.7). Since there is a large number of firms, the change of information structure for firm-specific events in any single firm has only a marginal impact on the vectors of expected aggregate consumption levels. Therefore, any individual manager's choice of information structure can be made as if the implicit state-contingent prices pertaining to the levels of expected aggregate consumption at $t=1$ and at $t=2$ do not depend on that choice. Thus, the cost and the benefits of changing information structures can be calculated using these prices. If the managers choose the information structures at $t=0$, then the benefit of an individual manager's change of information structure is the change in the market value at $t=0$ of the state-contingent investment at $t=1$ and the state-contingent output at $t=2$. Similarly, if the required resources of acquiring firm-specific information are a deduction in the firm's cash flow in the period from $t=0$ to $t=1$, i.e., a deduction in x_{j0} , then the cost of an information structure is the market value at $t=0$ of these required resources. Hence, the managers' information structures for firm-specific events are socially optimal, if the costs exceed the benefits of changing these information structures. In other words, the information structures are optimal, if they maximize the market values of firms at $t=0$ net of information acquisition costs.

In our economy, the concepts of costs and benefits of alternative information structures are well-defined. In general, this is only the case when investors are risk neutral (see, e.g., Demski (1980, ch. 3)). All economic agents are risk averse in our economy, but they are well-diversified at the dates, where the information structures influence the state-contingent dividends of firms (i.e., at $t=1$ and at $t=2$). This implies that the information acquisition decision of any individual manager has only a marginal effect on his own consumption plan and on that of any other economic agent. Since the utility functions are differentiable, all managers and all investors are risk neutral with respect to marginal changes in their state-contingent consumption plans. Thus, they can be considered as risk neutral with respect to any single manager's choice of information structure.

For efficient equilibrium consumption plans the managers' expected marginal utilities of state-contingent consumption are proportional to the implicit state-contingent prices pertaining to the levels of expected aggregate consumption (see Proposition 2.3). Their preferences for marginal changes in the their state-contingent consumption plans are therefore the same as those of investors. Since managers are only allowed to trade in well-diversified portfolios about which they have no superior information, their preferences for alternative information structures are determined by the resulting production plans. Hence, managers have no incentive to deviate from information structures for firm-specific events which maximize the market values of firms at $t=0$, and then, such information structures can be sustained as part of an equilibrium.

The investors' information about firm-specific events are determined by the information reported directly to them by managers and by the information that can be inferred from the dividends, given the investors' knowledge of the equilibrium production plan. The managers reporting firm-specific information to the investors is of no social value. Thus, if it requires resources to report information to the investors, all economic agents prefer that managers do not report any firm-specific information. The only information about firm-specific events available to investors is then the information that can be inferred from the dividends. Well-diversified portfolios will be of the same market value as they would have been if all information were available to investors. However, the market values of individual firms will in general deviate from their intrinsic values relative to the managers' information. Managers are not able to use their superior information about the firm-specific event of their own firm to obtain abnormal gains from speculative trading in their own firm, since they are only allowed to trade in well-diversified portfolios. Investors trade in well-diversified portfolios as well, but they do so voluntarily to insure themselves against firm-specific risk and to insure themselves against the differences between the market values and the intrinsic values of individual firms.

If the managers do not report firm-specific information to the investors, then any individual investor will realize that the managers have information which would enable him to gain from speculative trading, as long as

the other investors do not have that information. That is, the investors will be willing to spend resources in order to obtain private information about firm-specific events. An information structure to investors, which only reflects the information that can be inferred from the dividends, can therefore not be sustained as an equilibrium.

In that case, the investors will not restrict their portfolio plans to well-diversified portfolios. On the contrary, they will use resources to find over- and undervalued firms and invest accordingly in those firms. If only one investor engages in such activity, it might not influence the equilibrium market values of firms. However, all investors have incentives to obtain private information about firm-specific events, and then, the resulting changes in the supply and the demand of shares in firms will affect the equilibrium market values of firms. In the extreme, one could imagine that all available firm-specific information in the economy would be reflected in the equilibrium market values of firms, such that there would be no difference between the intrinsic and the market values of firms. That is, a fully revealing rational expectations equilibrium would obtain (see, e.g., Grossman (1981) for an introduction to this type of equilibria).

We do not formally pursue this line of argument here, but we note that this represents a socially inferior state of affairs, in that the investors spend resources to obtain socially useless information. Diamond (1985) analyzes this issue in a partly revealing rational expectations equilibrium model. The idea is that all economic agents can be made better off if the managers precommit themselves at $t=0$ to report their firm-specific information at $t=1$ such that the investors have no incentives to spend resources in order to obtain private firm-specific information. The gains arises because the investors would not want to spend resources in order to obtain private firm-specific information. Secondly, the risk sharing would be improved, since the investors would not have to take firm-specific risk as they would have to in order to obtain the gains from private information about firm-specific events. The costs are the resources used by managers to report their superior information to the investors. Clearly, these costs are lower than the cost incurred by investors if they have to obtain that information privately.

In this sense, it is of social value that the managers report the firm-specific information they obtain through the management of their firms.

PART II

Accounting Information and Moral Hazard
in
Efficient Capital Markets

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CHAPTER 6.

Introduction and Resumé of Part II

In the absence of insider trading and moral hazard problems it has been shown in Part I that reporting of firm-specific accounting information is of no social value in an efficient capital market. Investors are only concerned about current net dividends and information about general economic events that affect the composition and the market values of well-diversified portfolios. The basis for this result is that under the given assumptions it is possible to write compensation contracts which are both Pareto-efficient risk sharing contracts, and which provide the managers with economic incentives to use their private information in the production decisions in the same way as if that information was publicly available. In this setting a Pareto-efficient resource allocation with respect to the managers' superior information is obtained even if not all of this information is publicly available.

In Part II, one of the assumptions made in Part I is changed in order to analyze the demand for accounting information in an economy where it is impossible to align managers' and shareholders' preferences completely with respect to production decisions. The issue of motivating and controlling the managers of firms then becomes a central part of the analysis. This is known as the decision-influencing or stewardship demand for accounting information (see Demski and Feltham (1976) and Gjesdal(1981) for a discussion of this).

In order to model this economy it will be assumed, that the managers have a specific preference for certain production decisions in addition to the economic consequences of those decisions. If it is prohibitively expensive to report the production decisions credibly to the shareholders, a demand for reporting firm-specific accounting information is created. In particular, it is shown that this demand may take the form of a demand for a report on the cash flow of the previous period as well as a demand for an income measurement.

Managers of firms may have specific preferences for certain production decisions for a number of reasons. An assumption which has been used extensively in analytical models is that the manager of a firm has to supply effort in order to generate output and that he has a disutility for supplying more than the lowest possible amount. We adopt that assumption, such

that the production of a firm is determined by the state of nature, invested capital as well as of the manager's supply of effort.

If the manager's supply of effort is not observed by the shareholders, a moral hazard problem, well-known from the insurance literature, is created (e.g., the probability of a car accident depends on how carefully the insured drives the car). The heart of this problem is that the manager can only make a given supply of effort credible if he is given economic incentives through a compensation function (or bonus system) such that it is in his own interest to supply this level of effort.

In this agency model the optimal economic incentives are obtained through imposing more risk on the manager of the firm (the agent) than required by an optimal risk sharing equilibrium.¹ Therefore, the trade-off in the model is between improved economic incentives with respect to production decisions and an efficiency loss in the sharing of risk. The role of additional information in the standard agency model is that less risk has to be imposed on the agent to obtain the same supply of effort from the agent. In other words, the efficiency loss in risk sharing can be reduced on the basis of additional information (see Holmström (1979)).

The agency model is analyzed extensively in the managerial accounting literature.² However, analyses of the contracting relationships between top managers and shareholders have only received limited attention. Standard agency models have only one production factor which is the manager's supply of effort. This assumption is clearly too restrictive to capture an important aspect of the contracting relationship between the top manager and the shareholders of the firm, namely the process of acquiring new investment capital for the production process. Recognizing this fundamental aspect in an extended agency model, it is shown that the agency theory can be used to answer basic questions in accounting. That is, questions such as why the communication between the top management of the firm and the shareholders may take the form of an external accounting report, consisting of an audited report of the cash flow of the previous period and a report providing an income measurement.

If accounting information is considered as firm-specific information, there

is no demand for accounting information according to the model in Part I. Investors are only concerned about current net dividends which are equal to the difference between net cash flow of the previous period and current investment in the production process. It is important to recognize that investors are not concerned about the net cash flow of the previous period or current investments, they are only concerned about the difference. Hence, a report on the net cash flow of the previous period is of no value.

In the extended agency model, where the agent has a disutility for one of the production factors, namely his own supply of effort, such a demand exists. The reason is fairly obvious. When the manager's supply of effort is not observable to the shareholders, and when it can be substituted by capital from the shareholders, then the manager will clearly do so, unless he is motivated to do otherwise. The only way to create this motivation is to impose some firm-specific risk on the manager which is related to the sequence of net dividends. However, we know from Part I, that this is an inefficient risk sharing. We consider three types of accounting information which can reduce this inefficiency: Cash flow information reported by the manager (Chapter 8, section 8.1), cash flow information reported by an independent auditor (Chapter 8, section 8.2) and additional information reported by the manager (Chapter 9).

In Chapter 7 we formulate the incentive problem and characterize the optimal compensation functions for the case where the investors do not get any accounting information. The only variables, which can be observed by the shareholders, are the dividends. However, the optimal production decision rules are common-knowledge, and in particular, the optimal capital investment function used by the manager are known to the shareholders. Since the current dividend is equal to the cash flow of the previous period minus the capital investment, the observed dividend induces a partition on the set of the manager's private information. If this partition is complete, then the investors are in effect able to infer the private information received by the manager. Hence, when the manager is determining the optimal capital investment function he must take into account that different capital investment functions may reveal different information sets to the investors. That is, the capital investment decision has two roles to play, one as a productive decision and one as a communication device for his private in-

formation. This dual role of the capital investment decision is the basis for the analysis of the value of accounting information in Chapter 8 and Chapter 9.

In Chapter 8 we analyze a model where the cash flow of the previous period is the only private information the manager receives before he implements the production decisions. Cash flow accounting information is introduced in the model, first as reported by the manager (section 8.1) and next as reported by an independent auditor (section 8.2). The main issue analyzed in section 8.1 is whether it is of strict positive social value that the manager communicates the cash flow of the previous period to the investors.

First, the Revelation Principle is used to show that there is no loss of social value in assuming that the compensation functions are such that the manager reports the cash flow of the previous period truthfully and nothing more. Thus, in this case, it is shown that accrual accounting using any type of income measurement is of no value over cash flow accounting.

Secondly, we show in Proposition 8.6 that a necessary condition for a strict positive social value of communicating the cash flow of the previous period is that the optimal capital investment function with communication is such that the optimal dividend function is not an invertible function of the cash flow of the previous period. If the optimal dividend function with communication is an invertible function, then the optimal capital investment function is also optimal without communication. In this case, the investors are able to infer the manager's private information by observing the dividend and using their knowledge of the optimal capital investment function. On the other hand, if the optimal capital investment function without communication is such that the optimal dividend function is invertible, then this capital investment function might not be optimal with communication. The reason is that the optimal capital investment function without communication may be affected by its role as a communication device. That is, the manager may have to make a less efficient production decision in order to reveal his private information. In that case, it is of strict positive value for the manager to report the cash flow of the previous period, so that he can make more efficient capital investment decisions.

In Section 8.1.1 we give a number of examples that illustrate the general results. The examples show that one of the gains from revealing the manager's private information to the investors is that it facilitates a more efficient intertemporal risk sharing. Without communication of the cash flow of the previous period the trade-off is between the productive efficiency of the capital investment decision and the efficiency of the intertemporal risk sharing. That is, the manager will only reveal his private information through the dividend function if the loss of productive efficiency is less than the gains from a more efficient intertemporal risk sharing. With communication of the cash flow of the previous period, efficient intertemporal risk sharing can be achieved without affecting the productive efficiency of the capital investment decision.

In the previous analysis in Chapter 8 it has been assumed that the manager is not allowed to trade and, in particular, not allowed to lend or borrow money in the capital market. Hence, the manager can only obtain the optimal intertemporal risk sharing through the compensation scheme, and as shown by the examples in section 8.1.1, communication may be of value in that respect. Therefore, we consider in section 8.1.2 the consequences of allowing the manager to obtain the optimal intertemporal risk sharing through borrowing and lending in the capital market. In the case, where the manager receives perfect information before he implements the production decisions, it is shown in Proposition 8.8 that the manager can obtain the same gains from borrowing and lending in the capital market as he can with communication of his private information. This is so, even if his trades are not observable to the investors. Hence, without communication it is strictly preferred that the manager is able to borrow and lend in all those cases, where communication is of strict positive value without trading opportunities. With communication, the manager is indifferent as to whether or not he is able to trade.

These results depend critically on the assumption that the manager receives perfect information. If he does not receive perfect information and he is able to communicate, the manager strictly prefers to be able to precommit himself not to borrow or lend in the capital market (Proposition 8.9). The reason is that the optimal incentives in this case are obtained partly by leaving the manager with a residual desire to save some of his first period

compensation. Communication may still derive value from permitting a preferred intertemporal risk sharing, but the manager is not fully insured. Without communication the tradeoff is therefore between the gain from a fully efficient intertemporal risk sharing obtained through borrowing and lending, and the loss caused by the manager not being able to commit himself to retaining the optimal second best intertemporal risk sharing for incentive purposes. Example 8.5 demonstrates that cases exist without communication in which it is strictly preferred that the manager is able to borrow and lend. It is also demonstrated in that example that if the manager is not able to precommit himself not to borrow and lend, then communication can be valuable for incentive purposes, such that the manager does not have to make an inefficient production decision in order to reveal his private information.

In section 8.2 we consider the case where the cash flow of the previous period is reported by an independent auditor. The auditor always reports truthfully and it is assumed that he has incentives to do so. In effect, the cash flow of the previous period can be treated as public information. An audited report on the cash flow of the previous period might be of strict positive value even in the case where a report issued by the manager is of no value. The reason is that in this model we do not have any incentive problems associated with the reporting of cash flows, as we do when the manager is the one who is reporting the cash flow. When the cash flow of the previous period is public information, the capital investment can also be observed by the investors as the difference between the cash flow of the previous period and the observed dividend. Hence, in this model there is no moral hazard problem associated with the capital investment decision. However, the moral hazard problem associated with the effort decision has an impact on the optimal capital investment function. It is shown by an example that this can cause underinvestment as well as overinvestment depending on how the capital investment and the supply of effort are related in the production function.

In Chapter 9 we return to the general case where the manager observes the cash flow of the previous period as well as an additional information signal before he makes the production decisions. We assume that the cash flow of the previous period is public information, since cash flows are quite

easily checked by an auditor while this might not be the case with the manager's information about, e.g., future productivity of capital. Therefore, the analysis becomes similar to the analysis in section 8.1. The manager is now reporting the additional information instead of reporting the cash flow of the previous period, and the capital investment is now public information.

In section 9.1 we formulate the incentive problem and characterize the optimal contract. As in section 8.2 we show that although there is no moral hazard problem associated with the capital investment decision, the optimal capital investment function is affected by the moral hazard problem associated with the supply of effort. The incentive problem associated with the manager's communication of his private information also affects the optimal capital investment function. An example shows that overinvestment as well as underinvestment can occur since optimal investments depend on how the capital investment is related to both the supply of effort and the manager's private information. These examples show that there is no general result which states how optimal capital investments are affected by moral hazard problems associated with other production decisions and the manager's private information. It depends on the specific problem.

In section 9.2 we analyze the value of accounting information reported by the manager. This can be analyzed in a similar way as in section 8.1, such that a sufficient condition for no value of communication is that the optimal capital investment function with communication is an invertible function of the manager's private information given the cash flow of the previous period. However, if the capital investment function is smooth, this can only be the case if the manager's private information, ξ , is one-dimensional. The issue is then what happens in the general case where ξ is multi-dimensional.

The signals $\xi \in \Xi$ provide an abstract and complete representation of the manager's private information not conveyed by the cash flow of the previous period. However, accounting reports typically consist of a cash flow statement and an income measurement using some particular valuation scheme for the assets and the liabilities. One valuation scheme could be intrinsic values given the public information and the manager's private information.

An accounting principle can be viewed as a function from the set of signals received by the manager to a set of possible accounting reports. If that function is an invertible function on the set of signals received by the manager, then the set of accounting reports also provides a complete representation of the manager's information. Hence, the issue is whether a non-invertible accounting principle can achieve the same.

The main result in Chapter 9 is contained in Proposition 9.3. It gives sufficient conditions for a pre-specified accounting principle to yield the same gains from communication as full communication. The accounting principle must satisfy the following two conditions: First, the investors must be able to infer the relevant parts of the manager's private information from the combination of the accounting report and the observed capital investment through their knowledge of the accounting principle and the optimal capital investment function. Secondly, the optimal capital investment function must also be optimal if the manager were able to report all of his private information through the accounting report. If these two conditions are fulfilled then changing the accounting principle to a more informative accounting principle is of no value.

The first condition reflects that accounting information is not the only type of information available to the investors. Observable production decisions also provide the investors with information through their knowledge of the manager's production incentives, given his compensation functions and his private information structure. The second condition reflects that the accounting principle must allow the manager to communicate so much of his private information that the production decisions are not affected by their role as a communication device.

Part II is concluded in section 9.3 with some concluding remarks on the analysis in Part II, a discussion of some possible generalizations of the model, and some topics for further research.

Footnotes for Chapter 6.

1. The solution to the agency problem is in this case referred to as the second best solution as opposed to first best solution, which is the cooperative solution, where the production decisions as well as the risk sharing are unconstrained Pareto efficient.
2. See Baiman (1982, 1989) for surveys of this literature. Feltham (1984) surveys information economic research and agency theory with implications for financial accounting.

CHAPTER 7.

The Basic Model

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In this chapter we formulate the incentive problem in section 7.1 and characterize the optimal compensation functions in section 7.2 for the case where the investors do not get any accounting information. By comparing these characterizations to those obtained in Christensen (1981) it appears that the capital investment decision has two roles to play: one as a productive decision and one as a communication device for the manager's private information. This result is used in later chapters to study the value of accounting reports by which the manager is able to communicate his private information directly.

7.1. Description of the incentive problem.

The model, the assumptions, and the notation are very similar to the 2-period state-preference model analyzed in Part I. The main differences are the following. It is assumed that the production function for output at date $t=2$ can be simplified so that it only depends on the state, the capital investment and the supply of effort,

$$x_j(s_0, s_j, q_j, a_j),$$

where q_j is the amount of capital invested at date $t=1$ and a_j is the manager's supply of effort in the period from date $t=1$ to date $t=2$.¹ The amount of capital invested has a direct effect on the cash flow to the investors at date $t=1$, because the dividend is equal to the difference between the cash flow of the previous period, $x_{0j}(y_0, y_j^m)$, and the capital investment, $q_j(y_0, y_j^m)$:

$$d_j(y_0, y_j^I) = x_{0j}(y_0, y_j^m) - q_j(y_0, y_j^m)$$

The manager's supply of effort has no cash flow implications at date $t=1$. For simplicity, we assume that the support of the dividend at $t=2$ is independent of the supply of effort as well as of the amount of capital invested.

We assume that the manager has a disutility for the supply of effort:

(A5)' The managers' preferences are represented by utility functions which are additively separable in current consumption, future consumption and supply of effort, i.e.,²

$$U_j^m(c_j^m, a_j) = \sum_{s \in S} \{ u_{j1}^m(c_{j1}^m(s)) + u_{j2}^m(c_{j2}^m(s)) - H_j(a_j(s)) \} \cdot f(s)$$

where

$$u_{jt}^{m'}(\cdot) > 0, u_{jt}^{m''}(\cdot) < 0 \quad \text{for } t = 1, 2 \quad \text{and } H_j'(\cdot) > 0.$$

The investors' preferences are represented by time-additive utility functions over current and future consumption as in (A5).

The managers have a specific preference for one of the production factors, namely a disutility for supplying more than the lowest possible amount of effort. Assuming that the marginal productivity of capital as well as of effort are both strictly positive, an incentive problem for the managers' supply of effort is created. When the managers' supply of effort are not observable and managers are risk averse, then a full Pareto-efficient allocation is not attainable. Only a so-called second best solution is attainable (see Holmström (1979)). In this chapter, the model with no accounting information is characterized, and in the following chapters, the implications of introducing accounting information are analyzed.

The sequence of the contracting process between shareholders and managers is the same as in Part I, and it is shown in Figure 7.1.

The decision problem is that of choosing the contingent plan $(M_{j1}(\cdot), M_{j2}(\cdot), q_j(\cdot), a_j(\cdot))$ for all firms at time $t=0$. The plan must be sequentially rational, such that all plans are credible to all parties in the sequential game of making production decisions and paying the managers their management compensation. This means that the compensation functions $(M_{j1}(\cdot), M_{j2}(\cdot))$ can only be contingent on variables known to both the shareholders and the managers at the time of payment. Other plans are not enforceable. The production decision rules $(q_j(\cdot), a_j(\cdot))$, however, are plans contingent on public information as well as on the managers' private

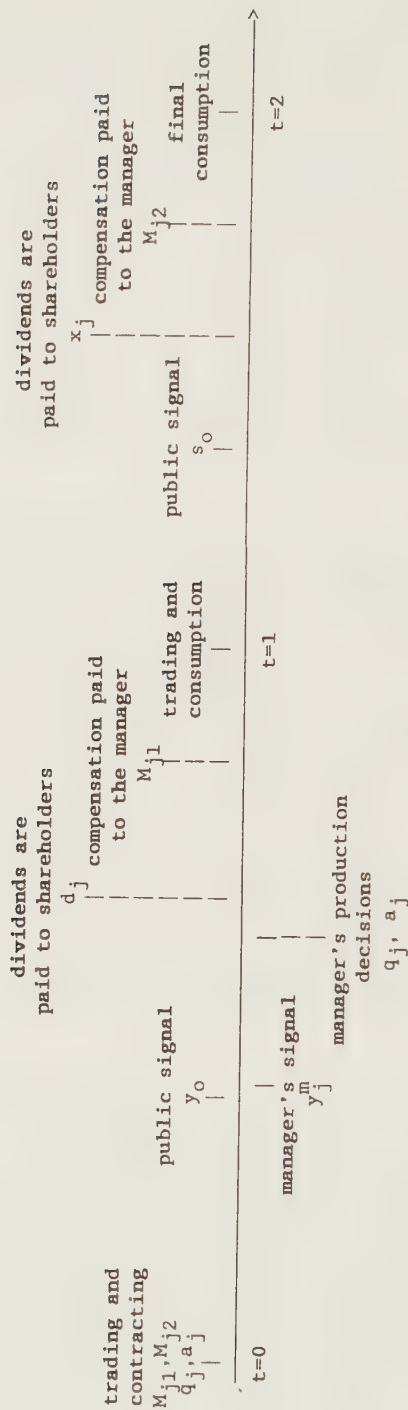


Figure 7.1: Sequence of the contracting process.

information. It is not possible nor optimal for the managers to commit themselves not to use their private information in the production decisions. Hence, in order to be credible to the shareholders, these rules and the compensation functions must be such that the managers have no incentive to deviate from these production decisions, when they get their private information y_j^m .

In the case of no accounting information only dividends and general information from other sources than accounting reports are known to both the shareholders and the managers. Hence, the feasible compensation functions are of the form:

$$M_{j1} = M_{j1}(y_0, d)$$

$$M_{j2} = M_{j2}(s_0, d, x)$$

where d and x are the dividends of all firms and y_0 and s_0 are economy-wide information. The focus of analysis is changed from the market as a whole to the incentive problem between the shareholders and the manager of an arbitrary firm. This can be done on the basis of the assumption that information not conveyed by the economy-wide information is firm-specific, and the assumption that firm-specific information for a particular firm does not convey information about the production conditions of any other firm (i.e., (A2)). This implies that only contracts based on economy-wide information and the dividends for the particular firm need to be considered (see Holmström (1982)). Hence, the subscript related to the firm can be omitted and the possible contracts can be written as

$$M_1 = M_1(y_0, d)$$

$$M_2 = M_2(s_0, d, x)$$

where d and x are the dividends for the firm in question.

In order to simplify the analysis of the contracting process we make the following assumptions:³

(B1) There are no economy-wide events.

(B2) The riskless interest rate is zero.

(B3) Managers are not endowed with shares in firms or allowed to trade in the capital market.

Assumption (B1) and the assumption of a sufficiently large number of independent firms (i.e., (A6)) imply that all risk can be diversified by investors. Since the shareholders have homogeneous beliefs (i.e., (A1)), they act as a risk-neutral syndicate relative to a particular firm (see Wilson (1968)). When the riskless interest rate is zero, (B2), the objective of a risk-neutral syndicate is to maximize the expected value of the payments to the syndicate. This allows a very compact representation of the shareholders' preferences, namely to maximize the expected value of the dividends net of the compensation to the manager. This, on the other hand, is the same as maximizing the market value of the firm at $t=0$, i.e.,

$$V_0 = E[E[d(\cdot) - M_1(d(\cdot)) | y^m, q(y^m), a(y^m)]] \\ + E[E[x(\cdot) - M_2(d(\cdot), x(\cdot)) | y^m, q(y^m), a(y^m)]],$$

where the inner expectation is the expected value conditional on the manager's information and given the production decision rules, $q(\cdot)$ and $a(\cdot)$. To get the unconditional expected value, expectation is taken over the manager's information.⁴

The Pareto-frontier for the manager and the shareholders can then be parameterized by the market value of the firm at time $t=0$, the time of contracting.

Assumption (B3) implies that the manager's consumption possibilities are given by his management compensation at $t=1$ and at $t=2$.

Hence, for a given market value of the firm at $t=0$, V , a Pareto-optimal plan can be defined as follows.

Definition 7.1

The contract $(M_1^*(d), M_2^*(d, x), q^*(y^m), a^*(y^m))$ is Pareto-optimal if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} & E[E[u_1(M_1(d(\cdot))) | y^m, q(y^m), a(y^m)]] \\ & + E[E[u_2(M_2(d(\cdot), x(\cdot))) | y^m, q(y^m), a(y^m)]] \\ & - E[H(a(\cdot))] \end{aligned}$$

subject to

$$\begin{aligned} & E[E[d(\cdot) - M_1(d(\cdot)) | y^m, q(y^m), a(y^m)]] \\ & + E[E[x(\cdot) - M_2(d(\cdot), x(\cdot)) | y^m, q(y^m), a(y^m)]] = \nabla \end{aligned}$$

$$\begin{aligned} \forall y^m: \{q(y^m), a(y^m)\} \in \operatorname{argmax}_{(q, a) \in Q \times A} & \{ E[u_1(M_1(d(\cdot))) | y^m, q, a] \\ & + E[u_2(M_2(d(\cdot), x(\cdot))) | y^m, q, a] - H(a) \} \end{aligned}$$

Since the utility functions of the investors no longer appear in the problem, the superscript on the manager's utility function is omitted.

The program defining the Pareto-optimal contracts is an optimal control problem in which the control variables are the compensation functions and the production decision rules. The interpretation of the problem is as follows. Before the manager gets any information, he selects these control variables such that his own expected utility is maximized. He does this under the constraint that the shareholders get a given expected utility corresponding to a given market value of the firm. Furthermore, the compensation functions must be such that when the manager gets his information y^m at $t=1$, it is in his self-interest to implement the production decisions specified by the decision rules $q(y^m)$ and $a(y^m)$.

It is the last constraint that captures the essence of the incentive problem. The decision rules $q(\cdot)$ and $a(\cdot)$ determined at $t=0$ are only credible to the shareholders, if the manager is given economic incentives through his compensation to implement these. This is known in the literature as incentive compatibility of the production decision rules, and it corresponds to the definition of incentive compatible production plans in Part I.

If the capital investment, q , and the manager's supply of effort, a , are public information, and if sufficient penalties are available, then the manager can commit to given production decisions at time $t=0$. However, even if q and a are observable such contracts are not optimal in general, since the manager is not able to change the production decisions according to useful information conveyed by his private information y^m . Only when it is known that no signal from the manager's private information structure conveys productivity information, can a full Pareto-optimal contract of the penalty type be written. In that case, the first best production decisions do not vary with the manager's private information. Hence, in the case with private information to the manager, the incentive problem is not only a matter of the observability of the manager's decisions (as in Holmström (1979)) but also a problem of how the information is used by the manager in his production decisions.

The formulation of the contracting problem in Definition 7.1 involves several precommitments both on the part of the manager and on the part of the shareholders. First, the manager precommits himself not to leave the firm, when he gets his information. This is a critical assumption, because, in general, the manager's expected utility conditional on his information depends on the particular signal he gets. In the absence of any moral hazard problems, as in Part I, this precommitment is not an issue. In that case, the optimal compensation functions do not depend on firm-specific information. Secondly, the manager and the shareholders precommit themselves not to recontract after time $t=0$. This is so even if it would be beneficial for both the manager and the shareholders at any later date. At time $t=1$, when the manager announces the dividend, all production decisions are made, and only the sharing of risk remains to be carried out. Since a full Pareto-optimal risk sharing is characterized by no firm-specific risk to any economic agent, it would be Pareto-preferred to recontract at that time. How-

ever, if no precommitment not to recontract is made, the manager has incentives to change the production decision rules $q(\cdot)$ and $a(\cdot)$. Because he is able to recontract, he will only supply the lowest possible amount of effort.

Definition 7.1 describes the general incentive problem with private information to the manager and no accounting information to the shareholders. Before the optimal contract is characterized in the following section the problem is reformulated according to the approach by Mirrlees (1974) and Holmström (1979). Furthermore, some additional assumptions of a technical nature are made. The state-space approach used in Part I is changed. The dividends' dependence of production decisions and the firm-specific signals and events are suppressed. Instead, the dividends are viewed as random variables with conditional distribution functions,

$$F(d|x_0, \xi, q(x_0, \xi), a(x_0, \xi))$$

$$F(d, x|x_0, \xi, q(x_0, \xi), a(x_0, \xi))$$

where the conditioning is on all available information and the production decision rules. The interpretation of ξ is that it measures the information to the manager not conveyed by the cash flow of the previous period. It is assumed, that (x_0, ξ) belongs to some subset, $X_0 \times \Xi$, of $\mathbb{R} \times \mathbb{R}^n$. This assumption is made in order to have a clear distinction between cash flow information and other types of information. Moreover, this formulation facilitates the mathematical analysis of the problem. The density functions corresponding to the distribution functions $F(\cdot)$ are denoted by $f(\cdot)$. From the definition of the dividend at time $t=1$, $d = x_0 - q$, the conditional density functions simplify to

$$f(d|x_0, \xi, q, a) = \begin{cases} 1 & \text{if } d = x_0 - q \\ 0 & \text{otherwise} \end{cases}$$

and

$$f(d, x|x_0, \xi, q, a) = \begin{cases} f(x|x_0, \xi, q, a) & \text{if } d = x_0 - q \\ 0 & \text{otherwise} \end{cases}$$

With this formulation of the uncertainty in the economy we can formulate the incentive problem in Definition 7.1 as follows.

Definition 7.2 Pareto-optimal contracts without accounting information.

The contract $(M_1^*(d), M_2^*(d,x), q^*(x_0, \xi), a^*(x_0, \xi))$ is Pareto-optimal given no accounting information if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} & \int_{X_0} \int_{\Xi} u_1(M_1(x_0 - q(x_0, \xi))) \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 \\ & - \int_{X_0} \int_{\Xi} H(a(x_0, \xi)) \cdot f(x_0, \xi) \, d\xi dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \int_{\Xi} \{ x_0 - q(x_0, \xi) - M_1(x_0 - q(x_0, \xi)) \} \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X \{ x - M_2(x_0 - q(x_0, \xi), x) \} \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 = 0 \end{aligned}$$

$$\forall (x_0, \xi) \in X_0 \times \Xi : \{q(x_0, \xi), a(x_0, \xi)\} \in \operatorname{argmax}_{(q, a) \in Q \times A} \{ u_1(M_1(x_0 - q))$$

$$+ \int_X u_2(M_2(x_0 - q, x)) \cdot f(x | x_0, \xi, q, a) \, dx - H(a) \}$$

This definition of the incentive problem will be used in the propositions concerning the value of accounting information in later chapters.

The self-selection constraint in the optimization problem makes it difficult to characterize the optimal contract. It is therefore assumed, that it can be substituted with its corresponding first order condition for maximum. This can be done when the manager's conditional expected utility is a concave function of the production choices (q,a) and it attains its maximum in the interior of $Q \times A$. Rogerson (1985b) gives sufficient conditions for the expected utility to be concave in the standard agency model. The first condition is a so-called monotone likelihood ratio condition. That condition implies the first order stochastic dominance conditions, i.e.,

$$\left. \frac{\partial F}{\partial q} (x | x_0, \xi, q, a) \right|_{\substack{q=q(x_0, \xi) \\ a=a(x_0, \xi)}} < 0$$

and

$$\left. \frac{\partial F}{\partial a} (x | x_0, \xi, q, a) \right|_{\substack{q=q(x_0, \xi) \\ a=a(x_0, \xi)}} < 0$$

These conditions correspond well to the assumption of a positive marginal productivity of investment capital as well as of the manager's supply of effort. The second condition is the convex distribution function condition which is a form of stochastic diminishing returns to scale. Whether these conditions are sufficient in our extended agency model is an open issue. Besides, these conditions are not necessary conditions even in the standard agency model. Therefore, we state as an assumption, that the first order approach is applicable. However, the proofs of all propositions concerning the value of accounting information in later chapters do not rely on this assumption.

Given this assumption, we can formulate the incentive problem as follows.

Definition 7.3 Pareto-optimal contracts without accounting information.

The contract $(M_1^*(d), M_2^*(d, x), q^*(x_0, \xi), a^*(x_0, \xi))$ is Pareto-optimal given no accounting information, if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} & \int_{X_0} \int_{\Xi} u_1(M_1(x_0 - q(x_0, \xi))) \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 \\ & - \int_{X_0} \int_{\Xi} H(a(x_0, \xi)) \cdot f(x_0, \xi) \, d\xi dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \int_{\Xi} \{ x_0 - q(x_0, \xi) - M_1(x_0 - q(x_0, \xi)) \} \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X \{ x - M_2(x_0 - q(x_0, \xi), x) \} \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 = 0 \end{aligned}$$

$$\begin{aligned} \forall (x_0, \xi) \in X_0 \times \Xi: & u_1'(M_1(x_0 - q(x_0, \xi))) \cdot M_{1q}(x_0 - q(x_0, \xi)) \\ & + \int_X u_2'(M_2(x_0 - q(x_0, \xi), x)) \cdot M_{2q}(x_0 - q(x_0, \xi), x) \\ & \quad \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \, dx \\ & + \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f_{aq}(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \, dx = 0 \end{aligned}$$

$$\begin{aligned} \forall (x_0, \xi) \in X_0 \times \Xi: & \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f_a(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \, dx \\ & - H'(a(x_0, \xi)) = 0 \end{aligned}$$

where the subscripts on the functions denote the partial derivatives with respect to the variable indicated by the subscript.

The first order approach used in this definition is not always applicable. An optimal solution to the program is not guaranteed to exist, and even if it does exist, there might be other contracts which are strictly Pareto-preferred to that solution. Therefore, to summarize the assumptions for this formulation of the incentive problem we make the assumption:

- (B4) There exist an optimal solution to the program in Definition 7.3, and there is no other contract which is strictly Pareto-preferred to that solution given no accounting information to the investors. Furthermore, the density functions and the contracts are differentiable and integrable in all arguments.

This completes the description of the basic model of the incentive problem with no accounting information.

7.2 Pareto-optimal compensation functions with no accounting information.

In this section we characterize the Pareto-optimal compensation functions for the incentive problem with no accounting information. That is, we give necessary conditions for the compensation functions to be Pareto-optimal. Furthermore, these characterizations are compared to those obtained in well-known agency models.

For the standard agency model Holmström (1979), among others, uses the technique of point-wise optimization of the Lagrangian for the incentive problem to characterize the optimal compensation functions. That is, characterizing stationary points for the Lagrangian for each value of the dividends. This solution technique cannot be used for the problem in Definition 7.3, because the self-selection constraint for the capital investment includes terms with the partial derivatives of the manager's compensation with respect to the capital investment, as e.g.,

$$\begin{aligned} M_{1q}(x_0 - q(x_0, \xi)) &= \frac{dM_1}{dd} \cdot \frac{\partial d}{\partial q} \Big|_{q=q(x_0, \xi)} \\ &= - M_{1d}(x_0 - q(x_0, \xi)) \end{aligned}$$

The more general technique of calculus of variations must be used.⁵ The following proposition characterizes Pareto-optimal compensation functions given that the incentive compatibility constraints can be substituted with the first order conditions. Since the proof does not provide any easy intuition into the basic elements of the incentive problem we have placed the proof in an appendix to Part II.

Proposition 7.1

Let $(M_1^*(d), M_2^*(d, x), q^*(x_0, \xi), a^*(x_0, \xi))$ be a Pareto-optimal contract for the incentive problem with no accounting information. Define the set of signals (x_0, ξ) which gives the same dividend d given the optimal capital investment function:

$$X_0 \times \Xi | d = \{ (x_0, \xi) \in X_0 \times \Xi \mid d^*(x_0, \xi) = x_0 - q^*(x_0, \xi) = d \text{ for } d \in D \}$$

If the set

$$\{ (x_0, \xi) \in X_0 \times \Xi | d_{x_0}^*(x_0, \xi) = 1 - q_{x_0}^*(x_0, \xi) = 0 \}$$

has zero probability with respect to the induced density function over the set $X_0 \times \Xi | d$, i.e. $f(x_0, \xi | d, q^*(x_0, \xi))$, then

$$(7.1) \quad 1/u_1^*(M_1^*(d)) = 1/\lambda \cdot \left\{ 1 + \frac{\int_{X_0 \times \Xi | d} \gamma_{x_0}^*(x_0, \xi) \cdot f(x_0, \xi) d\xi dx_0}{f(d | q^*)} + \frac{\int_{X_0 \times \Xi | d} \gamma(x_0, \xi) \cdot f_{x_0}(x_0, \xi) d\xi dx_0}{f(d | q^*)} \right\}$$

$$(7.2) \quad 1/u_2^*(M_2^*(d, x)) =$$

$$\begin{aligned} & 1/\lambda \cdot \left\{ 1 + \frac{\int_{X_0 \times \Xi | d} \gamma_{x_0}(x_0, \xi) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0}{f(d, x | q^*, a^*)} \right. \\ & + \frac{\int_{X_0 \times \Xi | d} \gamma(x_0, \xi) \cdot f_{x_0}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0}{f(d, x | q^*, a^*)} \\ & + \frac{\int_{X_0 \times \Xi | d} \gamma(x_0, \xi) \cdot a_{x_0}^*(x_0, \xi) \cdot f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0}{f(d, x | q^*, a^*)} \\ & + \frac{\int_{X_0 \times \Xi | d} \gamma(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0}{f(d, x | q^*, a^*)} \\ & \left. + \frac{\int_{X_0 \times \Xi | d} \mu(x_0, \xi) \cdot f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0}{f(d, x | q^*, a^*)} \right\} \end{aligned}$$

where λ , $\zeta(x_0, \xi)$ and $\mu(x_0, \xi)$ are optimal Lagrange-multipliers and

$$\gamma(x_0, \xi) \equiv \zeta(x_0, \xi) / d_{x_0}^*(x_0, \xi)$$

Proof: see the appendix.

The characterizations of the compensation functions given in Proposition 7.1 are unfortunately too complex to give an easy interpretation. Some intuition, however, can be gained from comparing them to results from well-known agency models.

As expected, the contract is only a second best solution, since the manager's compensation depends non-trivially on the dividends of the firm. That is, the manager has to take some firm-specific risk related to the sequence of dividends to make the production decision rules $q^*(\cdot)$ and $a^*(\cdot)$ credible to the shareholders. Furthermore, the form of the characterizations has some similarities to the communication model by Christensen (1981). In his model, the problem is to write a compensation contract for the manager that induces him to report his private information (ξ) truthfully to a principal. In our model, x_0 and ξ are partly revealed to the investors through the observable dividend at time $t=1$ and through the common-knowledge of the optimal capital investment decision rule $q^*(\cdot)$. This is explored further below.

Suppose for the moment that the set $X_0 \times \Xi | d$ is a singleton for some dividend d at time $t=1$. That is, there is only one pair of signals to the manager (x_0, ξ) that gives the same dividend d given the optimal capital investment decision rule $q^*(\cdot)$. In this case, the compensation functions are characterized by

$$(7.3) \quad 1/u'_1(M_1^*(d)) = 1/\lambda \cdot \left\{ 1 + \gamma_{x_0}(x_0, \xi) + \gamma(x_0, \xi) \cdot \frac{f_{x_0}(x_0, \xi)}{f(x_0, \xi)} \right\}$$

$$(7.4) \quad 1/u'_2(M_2^*(d, x)) =$$

$$\begin{aligned} & 1/\lambda \cdot \left\{ 1 + \gamma_{x_0}(x_0, \xi) + \gamma(x_0, \xi) \cdot \frac{f_{x_0}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right. \\ & \quad + \gamma(x_0, \xi) \cdot a_{x_0}^*(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \\ & \quad \left. + \gamma(x_0, \xi) \cdot \frac{f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right\} \end{aligned}$$

$$+ \mu(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \Bigg\}$$

The last two terms for the second period are the usual terms for a model with two productive decisions, namely, the derivative of the log-likelihood function with respect to each of the productive decisions times the respective optimal Lagrange-multipliers. Given that $X_0 \times \Xi | d$ is a singleton, the manager's private signals (x_0, ξ) are in effect also known to the shareholders when they observe the dividend d . These two terms can then be given the usual interpretation known from standard agency models, namely as a measure of the likelihood that the manager in fact implemented the production decisions $q^*(x_0, \xi)$ and $a^*(x_0, \xi)$ based on the observed dividend x in the second period and (x_0, ξ) . However, this interpretation should not be carried too far, since the investors know that the production decisions $q^*(x_0, \xi)$ and $a^*(x_0, \xi)$ will be implemented by the manager given the compensation functions. The intuition is, that the compensation must be related to these likelihood-ratios to assure that the manager in his self-interest implements the production decisions $q^*(x_0, \xi)$ and $a^*(x_0, \xi)$.

The other terms are due to the specific nature of the capital investment decision given through the relation

$$d = x_0 - q.$$

A marginal change in the capital investment has the same effect on the observable variable d on which the compensation functions are written as an opposite marginal change in the cash flow of the previous period. In effect, the manager selects his compensation at $t=1$ and a particular compensation function from the menu of compensation functions for the second period, i.e., an element of the vector function $\{M_2^*(\cdot, x)\}_{d \in D}$, by his choice of capital investment. This distinguishes the capital investment decision from the effort decision, because the effort decision has no impact on current dividends and on the support of future dividends.

This specific nature of the capital investment decision is the basis for our characterizations of the optimal compensation functions being similar to that obtained in the communication model by Christensen (1981). In his

one-period model (with no cash flow of the previous period and no capital investment decision) the issue is to write a compensation contract that induces an agent to report his private information (ξ) truthfully to a principal. This is obtained by offering the agent a menu of contracts parameterized by the agent's choice of message to the principal. Using the so-called Revelation Principle (see, e.g., Myerson (1979) and below) Christensen shows that there is no loss of social wealth in restricting the contracts to be truth-inducing, given the menu of contracts. The characterization of the compensation function obtained by Christensen is

$$\begin{aligned} 1/u'_2(M_2^*(\xi, x)) = & 1/\lambda \cdot \left\{ 1 - \rho_\xi(\xi) - \rho(\xi) \cdot \frac{f_\xi(\xi, x | a^*(\xi))}{f(\xi, x | a^*(\xi))} \right. \\ & - \rho(\xi) \cdot a_\xi^*(\xi) \cdot \frac{f_a(x, \xi | a^*(\xi))}{f(x, \xi | a^*(\xi))} \\ & \left. + \mu(\xi) \cdot \frac{f_a(x, \xi | a^*(\xi))}{f(x, \xi | a^*(\xi))} \right\} \end{aligned}$$

where $\rho(\xi)$ is the Lagrange-multiplier for the constraint, that the manager reports truthfully in his self-interest, given the menu of compensation functions $\{ M_2^*(\cdot, x) \}_{\xi \in \Xi}$. When $X_0 \times \Xi | d$ in our model is a singleton, the decision problem for the manager is almost identical to a communication setting, since the capital investment decision in our model and a message decision in a communication model both serve as devices for communicating the manager's private information. However, there is one essential difference, namely that the capital investment decision is a productive decision. That is, the probability distribution of output at date $t=2$ depends on this decision, while this is not the case for a message decision in a communication model. Thus, the capital investment decision plays two roles, one as a decision that communicates the manager's private information and one as a productive decision. The second role is reflected by an additional term compared to the characterization obtained by Christensen,

$$\gamma(x_0, \xi) \cdot \frac{f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))},$$

and the fact that the Lagrange multiplier for the capital investment decision constraint, $\zeta(x_0, \xi)$, is $\gamma(x_0, \xi)$ times $(1 - q_{x_0}^*(x_0, \xi))$ reflecting that a change in the cash flow of the previous period also changes the optimal capital investment.

The discussion so far has been based on the assumption, that $X_0 \times \Xi | d$ is a singleton. When this is not the case a weighted average is taken in (7.3) and (7.4) over the set of private signals (x_0, ξ) that gives the observable dividend d given the optimal capital investment decision rule $q^*(\cdot)$. The averaging reflects that the observable dividend d and the common-knowledge of $q^*(\cdot)$ only provide the investors with incomplete knowledge about the manager's private signals when $X_0 \times \Xi | d$ is not a singleton.

For a given optimal capital investment decision rule, $q^*(\cdot)$, the dividend d can be written as a function of the manager's private signals (x_0, ξ) ,

$$d = d^*(x_0, \xi) = x_0 - q^*(x_0, \xi).$$

Hence, the dividend function is defined on the set $X_0 \times \Xi$ which is at least a 2-dimensional set if the manager has more private information than the cash flow of the previous period (i.e., Ξ is not a singleton). Since we are assuming that $q^*(\cdot)$ is a smooth and differentiable function, the inverse of the dividend function is a correspondence rather than a function. That is, the set $X_0 \times \Xi | d$ cannot be a singleton, when the manager has more private information than the cash flow of the previous period. Therefore, the observable dividend and the common-knowledge of $q^*(\cdot)$ only provide incomplete information, in general, to the investors. This implies, that it should be possible for the manager to report more information to the investors through, e.g., accounting reports, than already conveyed by the dividend and the common-knowledge of the capital investment decision rule. This issue is analyzed further in Chapter 9.

When the manager has no other private information than the cash flow of the previous period (i.e., $X_0 \times \Xi = X_0$) it is possible that $X_0 | d$ is a singleton for all dividends d . In that case, the issue is, whether it is of positive value to let the manager report his cash flow information directly to the investors through an accounting report rather than conveying this information indirectly by the dividend and the common-knowledge of $q^*(\cdot)$. To get

some insight into that problem, a communication model will be analyzed in Chapter 8. In that model, the manager issues an accounting report based on his private knowledge of the cash flow of the previous period.

7.3 Conclusion.

The main issue of this chapter has been the definition of the incentive problem with no accounting information to the investors in section 7.1, and the characterization of the optimal compensation functions in section 7.2.

When investors do not get any accounting information from firms, only dividends can be contracted upon. It is assumed that there are no economy-wide events, such that the dividends are independent between firms. Given this assumption, the allocation problem in the economy can be separated into one incentive problem for each firm. In that case, the compensation functions for any single manager depend only on the dividends of his own firm. The incentive problem is formulated as the manager's decision problem. Before the manager gets any information, he selects the compensation functions and the production decision rules, such that his own expected utility is maximized. He does this under the constraint that a given market value of the firm must be obtained. Furthermore, the compensation functions must be such that he has no incentives to deviate from the production decision rules, when he gets his private information. This reflects that the production decision rules are incentive compatible, such that they will be reflected in the market value of the firm.

The manager's private information consists of his knowledge of the cash flow of the previous period and an additional information signal which aggregates the private information not conveyed by the cash flow of the previous period. In general, the manager's production decisions depend on both of these types of private information.

Characterizations of the optimal compensation functions are given in section 7.2. These show that the manager has to take some firm-specific risk related to the sequence of dividends in order to make the production decision rules incentive compatible. Comparing the characterizations to those

obtained in the communication model by Christensen (1981) shows that the capital investment decision has some similarities to the decision of communicating the manager's private information. This is due to the fact that the dividend at $t=1$ reveals some of the manager's private information through the common-knowledge of the optimal capital investment decision rule, and the fact that the dividend is equal to the cash flow of the previous period minus the capital investment. However, the capital investment decision is also a productive decision. It is this dual role of the capital investment decision that is the basis for the analysis of the value of cash flow accounting information in the next chapter. In chapter 9 this dual role is also the basis for the analysis of the value of accounting reports which also provide income measurements.

Footnotes for Chapter 7.

1. In Part I, a_j denoted general production decisions and q_j denoted the capital investment required to implement those decisions. In this part of the dissertation, the production technology is more simple in the sense that the production decisions can be represented by the amount invested, q_j , and the manager's supply effort, a_j , which does not require any capital investment.
2. We assume additive separability in consumption and supply of effort, because otherwise randomized contracts might be necessary to achieve Pareto optimality (see Gjesdal (1976)).
3. The assumptions that there are no economy-wide events and that the riskless interest rate is equal to zero are not critical assumptions for the analysis as long as the information about economy-wide events is publicly available and the capital market permits efficient risk sharing among investors. Atkinson and Feltham (1983) consider a model with firm-specific events as well as economy-wide events. The third assumption that the managers are not allowed to trade in the capital market is a critical assumption. This is due to the fact that in intertemporal models with moral hazard problems, the optimal incentives are in most cases obtained partly by leaving the manager with a desire to save some of his compensation (see Rogerson (1985a)). Hence, it is Pareto-preferred that the manager is able to precommit himself not to trade in the capital market. However, the manager might not be able to make such a precommitment. Moreover, when the manager has private information and he is not able to communicate that information to the investors, there exist cases where it is Pareto-preferred that the manager is able to trade in the capital market (see section 8.1.2).
4. Note that it is an implicit assumption in this formulation that the production decision rules and the manager's information structure are known to the shareholders.
5. An intuitive but also rigorous reference to this technique is Luenberger (1969). For an application to agency models see Christensen (1981).

CHAPTER 8.

Cash Flow Accounting Information

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In this chapter cash flow accounting information is introduced in the model. First as reported by the manager and next as reported by an independent auditor. Throughout this chapter it is assumed, that the cash flow of the previous period is the only private information the manager receives, before he chooses the production decisions. The main issue analyzed is whether cash flow accounting information can improve upon the second-best solution of the incentive problem with no accounting information. The main result is given in Proposition 8.6. It states that cash flow accounting information reported by the manager is of no value if the optimal dividend function with accounting information is an invertible function of the cash flow of the previous period.

8.1 Cash flow information reported by the manager.

In this section we analyze the social value of introducing an accounting report at $t=1$. The accounting report is made by the manager based on his observation of the cash flow of the previous period. In this situation, the manager has an additional decision variable, namely, what to report or communicate for each possible value of the cash flow of the previous period.

An accounting principle or a communication structure is defined as a function $m(\cdot)$ from X_0 to some set of messages M :

$$m: X_0 \rightarrow M$$

That is, for each given value of the cash flow of the previous period, x_0 , observed by the manager, an accounting report is issued consisting of a message $m(x_0) = m$. The message $m \in M$ can be almost anything. One possibility is that the message is simply the reported cash flow. Another possibility is that the accounting principle is an accrual accounting principle in which the cash flow of the previous period is reported together with values of the assets and liabilities of the firm. Hence, we do not restrict M to be an one-dimensional set.

The observable variables by the investors are the dividend d and the mes-

sage m at $t = 1$, and the final dividend x at $t = 2$. Feasible compensation functions can therefore be written on the form

$$M_1 = M_1(d, m)$$

$$M_2 = M_2(d, m, x)$$

Given that the manager has observed x_0 and reported m , his production decisions are denoted by

$$q = q(x_0, m)$$

$$a = a(x_0, m)$$

For given compensation functions, the investors know that the manager selects his three decision variables, the report and the two production decisions, so as to maximize his own expected utility conditional on his private information. The incentive problem with communication of cash flow information can then be formulated as

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ m(\cdot), q(\cdot), a(\cdot)}} & \int_{X_0} u_1(M_1(x_0 - q(x_0, m(x_0))), m(x_0)) \cdot f(x_0) \, dx_0 \\ & + \int_{X_0} \int_X u_2(M_2(x_0 - q(x_0, m(x_0))), m(x_0), x) \cdot f(x | x_0, q(x_0, m(x_0)), a(x_0, m(x_0))) \\ & \quad \cdot f(x_0) dx dx_0 \\ & - \int_{X_0} H(a(x_0, m(x_0))) \cdot f(x_0) \, dx_0 \end{aligned}$$

subject to

$$\int_{X_0} \{ x_0 - q(x_0, m(x_0)) - M_1(x_0 - q(x_0, m(x_0))), m(x_0) \} \cdot f(x_0) \, dx_0$$

$$+ \int_{X_0} \int_X \{ x - M_2(x_0 - q(x_0, m(x_0)), m(x_0), x) \} \cdot f(x | x_0, q(x_0, m(x_0)), a(x_0, m(x_0))) \\ \cdot f(x_0) \, dx dx_0 = \nabla$$

$$\forall x_0 \in X_0: m(x_0) \in \operatorname{argmax}_{m \in M} \{ u_1(M_1(x_0 - q(x_0, m), m))$$

$$+ \int_X u_2(M_2(x_0 - q(x_0, m), m, x)) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) \, dx - H(a(x_0, m)) \}$$

$$\forall x_0 \in X_0: \{q(x_0, m), a(x_0, m)\} \in \operatorname{argmax}_{(q, a) \in Q \times A} \{ u_1(M_1(x_0 - q, m))$$

$$+ \int_X u_2(M_2(x_0 - q, m, x)) \cdot f(x | x_0, q, a) \, dx - H(a) \} \quad \forall m \in M$$

If we compare this incentive problem to that in Definition 7.2 without accounting information, we have an additional incentive compatibility constraint. This constraint is that it is in the manager's self-interest to report according to the accounting principle $m(\cdot)$ whatever cash flow of the previous period he observes. The accounting principle is chosen such that it maximizes the manager's expected utility.

If it is costless to introduce the communication structure, then it is also weakly Pareto-preferred that the manager issues a report based on his observation of the cash flow of the previous period. As appears from the proof of the following proposition, this follows from the fact that the manager has the option not to make the compensation functions contingent on what he reports.

Proposition 8.1.

Denote the optimal contract for the incentive problem with communication of cash flow information by $C = (M_1(d, m), M_2(d, m, x), \bar{m}(x_0), \bar{q}(x_0, m), \bar{a}(x_0, m))$. This contract is weakly Pareto-preferred to the Pareto-optimal contract without accounting information $C^* = (M_1^*(d), M_2^*(d, x), q^*(x_0), a^*(x_0))$.

Proof:

The claim can be proven by showing that the communication contract C defined by

$$M_1(d,m) = M_1^*(d) \quad \forall m \in M, \forall d \in D$$

$$M_2(d,m,x) = M_2^*(d,x) \quad \forall m \in M, \forall d,x \in D \times X$$

$$m(x_0) = x_0 \quad \forall x_0 \in X_0$$

$$q(x_0,m) = q^*(x_0) \quad \forall m \in M, \forall x_0 \in X_0$$

$$a(x_0,m) = a^*(x_0) \quad \forall m \in M, \forall x_0 \in X_0$$

gives the same expected utility to the manager as the no accounting information contract, and that it is a feasible contract in the incentive problem with communication of cash flow information.

It is obvious that C and C* give the same expected utility to both the manager and the investor, since the production decisions and the x_0 -contingent compensation functions are pointwise identical for the two contracts.

Reporting the cash flow truthfully is incentive compatible, because his compensation and the production decisions do not depend on his report.

Furthermore, the production decision rules $q(\cdot)$ and $a(\cdot)$ are also incentive compatible, since

$$\begin{aligned} \forall x_0 \in X_0: \{q(x_0,m), a(x_0,m)\} = \{q^*(x_0), a^*(x_0)\} \in \operatorname{argmax}_{(q,a) \in Q \times A} \{ & u_1(M_1^*(x_0-q)) \\ & + \int_X u_2(M_2^*(x_0-q,x)) \cdot f(x|x_0,q,a) dx - H(a) \} \end{aligned}$$

$$\begin{aligned}
&= \operatorname{argmax}_{(q,a) \in Q \times A} \{ u_1(M_1(x_0 - q, m)) \\
&\quad + \int_X u_2(M_2(x_0 - q, m, x)) \cdot f(x | x_0, q, a) dx - H(a) \} \quad \forall m \in M
\end{aligned}$$

Q.E.D.

This well-known proposition (see, e.g., Christensen (1981)) shows that there can never be a social loss associated with the introduction of accounting information reported by the manager, if it is costless. On the other hand, when it is costly to report accounting information, then it must be of strict positive social value to communicate that information, if it is optimal to communicate the information. Strict positive gains can be due either to a more efficient risk sharing or to improved production decisions (or both). Neither the proposition nor its proof give any insight into what may cause communication to have strict positive social value. Unfortunately, that is one of the interesting questions.

Communication allows the manager to precommit himself to an optimally determined dividend function,

$$d = d(m),$$

such that the dividend must be $d(m)$ if m is reported. The form of the compensation functions that allows this precommitment is

$$M_1(d, m) = \begin{cases} M_1(m) & \text{if } d = d(m) \\ P & \text{otherwise} \end{cases}$$

$$M_2(d, m, x) = \begin{cases} M_2(m, x) & \text{if } d = d(m) \\ P & \text{otherwise} \end{cases}$$

where P is a sufficiently large penalty¹ that ensures that the manager will always select the capital investment

$$q(x_0, m) = x_0 - d(m)$$

when he observes x_0 and reports m .

It should be noted, that even if this precommitment is made, the capital investment is not observable to the shareholders, because

$$q(x_0, m) \neq m - d(m)$$

if the report m is different from the true cash flow of the previous period. This is due to the fact that even if the manager does not report the cash flow of the previous period truthfully, the dividend to the investors is still equal to the true cash flow of the previous period minus the capital investment. That is, the manager cannot use the cash flow of the firm for his own benefit. Using this possibility of precommitment, the incentive problem with communication of cash flow information can be reformulated as follows.

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ m(\cdot), d(\cdot), a(\cdot)}} & \int_{X_0} u_1(M_1(m(x_0))) \cdot f(x_0) \, dx_0 \\ & + \int_{X_0} \int_X u_2(M_2(m(x_0), x)) \cdot f(x|x_0, q(x_0, m(x_0)), a(x_0, m(x_0))) \cdot f(x_0) \, dx dx_0 \\ & - \int_{X_0} H(a(x_0, m(x_0))) \cdot f(x_0) \, dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \{ x_0 - q(x_0, m(x_0)) - M_1(m(x_0)) \} \cdot f(x_0) \, dx_0 \\ & + \int_{X_0} \int_X \{ x - M_2(m(x_0), x) \} \cdot f(x|x_0, q(x_0, m(x_0)), a(x_0, m(x_0))) \\ & \quad \cdot f(x_0) \, dx dx_0 = 0 \end{aligned}$$

$$\forall x_0 \in X_0: m(x_0) \in \operatorname{argmax}_{m \in M} \{ u_1(M_1(m))$$

$$+ \int_X u_2(M_2(m, x)) \cdot f(x|x_0, q(x_0, m), a(x_0, m)) \, dx - H(a(x_0, m)) \}$$

$$\forall x_0 \in X_0: a(x_0, m) \in \operatorname{argmax}_{a \in A} \left\{ \int_X u_2(M_2(m, x)) \cdot f(x | x_0, q(x_0, m), a) dx - H(a) \right\} \forall m \in M$$

$$\forall x_0 \in X_0: q(x_0, m) = x_0 - d(m)$$

The compensation functions are now written on just m and x , and the incentive compatibility restriction on the capital investment is substituted with the restriction on q from the precommitment to the dividend function $d(\cdot)$.

The following proposition shows that precommitment to an optimally determined dividend function is at least as good as no precommitment.

Proposition 8.2.

The optimal contract $\tilde{C} = (\tilde{M}_1(m), \tilde{M}_2(m, x), \tilde{m}(x_0), \tilde{d}(m), \tilde{q}(x_0, m), \tilde{a}(x_0, m))$ is weakly Pareto-preferred to the optimal contract $\bar{C} = (\bar{M}_1(d, m), \bar{M}_2(d, m, x), \bar{m}(x_0), \bar{q}(x_0, m), \bar{a}(x_0, m))$ without explicit precommitment.

Proof:

Let the contract C be defined by

$$M_1(m) = \begin{cases} \bar{M}_1(d, m) & \text{if } d = d(m) \\ P & \text{otherwise} \end{cases} \quad \forall m \in M, \forall d \in D$$

$$M_2(m, x) = \begin{cases} \bar{M}_2(d, m, x) & \text{if } d = d(m) \\ P & \text{otherwise} \end{cases} \quad \forall m \in M, \forall d, x \in D \times X$$

$$m(x_0) = \bar{m}(x_0) \quad \forall x_0 \in X_0$$

$$q(x_0, m) = \bar{q}(x_0, m) \quad \forall m \in M, \forall x_0 \in X_0$$

$$a(x_0, m) = \bar{a}(x_0, m) \quad \forall m \in M, \forall x_0 \in X_0$$

$$d(m) = x_0 - \bar{q}(x_0, m) \quad \forall m \in M, \forall x_0 \in X_0$$

It is easily seen that the contract with precommitment, C , gives both the manager and the investors the same explicit utility as the contract $\bar{C}$. What remains to be shown is that C is incentive compatible. The subset of feasible combinations of message decisions and production decisions in the problem with precommitment is also feasible in the problem without precommitment and, furthermore, the compensation functions for the two problems are identical on this subset. Hence, incentive compatibility of $\bar{m}(\cdot)$, $\bar{q}(\cdot)$ and $\bar{a}(\cdot)$ for the contract $\bar{C}$ implies that these decision rules are also incentive compatible for the contract C . Since $(m(\cdot), q(\cdot), a(\cdot))$ is identical to $(\bar{m}(\cdot), \bar{q}(\cdot), \bar{a}(\cdot))$ the contract C is a feasible solution to the problem with precommitment and gives the manager the same expected utility as the optimal contract without precommitment. Q.E.D.

The proposition gives that precommitment to an optimally determined dividend function, $d(\cdot)$, is at least as good as no precommitment. However, the expected utility to both the manager and the investors can be made arbitrarily close for the two problems. This is so because the manager has the option to choose the compensation function $M_1(d, m)$ and $M_2(d, m, x)$ as smooth approximations to the contract with precommitment, such that it is very expensive for the manager to deviate from $\bar{d}(m)$ given that m is reported.

The precommitment solution does not imply that the manager reports the cash flow of the previous period truthfully. However, the optimal report function $\bar{m}(\cdot)$ is common-knowledge. This implies that, if the investors receive a report m from the manager, then they also know that

$$x_0 \in \bar{m}^{-1}(m).$$

That is, the optimal report function induces a partition on the set of possible values of the cash flow of the previous period. Even if that partition is incomplete, i.e., $\bar{m}^{-1}(m)$ is not a singleton for all $m \in M$, it is possible to find a new contract, that motivates the manager to report the cash flow of the previous period truthfully and nothing more. This contract is defined by

$$M_1^*(x_0) = \tilde{M}_1 \circ \tilde{m}(x_0)$$

$$M_2^*(x_0, x) = \tilde{M}_2 \circ \tilde{m}(x_0, x)$$

and

$$d^*(x_0) = \tilde{d} \circ \tilde{m}(x_0)$$

such that

$$M_1^*(x_0) = \tilde{M}_1(m) \quad \forall x_0 \in \tilde{m}^{-1}(m), \quad \forall m \in M.$$

$$M_2^*(x_0, x) = \tilde{M}_2(m, x) \quad \forall x_0 \in \tilde{m}^{-1}(m), \quad \forall m, x \in M \times X.$$

and

$$d^*(x_0) = \tilde{d}(m) \quad \forall x_0 \in \tilde{m}^{-1}(m), \quad \forall m \in M.$$

This contract not only motivates the manager to report truthfully, but it also motivates him to implement the same x_0 -contingent production decisions and it gives the same expected utility to both the manager and the investors.

Proposition 8.3.

Let $\tilde{C} = (\tilde{M}_1(m), \tilde{M}_2(m, x), \tilde{m}(x_0), \tilde{d}(m), \tilde{q}(x_0, m), \tilde{a}(x_0, m))$ be the optimal contract for the incentive problem with communication and precommitment to an optimally determined dividend function and let $M_1^*(\cdot)$, $M_2^*(\cdot)$ and $d^*(\cdot)$ be defined as above. Then the contract $C^* = (M_1^*(m'), M_2^*(m', x), m^*(x_0), d^*(m'), q^*(x_0, m'), a^*(x_0, m'))$ where

$$m^*(x_0) = x_0$$

$$q^*(x_0, m') = \tilde{q}(x_0, m) = x_0 - \tilde{d}(m), \quad m' \in \tilde{m}^{-1}(m).$$

$$a^*(x_0, m') = \tilde{a}(x_0, m), \quad m' \in \tilde{m}^{-1}(m).$$

is also an optimal contract for the incentive problem with communication and precommitment.

Proof:

It must be shown that both the manager and the investors get the same expected utility for the two contracts and that C^* is incentive compatible. From the definition of C^* it follows, that

$$\begin{aligned}
 & \int_{X_0} u_1(\tilde{M}_1(\tilde{m}(x_0))) \cdot f(x_0) \, dx_0 \\
 & + \int_{X_0} \int_X u_2(\tilde{M}_2(\tilde{m}(x_0), x)) \cdot f(x | x_0, \tilde{q}(x_0, \tilde{m}(x_0)), \tilde{a}(x_0, \tilde{m}(x_0))) \cdot f(x_0) \, dx dx_0 \\
 & - \int_{X_0} H(\tilde{a}(x_0, \tilde{m}(x_0))) \cdot f(x_0) \, dx_0 \\
 & = \int_{X_0} u_1(M_1^*(x_0)) \cdot f(x_0) \, dx_0 \\
 & + \int_{X_0} \int_X u_2(M_2^*(x_0, x)) \cdot f(x | x_0, q^*(x_0, x_0), a^*(x_0, x_0)) \cdot f(x_0) \, dx dx_0 \\
 & - \int_{X_0} H(a^*(x_0, x_0)) \cdot f(x_0) \, dx_0
 \end{aligned}$$

where the first equality comes from the definition of $M_1^*(\cdot)$ and $M_2^*(\cdot)$ and the second equality comes from

$$q^*(x_0, x_0) = \tilde{q}(x_0, \tilde{m}(x_0))$$

$$a^*(x_0, x_0) = \tilde{a}(x_0, \tilde{m}(x_0)),$$

since $x_0 \in \tilde{m}^{-1}(\tilde{m}(x_0))$. Thus, the manager gets the same expected utility for both contracts. It can be shown in the same way that this also holds for the investors. Incentive compatibility of the contract C^* can be shown as follows.

Incentive compatibility of $\tilde{a}(\cdot)$ implies, that

$$\begin{aligned} a^*(x_0, m') &= \tilde{a}(x_0, m) \in \operatorname{argmax}_{a \in A} \left\{ \int_X u_2(\tilde{M}_2(m, x)) \cdot f(x|x_0, \tilde{q}(x_0, m), a) dx - H(a) \right\} \\ &= \operatorname{argmax}_{a \in A} \left\{ \int_X u_2(M_2^*(m', x)) \cdot f(x|x_0, q^*(x_0, m'), a) dx - H(a) \right\} \end{aligned}$$

since $m' \in \tilde{m}^{-1}(m)$ and $M_1^* = \tilde{M}_1 \circ \tilde{m}$, $M_2^* = \tilde{M}_2 \circ \tilde{m}$ and $d^* = \tilde{d} \circ \tilde{m}$. Then it remains to be shown that truthful reporting is incentive compatible. Again, incentive compatibility of $\tilde{m}(\cdot)$ implies, that

$$\begin{aligned} &u_1(M_1^*(x_0)) + \int_X u_2(M_2^*(x_0, x)) \cdot f(x|x_0, q^*(x_0, x_0), a^*(x_0, x_0)) dx - H(a^*(x_0, x_0)) \\ &= u_1(\tilde{M}_1(\tilde{m}(x_0))) + \int_X u_2(\tilde{M}_2(\tilde{m}(x_0, x))) \cdot f(x|x_0, \tilde{q}(x_0, \tilde{m}(x_0)), \tilde{a}(x_0, \tilde{m}(x_0))) dx \\ &\quad - H(\tilde{a}(x_0, \tilde{m}(x_0))) \end{aligned}$$

$$\geq u_1(\tilde{M}_1(m)) + \int_X u_2(\tilde{M}_2(m, x)) \cdot f(x|x_0, \tilde{q}(x_0, m), \tilde{a}(x_0, m)) dx - H(\tilde{a}(x_0, m)) \quad \forall m \in M$$

$$\begin{aligned} &= u_1(M_1^*(m')) + \int_X u_2(M_2^*(m', x)) \cdot f(x|x_0, q^*(x_0, m'), a^*(x_0, m')) dx \\ &\quad - H(a^*(x_0, m')), \quad m' \in \tilde{m}^{-1}(m), \quad \forall m \in M \end{aligned}$$

since $q^*(x_0, m') = \tilde{q}(x_0, m)$, $a^*(x_0, m') = \tilde{a}(x_0, m)$, $M_1^*(m') = \tilde{M}_1(m)$, $M_2^*(m', x) = \tilde{M}_2(m, x)$ and $a^*(x_0, m')$ is incentive compatible for the compensation function $M_1^*(\cdot)$ and $M_2^*(\cdot)$. Hence, the manager does not have any incentives to report $m' \neq x_0$. Q.E.D.

This proposition is an application of the Revelation Principle (see, e.g., Myerson (1979)). It states that there is no loss of social wealth in restricting the set of compensation functions to those which induce the manager to report the cash flow of the previous period truthfully. This result is not all that surprising. Given a menu of compensation functions parameterized by the manager's report, the manager's incentives to report different values of the cash flow of the previous period is completely known by the investors through their knowledge of his preferences and expectations. Then it is also possible to make a re-labelling of the menu of compensation functions such that he gets the same compensation function from reporting truthfully as he would get for the optimal report given the original menu of compensation functions.

The proposition has some interesting implications for the choice of accounting principles. First, full revelation of the manager's private information is always optimal. That is, no gains can be obtained from restricting the manager's reporting possibilities, even when the manager has an economic interest in the reported values. Secondly, issues like accrual accounting and smoothing of the earnings of the firm plays no role.

The Revelation Principle implies that nothing can be gained by reporting, e.g., the values of the assets and liabilities in addition to the cash flow.

The manager's private information is the cash flow of the previous period and the unobservable production decisions. However, the optimal production decision rules are known to the investors. Hence, any calculations of the values of the assets and liabilities that the manager can perform can also be made by the investors, given that the manager reports the true cash flow of the previous period. Therefore, a necessary condition for gains to

accrual accounting is that the manager has more private information than the cash flow of the previous period. We return to this in Chapter 9.

The manager has an interest in a smooth consumption stream (see Rogerson (1985a) and Proposition 8.7 below). However, this is a different statement than the statement that there are gains to smoothing the earnings of the firm. The proof of the Revelation Principle shows that any smoothing of the earnings of the firm might as well be made directly in the manager's compensation functions. That is, if an optimal accounting principle $m^*(x_0)$ is a smoothing of the earnings then cash flow reporting yields the same result, if the smoothing is made directly in the compensation functions: $M_1^*(m^*(x_0))$, $M_2^*(m^*(x_0), x)$. Hence, there are no gains to smoothing the earnings of the firm.²

On the basis of Proposition 8.3, the set of compensation functions will be restricted to those which induce the manager to report the cash flow of the previous period truthfully and nothing more. That is, the report selection constraint is formulated as

$$\forall x_0 \in X_0: x_0 \in \operatorname{argmax}_{m \in X_0} \{ u_1(M_1(m)) + \int_X u_2(M_2(m, x)) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) dx - H(a(x_0, m)) \}$$

In order to characterize the optimal contract it is furthermore assumed that this constraint can be substituted with its first order condition,

$$\begin{aligned} & u_1'(M_1(m)) \cdot M_{1m}(m) \\ & + \int_X u_2'(M_2(m, x)) \cdot M_{2m}(m, x) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) dx \\ & + \int_X u_2(M_2(m, x)) \cdot \{ f_q(x | x_0, q(x_0, m), a(x_0, m)) \cdot q_m(x_0, m) \\ & \quad + f_a(x | x_0, q(x_0, m), a(x_0, m)) \cdot a_m(x_0, m) \} dx \end{aligned}$$

$$- H'(a(x_0, m)) \cdot a_m(x_0, m) \Big|_{m=x_0} = 0$$

The effort selection constraint is again substituted with its first order condition, i.e.,

$$\int_X u_2'(M_2(m, x)) \cdot f_a(x | x_0, q(x_0, m), a(x_0, m)) dx - H'(a(x_0, m)) = 0$$

Inserting this in the report selection constraint we get that

$$\begin{aligned} & u_1'(M_1(m)) \cdot M_{1m}(m) \\ & + \int_X u_2'(M_2(m, x)) \cdot M_{2m}(m, x) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) dx \\ & + q_m(x_0, m) \cdot \int_X u_2'(M_2(m, x)) \cdot f_q(x | x_0, q(x_0, m), a(x_0, m)) dx \Big|_{m=x_0} = 0 \end{aligned}$$

Using the precommitment constraint, $q(x_0, m) = x_0 - d(m)$, this is equivalent to

$$\begin{aligned} & u_1'(M_1(x_0)) \cdot M_{1x_0}(x_0) \\ & + \int_X u_2'(M_2(x_0, x)) \cdot M_{2x_0}(x_0, x) \cdot f(x | x_0, q(x_0, x_0), a(x_0, x_0)) dx \\ & - d_{x_0}(x_0) \cdot \int_X u_2'(M_2(x_0, x)) \cdot f_q(x | x_0, q(x_0, x_0), a(x_0, x_0)) dx = 0 \end{aligned}$$

The effort selection constraint must hold for all $m \in M$ and in particular for $m = x_0$. Since $a_m(x_0, m)$ no longer appears in the report selection constraint, the effort selection constraint need only be considered at the point $q(x_0, x_0), a(x_0, x_0)$. If we write $q(x_0, x_0) = q(x_0)$ and $a(x_0, x_0) = a(x_0)$, the incentive problem with communication of cash flow information and precommitment to an optimally determined dividend function can be defined as follows.

Definition 8.1

Pareto-optimal contracts with communication of cash flow information and precommitment.

The contract $C^* = (M_1^*(m), M_2^*(m, x), m^*(x_0), d^*(m), q^*(x_0, m), a^*(x_0, m))$ is a truth-inducing Pareto-optimal contract if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ d(\cdot), a(\cdot)}} & \int_{X_0} u_1(M_1(x_0)) \cdot f(x_0) \, dx_0 \\ & + \int_{X_0} \int_X u_2(M_2(x_0, x)) \cdot f(x|x_0, q(x_0), a(x_0)) \cdot f(x_0) \, dx dx_0 \\ & - \int_{X_0} H(a(x_0)) \cdot f(x_0) \, dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \{ d(x_0) - M_1(x_0) \} \cdot f(x_0) \, dx_0 \\ & + \int_{X_0} \int_X \{ x - M_2(x_0, x) \} \cdot f(x|x_0, q(x_0), a(x_0)) \cdot f(x_0) \, dx dx_0 = 0 \end{aligned}$$

$$\begin{aligned} \forall x_0 \in X_0: & \quad u_1'(M_1(x_0)) \cdot M_{1x_0}(x_0) \\ & + \int_X u_2'(M_2(x_0, x)) \cdot M_{2x_0}(x_0, x) \cdot f(x|x_0, q(x_0), a(x_0)) \, dx \\ & - d_{x_0}(x_0) \cdot \int_X u_2(M_2(x_0, x)) \cdot f_q(x|x_0, q(x_0), a(x_0)) \, dx = 0 \end{aligned}$$

$$\forall x_0 \in X_0: \quad \int_X u_2(M_2(x_0, x)) \cdot f_a(x|x_0, q(x_0), a(x_0)) \, dx - H'(a(x_0)) = 0$$

$$\forall x_0 \in X_0: \quad q(x_0) = x_0 - d(x_0)$$

The set of compensation functions for this problem is characterized in the following proposition.

Proposition 8.4.

If $C^* = (M_1^*(m), M_2^*(m, x), m^*(x_0), d^*(m), q^*(x_0, m), a^*(x_0, m))$ is a truth-inducing Pareto-optimal contract for the incentive problem in Definition 8.1, then

$$(8.1) \quad 1/u_1'(M_1^*(x_0)) = 1/\lambda \cdot \left\{ 1 - \rho_{x_0}(x_0) - \rho(x_0) \cdot \frac{f_{x_0}(x_0)}{f(x_0)} \right\}$$

$$(8.2) \quad 1/u_2'(M_2^*(x_0, x)) = 1/\lambda \cdot \left\{ 1 - \rho_{x_0}(x_0) - \rho(x_0) \cdot \frac{f_{x_0}(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))} \right. \\ - \rho(x_0) \cdot \frac{f_q(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))} \\ - \rho(x_0) \cdot a_{x_0}^*(x_0) \cdot \frac{f_a(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))} \\ \left. + \mu(x_0) \cdot \frac{f_a(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))} \right\}$$

where λ , $\rho(x_0)$ and $\mu(x_0)$ are optimal Lagrange-multipliers.

Proof: see the appendix.

If (8.1) and (8.2) are compared to (7.1) and (7.2), then the averaging in (7.1) and (7.2) is no longer present when communication is introduced. The obvious reason for this is that the cash flow of the previous period is communicated to the investors and can hence be contracted upon. This cannot be done in the case of no communication, since, in general, the investors get only incomplete information about x_0 through the observable dividend and the common-knowledge of the optimal capital investment decision rule $q^*(\cdot)$. In the case where the dividend and $q^*(\cdot)$ reveal perfect information

about x_0 to the investors, the two characterizations are identical if $\rho(x_0) = -\gamma(x_0)$ and if the optimal dividend functions of the two problems are identical. Furthermore, as shown by the following proposition, communication is of no value if the compensation functions are unaffected by communication.

Proposition 8.5.

Let the optimal contracts for the incentive problems without and with communication of cash flow information be $C^* = (M_1^*(d), M_2^*(d, x), q^*(x_0), a^*(x_0))$ and $C^{**} = (M_1^{**}(m), M_2^{**}(m, x), m^{**}(x_0), d^{**}(m), q^{**}(x_0, m), a^{**}(x_0, m))$ respectively. Then C^{**} is weakly Pareto-preferred to C^* and C^{**} is strictly Pareto-preferred to C^* if there exists a set of (x_0, x) which has positive probability such that either

$$M_1^*(d^*(x_0)) \neq M_1^{**}(x_0) \quad \text{or} \quad M_2^*(d^*(x_0), x) \neq M_2^{**}(x_0, x)$$

where

$$d^*(x_0) = x_0 - q^*(x_0).$$

Proof:

Define the contract C by

$$M_1(m) = M_1^*(m - q^*(m))$$

$$M_2(m, x) = M_2^*(m - q^*(m), x)$$

$$m(x_0) = x_0$$

$$d(m) = m - q^*(m)$$

$$q(x_0, m) = x_0 - d^*(m)$$

$$a(x_0, x_0) = a^*(x_0)$$

This contract is a communication contract, where the manager precommits himself to the optimal dividend function for the incentive problem without accounting information. If truth-telling and $a^*(x_0)$ are incentive compatible then the contract gives the same expected utility to both the manager and the investors as the contract C^* . To show that C^{**} is weakly Pareto-preferred to C^* it remains to be shown that truth-telling and $a^*(x_0)$ are incentive compatible. First, the compensation functions are truth-inducing:

$$\begin{aligned}
 & u_1(M_1(m)) + \int_X u_2(M_2(m, x)) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) dx - H(a(x_0, m)) \\
 &= u_1(M_1^*(x_0 - q(x_0, m))) + \int_X u_2(M_2^*(x_0 - q(x_0, m), x)) \cdot f(x | x_0, q(x_0, m), a(x_0, m)) dx \\
 &\quad - H(a(x_0, m)) \\
 &\leq u_1(M_1^*(x_0 - q^*(x_0))) + \int_X u_2(M_2^*(x_0 - q^*(x_0), x)) \cdot f(x | x_0, q^*(x_0), a^*(x_0)) dx \\
 &\quad - H(a^*(x_0)) \\
 &= u_1(M_1(x_0)) + \int_X u_2(M_2(x_0, x)) \cdot f(x | x_0, q(x_0, x_0), a(x_0, x_0)) dx - H(a(x_0, x_0))
 \end{aligned}$$

The equalities follow from the definition of the contract C and the inequality follows from $q^*(x_0)$ and $a^*(x_0)$ being incentive compatible for the compensation functions $M_1^*(\cdot)$ and $M_2^*(\cdot)$. Hence, the manager reports truthfully. It then only has to be shown that $a(x_0, x_0) = a^*(x_0)$ is incentive compatible. This follows easily from $a^*(x_0)$ being incentive compatible for the compensation functions $M_1^*(\cdot)$ and $M_2^*(\cdot)$. This completes the proof of the first part of the proposition.

The contract C is a feasible contract for the incentive problem with communication and it gives both the manager and the investors the same expected utility as the contract C^* and the (x_0, x) -contingent compensation to the manager is the same for C and C^* . Hence, if the (x_0, x) -contingent compensation to the manager differs with positive probability for the contracts C^*

and C^{**} , then the contract C cannot be an optimal solution to the incentive problem with communication. This implies, that C^{**} is strictly preferred to C or equivalently C^{**} is strictly preferred C^* . Q.E.D.

In the proof of the proposition it was used that the contract defined by the optimal compensation functions and the dividend function for the incentive problem without accounting information

$$M_1(m) = M_1^* \circ d^*(m) \quad M_2(m,x) = M_2^* \circ d^*(m,x) \quad d(m) = m - q^*(m)$$

is a feasible truth-inducing communication contract. However, these compensation functions might not be optimal given the dividend function. Furthermore, the dividend function might not be optimal with communication. The following proposition gives a sufficient condition for the contract to be optimal with communication.

Proposition 8.6.

Let C^{**} and C^* be defined as in Proposition 8.5. If the dividend function, $d^{**}(\cdot)$, to which the manager precommits himself in the incentive problem with communication, is invertible, then communication is of no value and $d^*(\cdot) = d^{**}(\cdot)$. On the other hand, if communication has no value, then $d^{**}(\cdot)$ is invertible or $M_1^{**}(m') = M_1^{**}(m'') \forall m', m'' \in d^{**^{-1}}(d)$ for all dividends d for which $d^{**^{-1}}(d)$ is not a singleton.

Proof:

For $d^{**}(\cdot)$ invertible it has to be shown that a contract exists for the incentive problem without accounting information which is feasible and gives the same expected utility to the manager as C^{**} .

Write the compensation functions $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$ as

$$M_1^{**}(d,m) = \begin{cases} M_1^{**}(m) & \text{if } d = d^{**}(m) \\ P & \text{otherwise} \end{cases}$$

$$M_2^{**}(d,m,x) = \begin{cases} M_2^{**}(m,x) & \text{if } d = d^{**}(m) \\ P & \text{otherwise} \end{cases}$$

and define a contract C without accounting information by

$$M_1(d) = \begin{cases} \max_m M_1^{**}(d,m) & \text{if } d \in D^{**} \\ P & \text{otherwise} \end{cases}$$

$$M_2(d,x) = \begin{cases} \max_m M_2^{**}(d,m,x) & \text{if } d \in D^{**} \\ P & \text{otherwise} \end{cases}$$

$$q(x_0) = q^{**}(x_0) \quad \forall x_0 \in X_0$$

$$a(x_0) = a^{**}(x_0) \quad \forall x_0 \in X_0$$

where D^{**} is the range of the dividend function $d^{**}(\cdot)$.

For any given x_0 consider two arbitrary production decisions $\hat{q}$ and $\hat{a}$. Then

$$\begin{aligned} & u_1(M_1(x_0 - \hat{q})) + \int_X u_2(M_2(x_0 - \hat{q}, x)) \cdot f(x|x_0, \hat{q}, \hat{a}) dx - H(\hat{a}) \\ &= u_1(M_1^{**}(x_0 - \hat{q}, \hat{m})) + \int_X u_2(M_2^{**}(x_0 - \hat{q}, \hat{m}, x)) \cdot f(x|x_0, \hat{q}, \hat{a}) dx - H(\hat{a}) \end{aligned}$$

since there is a unique message, $\hat{m}$, for $x_0 - \hat{q} \in D^{**}$, which maximizes both $M_1^{**}(x_0 - \hat{q}, \cdot)$ and $M_2^{**}(x_0 - \hat{q}, \cdot, x)$ for any x when $d^{**}(\cdot)$ is invertible. For $x_0 - \hat{q}$ not in D^{**} the relation is trivial. The compensation functions $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$ induce the manager to report truthfully and to implement the production decisions $q^{**}(x_0)$ and $a^{**}(x_0)$. Hence,

$$u_1(M_1^{**}(x_0 - \hat{q}, \hat{m})) + \int_X u_2(M_2^{**}(x_0 - \hat{q}, \hat{m}, x)) \cdot f(x|x_0, \hat{q}, \hat{a}) dx - H(\hat{a})$$

$$\begin{aligned}
&\leq u_1(M_1^{**}(x_0 - q^{**}(x_0), x_0)) \\
&\quad + \int_X u_2(M_2^{**}(x_0 - q^{**}(x_0), x), x) \cdot f(x | x_0, q^{**}(x_0), a^{**}(x_0)) dx - H(a^{**}(x_0)) \\
&\leq u_1(M_1(x_0 - q^{**}(x_0))) + \int_X u_2(M_2(x_0 - q^{**}(x_0), x), x) \cdot f(x | x_0, q^{**}(x_0), a^{**}(x_0)) dx \\
&\quad - H(a^{**}(x_0)) \\
&= u_1(M_1(x_0 - q(x_0))) + \int_X u_2(M_2(x_0 - q(x_0), x), x) \cdot f(x | x_0, q(x_0), a(x_0)) dx - H(a(x_0))
\end{aligned}$$

where the second inequality follows from the definition of $M_1(\cdot)$ and $M_2(\cdot)$ and the last equality follows from the definition of $q(\cdot)$ and $a(\cdot)$. This establishes that the contract C is incentive compatible.

From the invertibility of $d^{**}(\cdot)$ it follows, that

$$\begin{aligned}
M_1^{**}(x_0 - q^{**}(x_0), m) &= \begin{cases} M_1^{**}(x_0) & \text{if } m = x_0 \\ P & \text{otherwise} \end{cases} \\
M_2^{**}(x_0 - q^{**}(x_0), m, x) &= \begin{cases} M_2^{**}(x_0, x) & \text{if } m = x_0 \\ P & \text{otherwise} \end{cases}
\end{aligned}$$

and from the definition of $M_1(\cdot)$ and $M_2(\cdot)$ this implies that

$$\begin{aligned}
M_1(x_0 - q^{**}(x_0)) &= M_1^{**}(x_0) \\
M_2(x_0 - q^{**}(x_0), x) &= M_2^{**}(x_0, x)
\end{aligned}$$

Thus, the (x_0, x) -contingent compensations to the manager are the same for the contracts C and C^{**} . Since the production decision rules are also identical, both the manager and the investors get the same expected utility for the two contracts. From Proposition 8.5 it then follows that C is indeed an optimal contract for the incentive problem without accounting information. This completes the proof of the first claim in the proposition.

Suppose, on the other hand, that communication is of no value. Define a

communication contract C as in Proposition 8.5, i.e.,

$$M_1(m) = M_1^* \circ d^*(m)$$

$$M_2(m) = M_2^* \circ d^*(m, x)$$

$$m(x_0) = x_0$$

$$d(m) = d^*(m)$$

$$q(x_0, m) = x_0 - d^*(m)$$

$$a(x_0, x_0) = a^*(x_0)$$

As shown in the proof of Proposition 8.5, this contract is a feasible communication contract and it gives the manager the same expected utility as the optimal contract without communication. Since communication is of no value this contract is also an optimal communication contract. That is, $C = C^{**}$. This implies, that $M_1^{**}(m) = M_1^*(d^*(m))$ for all m . The last claim in the proposition follows from $M_1(\cdot)$ being constant for $m \in d^{*-1}(d)$ and $d^{**-1}(d) = d^{*-1}(d)$. Q.E.D.

This proposition gives that a necessary condition for communication to be of strict positive value is that the optimal dividend function with communication is not invertible. It means that some levels of the cash flow of the previous period must exist for which the marginal change in the capital investment is greater than the marginal change in the cash flow of the previous period. This can occur, when the relative productivity of capital to effort increases sufficiently with x_0 .

If the dividend function with communication is invertible, then the dividend function without accounting information is also invertible. However, the other implication does not hold (cf. the examples below). Hence, even if the dividend function without accounting information is invertible, communication may be of strict positive value. Without communication, the

capital investment decision has two roles to play, one as a productive decision and one as a communication device for the manager's private information. Because the manager may prefer to communicate his private information to the investors (Proposition 8.1), it may be optimal for him to make the dividend function without accounting information invertible in order to reveal his private information. This can occur even if it would not be invertible, if the manager could communicate his private information directly to the investors through an accounting report. In these cases an increasing relative productivity of capital to effort with x_0 cannot be utilized to the same extent without accounting information as with an accounting report. In other words, communication allows a more efficient utilization of capital, since the capital investment decision rule is no longer restricted by its role of a communication device.

Melumad and Reichelstein (1987) give a similar result as Proposition 8.6 for a one-period agency model with a directly observable decision made by the agent. They examine the case where the observable decision is not able to distinguish between all of the agent's private signals. In our model, this is not the issue. The observable dividend distinguishes between all possible cash flows of the previous period, if we choose the same capital investment for all cash flows of the previous period, because the dividend function is then a linear increasing function of these cash flows. The issue is that due to the communication role of the capital investment decision, it can be optimal to determine the capital investment rule such that the observable dividend distinguishes between all cash flows of the previous period. Whether this will be optimal or not, depends of the loss of productive efficiency in doing so, and the gains that follows from revealing the manager's private information.

A case of special interest is the case where the manager gets perfect information before he reports the cash flow of the previous period and makes the production decisions. That is, when the dividend at $t=2$ is a deterministic function of x_0 , q and a :

$$x = x(x_0, q, a)$$

This case is of special interest, because it is shown by Melumad and Rei-

chelstein (1987) that communication is of no value in their one-period agency model for that case. The model by Holmström and Weiss (1985), which is one of the few agency analyses with capital investments, is also of that type. Our model is a two-period model in which the dynamics of the capital acquisition process is more transparent, and this has an effect on the value of communication and thereby on the value of accounting information.

When the manager gets perfect information at date $t=1$, he is able to pre-commit himself to dividend functions for both dates, $d(m)$ and $x(m)$. It can easily be shown, that this is weakly Pareto-preferred to no precommitment (as in Proposition 8.2) and that the Revelation Principle still applies (as in Proposition 8.3). Hence, it can be assumed without loss of generality that the optimal compensation functions are of the form

$$M_1^*(d,m) = \begin{cases} M_1^*(m) & \text{if } d = d^*(m) \\ P & \text{otherwise} \end{cases}$$

$$M_2^*(d,m,x) = \begin{cases} M_2^*(m) & \text{if } d = d^*(m) \text{ and } x = x^*(m) \\ P & \text{otherwise} \end{cases}$$

and that the manager reports truthfully, i.e., $m^*(x_0) = x_0$. Using this, the incentive problem with perfect information to the manager at date $t=1$ and communication can be formulated as follows.

Definition 8.2 Pareto-optimal contracts with perfect information to the manager at date $t=1$, communication and precommitment to optimally determined dividend functions for date $t=1$ and date $t=2$.

$C^{**} = (M_1^{**}(m), M_2^{**}(m), m^{**}(x_0), d^{**}(m), x^{**}(m), q^{**}(x_0, m), a^{**}(x_0, m))$ is a Pareto-optimal contract if it is a solution to the program

$$\max_{\substack{M_1(\cdot), M_2(\cdot) \\ d(\cdot), x(\cdot)}} \int_{X_0} \{ u_1(M_1(x_0)) + u_2(M_2(x_0)) - H(a(x_0)) \} \cdot f(x_0) dx_0$$

subject to

$$\int_{X_0} \{ d(x_0) - M_1(x_0) + x(x_0) - M_2(x_0) \} \cdot f(x_0) dx_0 = \nabla$$

$$\forall x_0 \in X_0: x_0 \in \operatorname{argmax} \{ u_1(M_1(m)) + u_2(M_2(m)) - H(a(x_0, m)) \}$$

$$\forall x_0 \in X_0: q(x_0, m) = x_0 - d(m)$$

$$\forall x_0 \in X_0: x(x_0, q(x_0, m), a(x_0, m)) = x(m)$$

The following proposition gives necessary and sufficient conditions for no value of communication for this incentive problem.

Proposition 8.7.

Let $C^{**} = (M_1^{**}(m), M_2^{**}(m), m^{**}(x_0), d^{**}(m), x^{**}(m), q^{**}(x_0, m), a^{**}(x_0, m))$ be an optimal solution to the incentive problem in Definition 8.2. Then

$$(a) \quad u_1'(M_1^{**}(m)) = u_2'(M_2^{**}(m)) \quad \forall m \in X_0$$

and

$$(b) \quad \text{communication is of no value if and only if } d^{**}(\cdot) \text{ is invertible or a dividend level at date } t=2, x, \text{ exists for all } d \text{ for which } d^{**-1}(d) \text{ is not a singleton, such that } d^{**-1}(d) \subseteq x^{**-1}(x).$$

Proof:

(a): First it will be shown that

$$u_1'(M_1^{**}(m)) = u_2'(M_2^{**}(m)) \quad \forall m \in X_0.$$

Assume that a message $\hat{m}$ exists for which this is not the case. Since the manager is strictly risk-averse we can choose variations $(\delta_1(\hat{m}), \delta_2(\hat{m}))$ in the compensations for the message $\hat{m}$ such that

$$\delta_1(\hat{m}) + \delta_2(\hat{m}) < 0$$

and

$$\begin{aligned} & u_1(M_1^{**}(\hat{m}) + \delta_1(\hat{m})) + u_2(M_2^{**}(\hat{m}) + \delta_2(\hat{m})) - [u_1(M_1^{**}(\hat{m})) + u_2(M_2^{**}(\hat{m}))] \\ &= u_1'(M_1^{**}(\hat{m})) \cdot \delta_1(\hat{m}) + u_2'(M_2^{**}(\hat{m})) \cdot \delta_2(\hat{m}) \\ &= 0 \end{aligned}$$

Since the manager's utility is unaffected by these variations for all messages, truthtelling is still optimal and, thus, the production decisions are also unaffected. The expected utility of the manager is unaffected by the variations, but the payment to the investors increases for $x_0 = \hat{m}$, since the total payment to the manager decreases ($\delta_1(\hat{m}) + \delta_2(\hat{m}) < 0$) and the production decisions are unaffected. Hence, if the manager's marginal utility of compensation in the two periods differs with positive probability, variations in the compensations can be made, which leaves the manager's expected utility unaffected, but increases the expected utility of the investors. This contradicts the optimality of $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$, since the investors' expected utility constraint obviously must be binding for the optimal contract. This establishes that the marginal utility of compensation is the same in both periods with probability one.

(b, Only if): Assume that communication is of no value. Define the contract C by

$$M_1(m) = M_1^*(d^*(m))$$

$$M_2(m) = M_2^*(d^*(m), x^*(m))$$

$$m(x_0) = x_0$$

$$d(m) = d^*(m) = m - q^*(m)$$

$$x(m) = x^*(m) = x(m, q^*(m), a^*(m))$$

$$q(x_0, m) = x_0 - d(m)$$

$$x(x_0, q(x_0, m), a(x_0, m)) = x(m)$$

where $C^* = (M_1^*(d), M_2^*(d, x), q^*(x_0), a^*(x_0))$ is the optimal contract without communication. Contract C is a communication contract and in order to show feasibility of this contract, it only has to be shown that truthtelling is incentive compatible. Consider an arbitrary x_0 :

$$\begin{aligned} & u_1(M_1(m)) + u_2(M_2(m)) - H(a(x_0, m)) \\ &= u_1(M_1^*(d^*(m))) + u_2(M_2^*(d^*(m), x^*(m))) - H(a(x_0, m)) \\ &\leq u_1(M_1^*(d^*(x_0))) + u_2(M_2^*(d^*(x_0), x^*(x_0))) - H(a^*(x_0)) \\ &= u_1(M_1(x_0)) + u_2(M_2(x_0)) - H(a(x_0, x_0)) \end{aligned}$$

since $q^*(x_0)$ and $a^*(x_0)$ are the preferred production decisions by the manager, given the compensation function $M_1^*(\cdot)$ and $M_2^*(\cdot)$. This gives feasibility of C and since communication is of no value, C is also an optimal communication contract, $C = C^{**}$, implying, that $d^{**}(\cdot) = d^*(\cdot)$. Hence, $d^{**}(\cdot)$ must either be invertible or

$$M_1^{**}(m') = M_1^{**}(m'') \quad \forall \quad m', m'' \in d^{**^{-1}}(d)$$

for those d for which $d^{**^{-1}}(d)$ is not a singleton. The equality of the marginal utility of compensation in both periods implies, that

$$M_2^{**}(m') = M_2^{**}(m'') \quad \forall \quad m', m'' \in d^{**^{-1}}(d)$$

To prove the claim it must be shown, that

$$x^{**}(m') = x^{**}(m'') \quad \forall \quad m', m'' \in d^{**^{-1}}(d)$$

Assume without loss of generality that $x^{**}(m') > x^{**}(m'')$. This implies for $x_0 = m'$, that

$$\begin{aligned}
& u_1(M_1^{**}(x_0)) + u_2(M_2^{**}(x_0)) - H(a^{**}(x_0)) \\
& = u_1(M_1^{**}(m'')) + u_2(M_2^{**}(m'')) - H(a^{**}(x_0)) \\
& < u_1(M_1^{**}(m'')) + u_2(M_2^{**}(m'')) - H(a^{**}(x_0, m''))
\end{aligned}$$

since for $x^{**}(x_0) > x^{**}(m'')$ it follows from the positive marginal productivity of effort that $a^*(x_0, x_0) > a^*(x_0, m'')$. Thus, reporting m'' instead of x_0 reduces the required effort without affecting the compensation. This contradicts the optimality of $m^{**}(x_0) = x_0$. Hence, $x^{**}(m') = x^{**}(m'')$
 $\forall m', m'' \in d^{**^{-1}}(d)$. **Q.E.D. (only if).**

(b,if): For $d^{**}(\cdot)$ invertible a similar proof as given for Proposition 8.6 can be applied. Therefore, choose an arbitrary d , such that $d^{**^{-1}}(d)$ is not a singleton. Then

$$\begin{aligned}
d^{**}(m') &= d^{**}(m'') \quad \forall m', m'' \in d^{**^{-1}}(d) \\
x^{**}(m') &= x^{**}(m'') \quad \forall m', m'' \in d^{**^{-1}}(d)
\end{aligned}$$

The first part of the proof is to show, that

$$M_1^{**}(m') = M_1^{**}(m'') \quad \forall m', m'' \in d^{**^{-1}}(d)$$

Assume on the contrary, that $M_1^{**}(m') < M_1^{**}(m'')$ without loss of generality. Using that the marginal utility of compensation is the same in both periods, we have that $M_2^{**}(m') < M_2^{**}(m'')$ and for $x_0 = m'$ that

$$\begin{aligned}
& u_1(M_1^{**}(x_0)) + u_2(M_2^{**}(x_0)) - H(a^{**}(x_0)) \\
& < u_1(M_1^{**}(m'')) + u_2(M_2^{**}(m'')) - H(a^{**}(x_0)) \\
& = u_1(M_1^{**}(m'')) + u_2(M_2^{**}(m'')) - H(a^{**}(x_0, m''))
\end{aligned}$$

since the required effort is independent of the message, when the required dividend $x^{**}(\cdot)$ is independent of the message. This contradicts the optimality of truthtelling. Hence, $M_1^{**}(m') = M_1^{**}(m'') \quad \forall m', m'' \in d^{**^{-1}}(d)$.

The last part of the proof is to show that a contract without accounting information exists, which gives the manager the same expected utility as C^{**} . Write the compensation function $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$ as

$$M_1^{**}(d, m) = \begin{cases} M_1^{**}(m) & \text{if } d = d^{**}(m) \\ P & \text{otherwise} \end{cases}$$

$$M_2^{**}(d, m, x) = \begin{cases} M_2^{**}(m) & \text{if } d = d^{**}(m) \text{ and } x = x^{**}(m) \\ P & \text{otherwise} \end{cases}$$

and define a contract without accounting information, C , by

$$M_1(d) = \begin{cases} \max_m M_1^{**}(d, m) & \text{if } d \in D^{**} \\ P & \text{otherwise} \end{cases}$$

$$M_2(d, x) = \begin{cases} \max_m M_2^{**}(d, m, x) & \text{if } d \in D^{**} \text{ and } x \in X^{**} \\ P & \text{otherwise} \end{cases}$$

$$q(x_0) = q^{**}(x_0)$$

$$a(x_0) = a^{**}(x_0)$$

where D^{**} and X^{**} are the ranges of the dividend functions $d^{**}(\cdot)$ and $x^{**}(\cdot)$ respectively.

For any given x_0 consider two arbitrary production decisions $\hat{q}$ and $\hat{a}$, such that $x_0 - \hat{q} \in D^{**}$ and $x(x_0, \hat{q}, \hat{a}) \in X^{**}$. Then

$$\begin{aligned} & u_1(M_1(x_0 - \hat{q})) + u_2(M_2(x_0 - \hat{q}, x(x_0, \hat{q}, \hat{a}))) - H(\hat{a}) \\ &= u_1(M_1^{**}(x_0 - \hat{q}, \hat{m}_1)) + u_2(M_2^{**}(x_0 - \hat{q}, \hat{m}_2, x(x_0, \hat{q}, \hat{a}))) - H(\hat{a}) \\ &= u_1(M_1^{**}(x_0 - \hat{q}, \hat{m}_2)) + u_2(M_2^{**}(x_0 - \hat{q}, \hat{m}_2, x(x_0, \hat{q}, \hat{a}))) - H(\hat{a}) \end{aligned}$$

where $\hat{m}_1$ maximizes $M_1^{**}(\cdot)$ and $\hat{m}_2$ maximizes $M_2^{**}(x_0 - \hat{q}, \cdot, x(x_0, \hat{q}, \hat{a}))$. The second equality follows from $M_1^{**}(x_0 - \hat{q}, m') = M_1^{**}(x_0 - \hat{q}, m'')$ for all $m', m'' \in d^{**^{-1}}(x_0 - \hat{q})$. Truthtelling for $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$ implies that

$$\begin{aligned}
& u_1(M_1^{**}(x_0 - \hat{q}, \hat{m}_2)) + u_2(M_2^{**}(x_0 - \hat{q}, \hat{m}_2, x(x_0, \hat{q}, \hat{a}))) - H(\hat{a}) \\
& \leq u_1(M_1^{**}(x_0 - q^{**}(x_0), x_0)) + u_2(M_2^{**}(x_0 - q^{**}(x_0), x_0, x(x_0, q^{**}(x_0), a^{**}(x_0)))) \\
& \quad - H(a^{**}(x_0)) \\
& \leq u_1(M_1(x_0 - q^{**}(x_0))) + u_2(M_2(x_0 - q^{**}(x_0), x(x_0, q^{**}(x_0), a^{**}(x_0)))) \\
& \quad - H(a^{**}(x_0))
\end{aligned}$$

Since a penalty will be the consequence of choosing $\hat{q}$ and $\hat{a}$ for which $(x_0 - \hat{q}, x(x_0, \hat{q}, \hat{a}))$ is not in $D^{**} \times X^{**}$ it follows that $q^{**}(x_0)$ and $a^{**}(x_0)$ are incentive compatible for the contract C. Furthermore, the x_0 -contingent compensation to the manager is the same for the contract C as for the contract C^{**} , since

$$M_1(x_0 - q^{**}(x_0)) = M_1^{**}(x_0 - q^{**}(x_0), m) \quad \forall m \in d^{**-1}(x_0 - q^{**}(x_0))$$

and since truthtelling, $m^{**}(x_0) = x_0$, implies that

$$\begin{aligned}
& M_2(x_0 - q^{**}(x_0), x(x_0, q^{**}(x_0), a^{**}(x_0))) \\
& = M_2^{**}(x_0 - q^{**}(x_0), x_0, x(x_0, q^{**}(x_0), a^{**}(x_0)))
\end{aligned}$$

This shows that C is feasible and that C gives the manager the same expected utility as C^{**} . **Q.E.D. (if)**

This proposition gives a complete characterization of the cases where communication is of no value for the incentive problem where the manager gets perfect information. If we take the negation of the statement in condition (b), we get as a corollary to the proposition the following necessary and sufficient conditions for communication to be of strict positive value.

Corollary.

Communication of cash flow information is of strict positive value if and only if there exists a set of cash flows of the previous period $SX_0 \subset X_0$ which has a positive probability, for which there for each $x_0 \in SX_0$ exists another $x'_0 \in SX_0$, such that

$$d^{**}(x_0) = d^{**}(x'_0)$$

and

$$x^{**}(x_0) \neq x^{**}(x'_0).$$

The value of communication of the cash flow of the previous period is in this case due to a more efficient allocation of the manager's compensation over the two periods. That is, communication facilitates an optimal smoothing of the manager's consumption over the two periods, which is preferred as shown by the first statement in the proposition.³ The argument is as follows. If it is optimal that the dividend at $t=2$ should be different for x_0 and x'_0 , then the compensations at $t=2$ must also be different, since otherwise, it is optimal for the manager always to report a cash flow of the previous period corresponding to the lower of the dividends at $t=2$ and then supply the amount of effort required to get the lower of those dividends. Since an optimal allocation of the manager's consumption possibilities requires that the manager's marginal utility of consumption is the same in both periods, the compensations at $t=1$ must also be different. If the optimal capital investments required to give the optimal dividends at $t=2$ gives the same dividend at $t=1$, then communication of the cash flow of the previous facilitates that the manager gets different compensations at $t=1$. Without communication, the compensations at $t=1$ must be equal for x_0 and x'_0 , since the compensations must be measurable with respect to the dividend at $t=1$.

It should be noted that the value of communication due to facilitating an optimal smoothing of the manager's compensation through time is not a case

for smoothing the earnings of the firm. As shown by the Revelation Principle (Proposition 8.3) the smoothing of the manager's consumption stream might as well be made directly in the compensation functions as in the accounting report on which the manager's compensation is based.

The main results of this section are contained in the propositions 8.6 and 8.7. The idea of the results is that if the optimal dividend function with communication is an invertible function of the manager's private information, then there is no conflict between the two roles played by the capital investment decision. The capital investment decision can at the same time serve as an efficient communication device for the manager's private information and as an efficient productive decision. If the optimal dividend function is not an invertible function of the manager's private information, then there might be a conflict between the two roles, which gives a strict positive value to communication. The following examples illustrates these results.

8.1.1 Examples.

Throughout the examples, it is assumed that the manager gets perfect information at date $t=1$. The focus in the examples is the role of the capital investment decision in communicating the manager's private information to the investors and its role of a productive decision.

Example 8.1: Communication of cash flow information is of no value.

The cash flow of the previous period can take one of two values with equal probability, either $x_{01} = 1$ or $x_{02} = 2$, and it provides the manager with perfect information:

$$x(x_{01}, q, a) = \begin{cases} 2 + a & \text{if } q \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(x_{02}, q, a) = \begin{cases} 4 + a & \text{if } q \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $u_1(\cdot) = u_2(\cdot) = 2 \cdot (\cdot)^{\frac{1}{2}}$. From Proposition 8.7 this implies, that $M_1(\cdot) = M_2(\cdot)$ when communication is allowed. The manager's disutility for effort is given by

$$H(a) = \begin{cases} 2 \cdot a^2 & \text{if } a \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

It can be seen very easily, that the optimal capital investment is greater than or equal to one for both signals. The incentive problem with communication can then be formulated as

$$\begin{aligned} \max \quad & 2 \cdot (M(x_{01}))^{\frac{1}{2}} + 2 \cdot (M(x_{02}))^{\frac{1}{2}} - a(x_{01})^2 - a(x_{02})^2 \\ & M(\cdot) \\ & d(\cdot), x(\cdot) \end{aligned}$$

subject to

$$\frac{1}{2} \cdot \{ d(x_{01}) + d(x_{02}) + x(x_{01}) + x(x_{02}) \} - M(x_{01}) - M(x_{02}) \geq \nabla$$

$$x_{01}: 2 \cdot (M(x_{01}))^{\frac{1}{2}} - a(x_{01})^2 \geq 2 \cdot (M(x_{02}))^{\frac{1}{2}} - a(x_{01}, x_{02})^2$$

$$x_{02}: 2 \cdot (M(x_{02}))^{\frac{1}{2}} - a(x_{02})^2 \geq 2 \cdot (M(x_{01}))^{\frac{1}{2}} - a(x_{02}, x_{01})^2$$

$$d(x_{01}) = x_{01} - q(x_{01}) = x_{02} - q(x_{02}, x_{01}) \leq 0$$

$$d(x_{02}) = x_{02} - q(x_{02}) = x_{01} - q(x_{01}, x_{02}) \leq 1$$

$$x(x_{01}) = 2 + a(x_{01}) = 4 + a(x_{02}, x_{01})$$

$$x(x_{02}) = 4 + a(x_{02}) = 2 + a(x_{01}, x_{02})$$

where the disutility terms drop out, when the respective effort levels are less than or equal to zero. Since a capital investment higher than one as its only consequence has a negative effect on the expected utility to the investors, the optimal capital investment is one for both signals and then $d^{**}(x_{01}) = 0$ and $d^{**}(x_{02}) = 1$.

For a market value of the firm at date $t=0$ of $\nabla = 3.5$ the optimal solution to the incentive problem can be found by solving the Kuhn-Tucker conditions to be

$$M^{**}(x_{01}) = 0.191 \quad d^{**}(x_{01}) = 0 \quad x^{**}(x_{01}) = 2.511$$

$$M^{**}(x_{02}) = 0.294 \quad d^{**}(x_{02}) = 1 \quad x^{**}(x_{02}) = 4.461$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 1.485$$

$$\nabla = 3.5$$

The optimal dividend function with communication is invertible and hence,

communication of cash flow information is of no value (Proposition 8.7). The reason is that the contract without communication,

$$M_1^*(d) = \begin{cases} M^{**}(x_{01}) & \text{if } d = 0 \\ M^{**}(x_{02}) & \text{if } d = 1 \\ P & \text{otherwise} \end{cases}$$

$$M_2^*(d,x) = \begin{cases} M^{**}(x_{01}) & \text{if } d = 0 \text{ and } x = x^{**}(x_{01}) \\ M^{**}(x_{02}) & \text{if } d = 1 \text{ and } x = x^{**}(x_{02}) \\ P & \text{otherwise} \end{cases}$$

$$q^*(x_{01}) = 1 \quad q^*(x_{02}) = 1$$

$$a^*(x_{01}) = a^{**}(x_{01}) \quad a^*(x_{02}) = a^{**}(x_{02})$$

is both feasible and gives the manager the same expected utility as the contract with communication.

This example illustrates the case where there is no conflict between the communication role and the productive role of the capital investment decision. Therefore, without an explicit communication structure the capital investment can be selected on the basis of productive considerations alone. At the same time, the optimal dividend function is as efficient as an explicit communication structure in communicating the manager's private information.

Example 8.2: Communication of cash flow information is of strict positive value.

Let the situation be as in Example 8.1 except that the capital investment must be at least equal to 2 for x_{02} to generate any output at date $t=2$:

$$x(x_{02}, q, a) = \begin{cases} 4 + a & \text{if } q \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

The optimal solution with communication is now

$$M^{**}(x_{01}) = 0.046 \quad d^{**}(x_{01}) = 0 \quad x^{**}(x_{01}) = 2.805$$

$$M^{**}(x_{02}) = 0.164 \quad d^{**}(x_{02}) = 0 \quad x^{**}(x_{02}) = 4.617$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 0.210$$

$$\nabla = 3.5$$

Since $d^{**}(x_{01}) = d^{**}(x_{02}) = 0$ and $x^{**}(x_{01}) \neq x^{**}(x_{02})$ it follows from the Corollary to Proposition 8.7 that communication is of strict positive value in this example. If the same dividend function is proposed without communication, $d^*(x_0) = 0$, $x_0 = x_{01}$, x_{02} , then the compensation to the manager at date $t=1$ must be the same for x_{01} and x_{02} , and this leads to a nonoptimal allocation of the manager's compensation over the two periods, since $M^{**}(x_{01}) \neq M^{**}(x_{02})$ (see the first claim in Proposition 8.7). This is the restriction that without communication, the compensation to the manager at date $t=1$ must be measurable with respect to the dividend at date $t=1$.

If the capital investment can be varied continuously, the value of communication is negligible in Example 8.2. Consider for a given $\varepsilon > 0$ the following contract without communication:

$$M_1(d) = \begin{cases} M^{**}(x_{01}) & \text{if } d = 0 \\ M^{**}(x_{02}) & \text{if } d = -\varepsilon \\ P & \text{otherwise} \end{cases}$$

$$M_2(d, x) = \begin{cases} M^{**}(x_{01}) & \text{if } d = 0 \text{ and } x = x^{**}(x_{01}) \\ M^{**}(x_{02}) & \text{if } d = -\varepsilon \text{ and } x = x^{**}(x_{02}) \\ P & \text{otherwise} \end{cases}$$

$$q(x_{01}) = 1 \quad q(x_{02}) = 2 + \varepsilon$$

$$a(x_{01}) = a^{**}(x_{01}) \quad a(x_{02}) = a^{**}(x_{02})$$

If the incentive problem is formulated as the investors' optimization problem, then it can easily be seen, that this contract is feasible. That is, it is incentive compatible and it gives the manager the same expected utility as the communication contract in Example 8.2. The investors' expected utility is $\bar{V}$ minus the expected over-investment for x_{02} , i.e. $\bar{V} - \frac{1}{2} \cdot \varepsilon$. By choosing ε sufficiently small, the expected utility of the investors can be made arbitrarily close to $\bar{V}$ and hence, the value of communication is infinitesimally small.

It is a general result, that the value of communication is negligible, when the cash flow of the previous period can take a finite number of values only, and the capital investment can be varied continuously. The optimal capital investment decision rule with communication only needs an infinitesimally small change to make the dividend function invertible in this case. Therefore, the loss in the productive efficiency of the capital investment decision rule is negligible. On the other hand, if the cash flow of the previous period can take a continuum of values, then small changes in the capital investment decision rule may not be sufficient to make the dividend function invertible. It may not even be optimal to make it invertible. The loss in productive efficiency of the capital investment might be too high compared to the gain from revealing the manager's private information at date $t=1$. To illustrate this in a similar example as above, it is assumed that also the capital investment can only take a finite number of values.

Example 8.3: Communication through the dividend function.

Make the following change in the productive function:

$$x(x_{01}, q, a) = \begin{cases} 2 + a & \text{if } q = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(x_{02}, q, a) = \begin{cases} 4 + a & \text{if } q = 2 \text{ or } q = 2.05 \\ 0 & \text{otherwise} \end{cases}$$

With a communication structure, the optimal solution to this incentive problem is the same as in Example 8.2. Without communication, the manager

has the option to make the dividend function invertible by choosing $q(x_{02}) = 2.05$. If he does this, it is easily seen that the incentive problem is equivalent to the incentive problem in Example 8.2 with a required market value of ∇ plus the expected over-investment for x_{02} , i.e. $\nabla + \frac{1}{2} \cdot 0.05$. The optimal solution is

$$M_1(d) = \begin{cases} 0.042 & \text{if } d = 0 \\ 0.160 & \text{if } d = -0.05 \\ P & \text{otherwise} \end{cases}$$

$$M_2(d, x) = \begin{cases} 0.042 & \text{if } d = 0 \text{ and } x = 2.828 \\ 0.160 & \text{if } d = -0.05 \text{ and } x = 4.625 \\ P & \text{otherwise} \end{cases}$$

$$q(x_{01}) = 1 \quad q(x_{02}) = 2.05$$

$$a(x_{01}) = 0.828 \quad a(x_{02}) = 0.625$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 0.134$$

$$\nabla = 3.5$$

The manager's expected utility is reduced from 0.210 with communication to 0.134 without communication when $q(x_{02}) = 2.05$. This reflects that a less efficient capital investment decision rule results, when the manager's private information is communicated indirectly through the dividend function, instead of directly through a communication structure.

Example 8.4: Private information not revealed to the investors at date $t=1$.

The other option available to the manager, is not to reveal his private information at date $t=1$ by choosing $q(x_{02}) = 2$ and then $d(x_{01}) = d(x_{02}) = 0$. In that case, the manager's compensation at date $t=1$ must be the same for both levels of the cash flow of the previous period, but he is still

able to precommit himself to only two possible values of the dividend at date $t=2$. Therefore, the incentive problem looks like:

$$\max_{M_1(\cdot), M_2(\cdot)} 2 \cdot (M_1(0))^{\frac{1}{2}} + (M_2(0, x(x_{01})))^{\frac{1}{2}} + (M_2(0, x(x_{02})))^{\frac{1}{2}} - a(x_{01})^2 - a(x_{02})^2$$

subject to

$$\frac{1}{2} \cdot \{ x(x_{01}) + x(x_{02}) \} - M_1(0) - \frac{1}{2} \cdot \{ M_2(0, x(x_{01})) - M_2(0, x(x_{02})) \} \geq \nabla$$

$$x_{01}: (M_2(0, x(x_{01})))^{\frac{1}{2}} - a(x_{01})^2 \geq (M_2(0, x(x_{02})))^{\frac{1}{2}} - a(x_{01}, x_{02})^2$$

$$x_{02}: (M_2(0, x(x_{02})))^{\frac{1}{2}} - a(x_{02})^2 \geq (M_2(0, x(x_{01})))^{\frac{1}{2}} - a(x_{02}, x_{01})^2$$

$$x(x_{01}) = 2 + a(x_{01}) = 4 + a(x_{02}, x_{01})$$

$$x(x_{02}) = 4 + a(x_{02}) = 2 + a(x_{01}, x_{02})$$

The optimal solution is:

$$M_1(d) = \begin{cases} 0.088 & \text{if } d = 0 \\ P & \text{otherwise} \end{cases}$$

$$M_2(d, x) = \begin{cases} 0.020 & \text{if } d = 0 \text{ and } x = 2.844 \\ 0.203 & \text{if } d = 0 \text{ and } x = 4.555 \\ P & \text{otherwise} \end{cases}$$

$$q(x_{01}) = 1 \quad q(x_{02}) = 2$$

$$a(x_{01}) = 0.844 \quad a(x_{02}) = 0.555$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 0.165$$

$$\nabla = 3.5$$

Since the manager's expected utility for $q(x_{02}) = 2$, (0.165), is higher than the expected utility for $q(x_{02}) = 2.05$, (0.134), the optimal capital investment decision rule is not to communicate through the dividend function, i.e. $d^*(x_{01}) = d^*(x_{02}) = 0$. The reason is that it is too costly in terms of the loss of productive efficiency. The value of communication is therefore the manager's expected utility in Example 8.2 minus the manager's expected utility in Example 8.4, i.e. $0.210 - 0.165 = 0.045$.

Whether it is optimal to communicate through the dividend function or not clearly depends on the loss of productive efficiency. If the over-investment for x_{02} in Example 8.3 could be reduced to 0.01, it would be optimal to communicate through the dividend function, since in that case the manager's expected utility would be 0.198, which is higher than the expected utility from not revealing his private information at date $t=1$.

These examples illustrate the main results of this section. First, a necessary condition for communication to be of strict positive value is that the optimal dividend function with communication is not invertible. Secondly, if communication has a strict positive value, the optimal dividend function without communication may or may not be invertible. Whether it will be invertible or not depends on the gains to revealing the manager's information and the loss of productive efficiency in doing so.

The gain to communication arises in the case with perfect information to the manager and, thus, in this example, only as a result of a preferred intertemporal risk sharing. In the next section we consider the case, where the manager is able to obtain an optimal intertemporal risk sharing through trading in the capital market and, hence, not only through the compensation scheme as in this example.

8.1.2 Communication and the value of trading.

In the previous analysis of this chapter it has been assumed that the manager is not allowed to trade and, in particular, not allowed to borrow and lend in the capital market (i.e., assumption (B3)). In this section we consider the consequences of allowing the manager to borrow and lend.

It appeared from the analysis in the previous sections that one of the gains to communication is that it facilitates an optimal smoothing of the manager's consumption over the two periods. In the case with perfect information to the manager and communication of his private information, the optimal compensations are such that the manager has no incentive to borrow or lend money in the capital market, if he was allowed to do that. This follows from the fact that for each signal received by the manager, the first order condition for optimal borrowing and lending is that the marginal utility of consumption are the same in both periods, and this condition is already satisfied without trading opportunities, i.e., condition (a) in Proposition 8.7. Hence, in this case it makes no difference, whether the manager is able to trade or not. However, it makes a difference, if the manager is not able to communicate his private information, or if he does not receive perfect information. First, we consider the case where the manager receives perfect information, but he is not able to communicate that information.

If borrowing and lending is not allowed, the manager may in this case have to make an inefficient capital investment decision in order to reveal his private information, such that he can make an optimal smoothing of his consumption over the two time periods (cf. the examples in the previous section). However, if he is able to borrow and lend, he can still use the optimal capital investment decision rule with communication, if he obtains the consumption smoothing through trading in the capital market. In fact, as shown below, it is strictly Pareto-preferred that the manager is able to borrow and lend if and only if communication is of strict positive value without trading opportunities.

If we assume that the manager's trades cannot be observed by the investors,

the incentive problem with trading opportunities for the manager and no communication can be defined as follows.

Definition 8.3 Pareto-optimal contracts with perfect information to the manager at $t=1$, unobservable trading opportunities and no communication.

The contract $C^* = (M_1^*(d), M_2^*(d, x), q^*(x_0), a^*(x_0), \delta^*(x_0))$ is a Pareto-optimal contract if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot), \delta(\cdot)}} \quad & \int_{X_0} u_1(M_1(d(x_0)) - \delta(x_0)) \cdot f(x_0) dx_0 \\ & + \int_{X_0} u_2(M_2(d(x_0), x(x_0)) + \delta(x_0)) \cdot f(x_0) dx_0 \\ & - \int_{X_0} H(a(x_0)) \cdot f(x_0) dx_0 \end{aligned}$$

subject to

$$\int_{X_0} \{ d(x_0) - M_1(d(x_0)) + x(x_0) - M_2(d(x_0), x(x_0)) \} \cdot f(x_0) dx_0 = \nabla$$

$$\begin{aligned} \forall x_0 \in X_0 : \{ q(x_0), a(x_0), \delta(x_0) \} \in \operatorname{argmax}_{(q, a, \delta) \in Q \times A \times \Delta} \{ & u_1(M_1(x_0 - q) - \delta) \\ & + u_2(M_2(x_0 - q, x(x_0, q, a)) + \delta) - H(a) \} \end{aligned}$$

where

$$d(x_0) = x_0 - q(x_0)$$

$$x(x_0) = x(x_0, q(x_0), a(x_0))$$

and $\delta(x_0)$ is the amount the manager lends out at $t=1$ given x_0 (or if negative, the amount he borrows).⁴

The difference between this incentive problem and the corresponding incentive problem without trading opportunities is that the manager's consumption plan is now determined by both the compensation he gets and by his optimal x_0 -contingent borrowing/lending plan. Moreover, the borrowing/lending plan is required to be incentive compatible.

The following proposition shows that the optimal value of the program in Definition 8.3 is the same as the optimal value of the program in Definition 8.2, i.e., the corresponding incentive problem with communication of the manager's private information and no trading opportunities.

Proposition 8.8

If $C^* = (M_1^*(d), M_2^*(d, x), q^*(x_0), a^*(x_0), \delta^*(x_0))$ is an optimal solution to the incentive problem in Definition 8.3, and if $C^{**} = (M_1^{**}(m), M_2^{**}(m), m^{**}(x_0), d^{**}(m), x^{**}(m), q^{**}(x_0, m), a^{**}(x_0, m))$ is an optimal solution to the program in Definition 8.2, then the manager is indifferent between the two contracts.

Proof:

Given the contract C^{**} , the manager has no incentive to borrow or lend, since the compensations are such that the first order condition for optimal borrowing and lending is satisfied for each x_0 , i.e., condition (a) in Proposition 8.7. Therefore, the manager is indifferent between the contract C^{**} and the optimal contract for the incentive problem with both communication and trading opportunities. Hence, the contract C^{**} is weakly Pareto-preferred to C^* , because communication is never of negative value. The proposition can therefore be proved by showing that there exists a feasible contract for the incentive problem in Definition 8.3 that gives the manager the same expected utility as the contract C^{**} .

Let the compensation at $t=1$ be some arbitrary positive constant M_1 for all dividends which are possible given the optimal dividend function with communication, i.e.,

$$M_1(d) = \begin{cases} M_1 & \text{if } d^{**^{-1}}(d) \neq \emptyset \\ P & \text{otherwise} \end{cases}$$

In order to assure that the manager gets the same x_0 -contingent consumption plan at $t=1$ as he gets with the contract C^{**} , define the amount he lends out at $t=1$ given x_0 by

$$\delta(x_0) = M_1 - M_1^{**}(x_0) \quad \forall x_0 \in X_0.$$

It was shown in the proof of Proposition 8.7 (i.e., the proof of the if statement in condition (b)) that if the set $d^{**,-1}(d) \cap x^{**,-1}(x)$ is not a singleton, then

$$M_1^{**}(x_0) = M_1^{**}(x'_0) \quad \forall x_0, x'_0 \in d^{**,-1}(d) \cap x^{**,-1}(x)$$

$$M_2^{**}(x_0) = M_2^{**}(x'_0) \quad \forall x_0, x'_0 \in d^{**,-1}(d) \cap x^{**,-1}(x)$$

Hence, the compensations at $t=2$ can be defined by

$$M_2(d, x) = \begin{cases} M_2^{**}(x_0) - \delta(x_0) & \text{for } x_0 \in d^{**,-1}(d) \cap x^{**,-1}(x) \neq \emptyset \\ p & \text{otherwise} \end{cases}$$

If the productions decision rules are the same as for the contract C^{**} , then the x_0 -contingent consumption plan at $t=2$ is also the same as for the contract C^{**} . Hence, let the production decision rules be defined by

$$q(x_0) = q^{**}(x_0)$$

$$a(x_0) = a^{**}(x_0)$$

To prove the proposition we must show that the contract defined above, i.e., $C = (M_1(d), M_2(d, x), q(x_0), a(x_0), \delta(x_0))$, is a feasible contract for the incentive problem in Definition 8.3, and that it gives the manager the same expected utility as the contract C^{**} .

Since the consumption plans and the effort decision rules are identical for the contracts C and C^{**} , the manager gets the same expected utility with both contracts. The market value of the firm at $t=0$ is also the same for the two contracts, since the production decision rules are the same, and since the total compensation paid to the manager is the same for each x_0 , i.e.,

$$\begin{aligned}
M_1(d(x_0)) + M_2(d(x_0), x(x_0)) &= M_1 + M_2^{**}(x_0) - \delta(x_0) \\
&= M_1 + M_2^{**}(x_0) - [M_1 - M_1^{**}(x_0)] \\
&= M_1^{**}(x_0) + M_2^{**}(x_0).
\end{aligned}$$

Hence, it remains to be shown that $\{q(x_0), a(x_0), \delta(x_0)\}$ is incentive compatible. Consider an arbitrary $x_0 \in X_0$, and an arbitrary amount to lend $\hat{\delta}$. Let also arbitrary production decisions $\hat{q}, \hat{a}$ be given. Assume without loss of generality that these production decisions are such that there exists a cash flow of the previous period, x'_0 , for which

$$x'_0 \in d^{**-1}(x_0 - \hat{q}) \cap x^{**-1}(x(x_0, \hat{q}, \hat{a}))$$

Otherwise, the manager would receive a penalty at $t=2$. Hence, from the precommitment to the dividend functions for the contract C^{**} it follows that there exists a cash flow of the previous period, x'_0 , such that

$$\hat{q} = q^{**}(x_0, x'_0)$$

$$\hat{a} = a^{**}(x_0, x'_0)$$

We now have from the definition of the contract C that

$$\begin{aligned}
&u_1(M_1(x_0 - \hat{q}) - \hat{\delta}) + u_2(M_2(x_0 - \hat{q}, x(x_0, \hat{q}, \hat{a})) + \hat{\delta}) - H(\hat{a}) \\
&= u_1(M_1(d^{**}(x'_0)) - \hat{\delta}) + u_2(M_2(d^{**}(x'_0), x^{**}(x'_0)) + \hat{\delta}) - H(a^{**}(x_0, x'_0)) \\
&= u_1(M_1 - \hat{\delta}) + u_2(M_2^{**}(x'_0) - \delta(x'_0) + \hat{\delta}) - H(a^{**}(x_0, x'_0))
\end{aligned}$$

From condition (a) in Proposition 8.7, i.e.,

$$u'_1(M_1^{**}(m)) = u'_2(M_2^{**}(m)) \quad \forall m \in X_0,$$

we get that $\delta(x'_0)$ is weakly preferred to $\hat{\delta}$. That is,

$$u_1(M_1 - \hat{\delta}) + u_2(M_2^{**}(x'_0) - \delta(x'_0) + \hat{\delta}) - H(a^{**}(x_0, x'_0))$$

$$\begin{aligned}
&\leq u_1(M_1 - \delta(x'_0)) + u_2(M_2^{**}(x'_0) - \delta(x'_0) + \delta(x'_0)) - H(a^{**}(x_0, x'_0)) \\
&= u_1(M_1^{**}(x'_0)) + u_2(M_2^{**}(x'_0)) - H(a^{**}(x_0, x'_0))
\end{aligned}$$

Since truthtelling is incentive compatible for the contract C^{**} we get that

$$\begin{aligned}
&u_1(M_1^{**}(x'_0)) + u_2(M_2^{**}(x'_0)) - H(a^{**}(x_0, x'_0)) \\
&\leq u_1(M_1^{**}(x_0)) + u_2(M_2^{**}(x_0)) - H(a^{**}(x_0))
\end{aligned}$$

Finally, we get from the definition of the contract C that

$$\begin{aligned}
&u_1(M_1^{**}(x_0)) + u_2(M_2^{**}(x_0)) - H(a^{**}(x_0)) \\
&= u_1(M_1 - \delta(x_0)) + u_2(M_2(x_0 - q(x_0), x(x_0, q(x_0), a(x_0))) + \delta(x_0)) - H(a(x_0))
\end{aligned}$$

and this proves that $\{ q(x_0), a(x_0), \delta(x_0) \}$ is incentive compatible. **Q.E.D.**

This proposition shows that, in the case with perfect information to the manager before he implements the production decisions, trading opportunities are a perfect substitute for communication of the manager's private information. Of course, this is due to the fact that communication in this case only derives its value from permitting an optimal intertemporal risk sharing, and that it is a matter of indifference whether this intertemporal risk sharing is obtained through the compensation scheme or through borrowing and lending in the capital market. An immediate consequence of the proposition is the following characterization of the cases in which it is of strict positive value that the manager is able borrow and lend.

Corollary.

With no communication, it is of strict positive value that the manager is able to borrow and lend in the capital market if and only if communication is of strict positive value with no trading opportunities.

The results in Proposition 8.8 and its corollary depend critically on the assumption that the manager receives perfect information before he implements the production decisions. If there exists a remaining uncertainty about the compensation at $t=2$ after the manager has observed the cash flow of the previous period, then it can still be valuable to reveal the manager's private information due to the fact that it permits intertemporal risk sharing. However, although it is possible to make a fully efficient intertemporal risk sharing without trading opportunities, the production incentives can be improved if the manager only obtains a second best intertemporal risk sharing. In fact, the gain of improved production incentives is always higher than the loss of only obtaining a second best intertemporal risk sharing. This is shown in following proposition.⁵

Proposition 8.9

Let $C^{**} = (M_1^{**}(m), M_2^{**}(m, x), m^{**}(x_0), d^{**}(m), q^{**}(x_0, m), a^{**}(x_0, m))$ be an optimal contract for the incentive problem in Definition 8.1 with communication of cash flow information. If there are at least two possible values of the compensation in the second period which have positive probability given x_0 , then

$$u_1'(M_1^{**}(x_0)) < E[u_2'(M_2^{**}(x_0, x)) \mid x_0, q^{**}(x_0), a^{**}(x_0)],$$

i.e., the manager has an incentive to lend out money given x_0 . If there is only one value of the compensation in the second period which is possible given x_0 , then

$$u_1'(M_1^{**}(x_0)) = E[u_2'(M_2^{**}(x_0, x)) \mid x_0, q^{**}(x_0), a^{**}(x_0)],$$

i.e., the manager has no incentive to lend out or to borrow money given x_0 .

Proof:

For each $x_0 \in X_0$ consider variations $\theta(x_0)$ in the manager's utilities at $t=1$ and at $t=2$,

$$u_1(M_1(x_0)) = u_1(M_1^{**}(x_0)) - \theta(x_0)$$

$$u_2(M_2(x_0, x)) = u_2(M_2^{**}(x_0, x)) + \theta(x_0)$$

Implicitly, these variations define x_0 -contingent variations in the compensations at $t=1$ and (x_0, x) -contingent variations in the compensations at $t=2$.

The compensation scheme $\{ M_1(\cdot), M_2(\cdot) \}$ induces truthtelling and the same production decisions as $\{ M_1^{**}(\cdot), M_2^{**}(\cdot) \}$. To see this, consider for any given x_0 arbitrary production decisions q , a , and message m , such that $m \in d^{**,-1}(x_0 - q)$. Then it follows from the incentive compatibility of C^{**} that

$$\begin{aligned} & u_1(M_1(m)) + \int_X u_2(M_2(m, x)) \cdot f(x|x_0, q, a) dx - H(a) \\ &= u_1(M_1^{**}(m)) - \theta(m) + \int_X \{ u_2(M_2^{**}(m, x)) + \theta(m) \} \cdot f(x|x_0, q, a) dx - H(a) \\ &= u_1(M_1^{**}(m)) + \int_X u_2(M_2^{**}(m, x)) \cdot f(x|x_0, q, a) dx - H(a) \\ &\leq u_1(M_1^{**}(x_0)) + \int_X u_2(M_2^{**}(x_0, x)) \cdot f(x|x_0, q^{**}(x_0), a^{**}(x_0)) dx - H(a^{**}(x_0)) \\ &= u_1(M_1^{**}(x_0)) - \theta(x_0) \\ &\quad + \int_X \{ u_2(M_2^{**}(x_0, x)) + \theta(x_0) \} \cdot f(x|x_0, q^{**}(x_0), a^{**}(x_0)) dx - H(a^{**}(x_0)) \\ &= u_1(M_1(x_0)) + \int_X u_2(M_2(x_0, x)) \cdot f(x|x_0, q^{**}(x_0), a^{**}(x_0)) dx - H(a^{**}(x_0)) \end{aligned}$$

Hence, $q^{**}(\cdot)$, $a^{**}(\cdot)$ and truthtelling are incentive compatible for the compensation scheme $\{ M_1(\cdot), M_2(\cdot) \}$.

The expected net dividends to the investors only depend on the expected compensation to the manager, since the production decisions are not affect-

ed by the variations in the manager's utilities. Hence, if the expected compensation to the manager is minimized with respect to the variations $\theta(x_0)$, then it must attain its minimum for $\theta(x_0) = 0$ with probability one, since the market value constraint is binding for the optimal contract C^{**} . The expected compensation to the manager is given by

$$E[u_1^{-1}(u_1(M_1^{**}(x_0)) - \theta(x_0)) \\ + E[u_2^{-1}(u_2(M_2^{**}(x_0, x)) + \theta(x_0)) \mid x_0, q^{**}(x_0), a^{**}(x_0)]]$$

Pointwise minimization with respect to $\theta(x_0)$ gives the following first order condition at $\theta(x_0) = 0$.

$$1/u_1'(M_1^{**}(x_0)) = E[1/u_2'(M_2^{**}(x_0, x)) \mid x_0, q^{**}(x_0), a^{**}(x_0)]$$

If there is only one value of the compensation in the second period which is possible given x_0 , i.e., $F(M_2^{**}(x_0, x) \mid x_0, q^{**}(x_0), a^{**}(x_0))$ is a degenerated distribution function, then

$$u_1'(M_1^{**}(x_0)) = E[u_2'(M_2^{**}(x_0, x)) \mid x_0, q^{**}(x_0), a^{**}(x_0)].$$

Otherwise, since $1/u_2'$ is a strictly convex function of u_2' it follows from Jensen's inequality that

$$u_1'(M_1^{**}(x_0)) < E[u_2'(M_2^{**}(x_0, x)) \mid x_0, q^{**}(x_0), a^{**}(x_0)].$$

The first order condition for an optimal lending decision given x_0 is that the marginal utility of consumption in the first period is equal to the expected marginal utility of consumption in the second period conditional on x_0 , i.e.,

$$u_1'(M_1^{**}(x_0) - \delta^{**}(x_0)) = E[u_2'(M_2^{**}(x_0, x) + \delta^{**}(x_0)) \mid x_0, q^{**}(x_0), a^{**}(x_0)]$$

where $\delta^{**}(x_0)$ is the optimal amount to lend out in the first period given x_0 . Hence, unless the manager has perfect information at $t=1$ about the second period compensation, he has an incentive to lend out some of his first period compensation, i.e., $\delta^{**}(x_0) > 0$. Q.E.D.

The proposition demonstrates that in the case with communication, it is valuable that the manager is able to precommit himself not to trade in the capital market. Moreover, if communication is of no value without trading opportunities, e.g., if $d^{**}(\cdot)$ is invertible, then it is also valuable that the manager is able to precommit himself not to trade in the case without communication. The reason is that the optimal incentives with respect to production decisions are obtained partly by imposing only a second best intertemporal risk sharing on the manager, and the fact that if the manager can borrow and lend and his trades are not observable to the shareholders, then he is not able to precommit himself not to undo the second best intertemporal risk sharing through trading.

Example 8.5 below demonstrates that if communication is of strictly positive value without trading opportunities, then cases exist with no communication, in which it is strictly valuable that the manager can trade. The tradeoff is between the gain from a fully efficient intertemporal risk sharing through trading and the loss due to the manager not being able to commit himself to the optimal second best intertemporal risk sharing for incentive purposes. It is also demonstrated in the example that if the manager is not able to commit himself not to trade, then communication might still be valuable. This follows from the fact that communication is not only valuable because it permits intertemporal risk sharing, but also because it allows preferred incentives.

Example 8.5

The cash flow of the previous period takes the value of either $x_{01} = 20$ or $x_{02} = 25$ with equal probability. Let $u_i(c_i) = (c_i)^{\frac{1}{2}}$, $i = 1, 2$. The possible effort levels are high and low, i.e., $a \in \{h, l\}$, with disutilities $H(h) = 2$ and $H(l) = 0$. Let the market value of the firm at $t=0$ be equal to $V = 30$. The matrix of conditional probabilities for the cash flow at $t = 2$ is given in the tableau below.

	$P(x x_0, q, a)$	
	$x = 20$	$x = 60$
$x_{01} = 20, q = 10, a = h, 1$	0.9	0.1
$x_{02} = 25, q = 15, a = h$	0.1	0.9
$x_{02} = 25, q = 17, a = h$	0.1	0.9
$x_{02} = 25, q = 15, a = 1$	0.9	0.1

We consider four incentive problems distinguished by whether the manager can trade and whether he can communicate his private information. In all four incentive problems the optimal production plans are as follows.

$$q^*(x_{01}) = 10, \quad d^*(x_{01}) = 10, \quad a^*(x_{01}) = 1$$

$$q^*(x_{02}) = 15, \quad d^*(x_{02}) = 10, \quad a^*(x_{02}) = h$$

Note that in the no communication cases it is not optimal for the manager to reveal his private information for x_{02} through the dividend, i.e., by choosing $q(x_{02}) = 17$. That is, the efficiency loss of the capital investment is higher than the gain from revealing the information.

The optimal compensation schemes, the optimal trading plans, and the resulting consumption plans for each of the four incentive problems are given below.

No trading opportunities, no communication:

	$c_1^* = M_1^*$	$c_2^* = M_2^*$	
		$x = 20$	$x = 60$
$x_{01} = 20$	9.22	3.19	18.37
$x_{02} = 25$	9.22	3.19	18.37

No trading opportunities, with communication:

	$c_1^* = M_1^*$		$c_2^* = M_2^*$	
	$x = 20$		$x = 60$	
$x_{01} = 20$	6.76		6.76	6.76
$x_{02} = 25$	12.96		1.82	14.82

With trading opportunities, no communication:

	M_1^*		M_2^*		δ^*	c_1^*	c_2^*	
	x = 20		x = 60				x = 20	x = 60
$x_{01} = 20$	13.35	- 1.33	14.64		7.04	6.31	5.71	21.68
$x_{02} = 25$	13.35	- 1.33	14.64		2.61	10.74	1.28	17.25

With trading opportunities, with communication:

It is easy to show that an indeterminacy of the optimal trading plan exists in this case, such that it can be assumed without loss of generality that $\delta^*(x_0) = 0$ for all levels of the cash flow of the previous period x_0 .

	M_1^*	M_2^*		δ^*	c_1^*	c_2^*	
		x = 20	x = 60			x = 20	x = 60
$x_{01} = 20$	6.67	6.67	6.67	0.00	6.67	6.67	6.67
$x_{02} = 25$	10.89	1.32	17.37	0.00	10.89	1.32	17.37

The manager's expected utilities for each of the four incentive problems are as follows.

	No communication	With communication.
No trading opportunities	5.07	5.20
With trading opportunities	5.13	5.17

Hence, trading opportunities are strictly valuable without communication in this example. It is due to the fact that it is too expensive for the manager to reveal his information through the dividend at $t=1$, i.e., choosing $q(x_{02} = 25) = 17$. Therefore, he gets the same compensation for both signals at $t=1$. However, if he is not allowed to trade, there is a substantial difference between the expected marginal utilities of the second period consumption, conditional on each of the two possible signals. Using the optimal compensation scheme given above for the "no trading opportunities, no communication" case, these marginal utilities can be determined to be as follows.

$$u_1'(c_1^*(x_{01} = 20)) = u_1'(c_1^*(x_{02} = 25)) = 0.16$$

$$E[u_2'(c_2^*) \mid x_{01} = 20, q^*(x_{01} = 20) = 10, a^*(x_{01} = 20) = 1] = 0.26$$

$$E[u_2'(c_2^*) \mid x_{02} = 25, q^*(x_{02} = 25) = 15, a^*(x_{02} = 25) = h] = 0.13$$

Hence, there is a very inefficient intertemporal risk sharing. That is, the manager has strong incentives to lend given $x_{01} = 20$ and strong incentives to borrow given $x_{02} = 25$.

The example also illustrates that communication can be valuable with trading opportunities, and that communication without trading opportunities dominates the other incentive problems, cf. Proposition 8.9.

In order to illustrate the role of the capital investment decision in this scenario, consider the case where the overinvestment for $x_{02} = 25$ is reduced to one unit, i.e., $q(x_{02} = 25) = 16$, other things being equal. In this case, it is optimal for the manager to reveal his private information through the dividends at $t=1$ without communication and trading opportunities, i.e., $q^*(x_{02} = 25) = 16$. The manager's expected utility for the "no trading opportunities, no communication" case is now equal to 5.12, whereas the optimal contracts for the other three programs are unchanged. By revealing his private information through the dividends at $t=1$, the manager now gets the optimal second best intertemporal risk sharing for incentive purposes. However, he obtains this at the expense of an inefficient capital investment for $x_{02} = 25$. In fact, he prefers a fully efficient intertemporal risk sharing obtained through trading to revealing his private information by making inefficient capital investments. This illustrates that without communication, trading opportunities might lead to more efficient capital investments even if the manager's trades cannot be observed by the investors.

The analysis in this section shows that unobservable trading opportunities for the manager are a perfect substitute for communication in the case where the manager receives perfect information. If the manager does not receive perfect information, and he is not able to communicate, then these trading opportunities can be of strictly positive as well as of strictly negative value. However, if the manager can communicate his private information, it is always preferred that the manager is able to commit himself not to borrow or lend (Proposition 8.9). Therefore, we continue the analysis assuming that the manager is able to make such a precommitment. This implies that the value of communication in the following analysis can be due to facilitating intertemporal risk sharing as well as to allowing improved incentives.

8.2 Audited cash flow information.

In the previous section cash flow accounting information is reported by the manager. Compensation functions are written such that he in his own interest reports truthfully. The main result of that section is that the dividend function with accounting information must be non-invertible to give manager-reported accounting information a strict positive social value. Otherwise, the observability of the dividend and the common-knowledge of the optimal capital investment decision rule are as efficient as accounting information in communicating the manager's private information.

In the previous analysis, the dividend at $t=1$ is equal to the cash flow of the previous period net of the capital investment regardless of what the manager reports to the investors. Hence, there is an implicit assumption that there is an auditor who audits the cash flow of the firm. Otherwise, the manager would have strong incentives not to pay the full amount to the investors and use the rest for his own benefit. However, the auditor does not report the result of his audit to the investors. The investors are therefore not able to observe e.g. whether a low dividend is caused by a high capital investment or a low cash flow of the previous period. If the manager is not motivated to reveal his private information about the cash flow he can substitute effort for higher capital investments and claim that the dividend is low because of a low cash flow. Therefore, there is a moral hazard problem associated with the capital investment decision.

In this section we describe the incentive problem for the case where the auditor does report the cash flow of the previous period to the investors. The auditor always reports truthfully and it is assumed that he has incentives to do so. In effect, the cash flow of the previous period can be treated as public information.

This view of accounting information is also consistent with a scenario where the manager prepares a report which, with some positive probability, will be investigated by an auditor. It is assumed that sufficient penalties are available, if the auditor detects the manager in lying, such that the threat of being detected will deter the manager from lying. Hence, it is

weakly Pareto preferred that the auditor reports the cash flow, if there is no direct reporting costs.

When both the cash flow of the previous period and the dividend at date $t=1$ are directly observable to the investors, the capital investment is also observable, since it is by definition equal to the difference between the cash flow and the dividend. This means, that there is no longer a moral hazard problem associated with the capital investment decision. Therefore, the incentive problem becomes similar to that of the standard agency model.

With the cash flow of the previous period as the only information received by the manager before the production decisions are implemented, the optimal capital investment decisions and the optimal effort decisions will be functions of x_0 , i.e., $q=q(x_0)$ and $a=a(x_0)$. Since the capital investment is observable we can assume without loss of generality that the manager pre-commits to an optimally determined x_0 -contingent capital investment decision rule. Hence, the compensation functions can be written on the form:

$$M_1(x_0, d, q) = \begin{cases} M_1(x_0) & \text{if } d = x_0 - q \text{ and } q = q(x_0) \\ P & \text{otherwise} \end{cases}$$

$$M_2(x_0, d, q, x) = \begin{cases} M_2(x_0, x) & \text{if } d = x_0 - q \text{ and } q = q(x_0) \\ P & \text{otherwise} \end{cases}$$

The incentive problem can then be formulated as follows.

Definition 8.4. Pareto-optimal contracts with cash flow information reported by an independent auditor.

The contract $C^* = (M_1^*(x_0), M_2^*(x_0, x), q^*(x_0), a^*(x_0))$ is a Pareto-optimal contract if it is a solution to the program:

$$\max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} \int_{X_0} u_1(M_1(x_0)) \cdot f(x_0) dx_0$$

$$\begin{aligned}
& + \int_{X_0} \int_X u_2(M_2(x_0, x)) \cdot f(x|x_0, q(x_0), a(x_0)) \cdot f(x_0) \, dx dx_0 \\
& - \int_{X_0} H(a(x_0)) \cdot f(x_0) \, dx_0
\end{aligned}$$

subject to

$$\begin{aligned}
& \int_{X_0} \{ x_0 - q(x_0) - M_1(x_0) \} \cdot f(x_0) \, dx_0 \\
& + \int_{X_0} \int_X \{ x - M_2(x_0, x) \} \cdot f(x|x_0, q(x_0), a(x_0)) \cdot f(x_0) \, dx dx_0 = 0
\end{aligned}$$

$$\forall x_0 \in X_0: \quad \int_X u_2(M_2(x_0, x)) \cdot f_a(x|x_0, q(x_0), a(x_0)) \, dx - H'(a(x_0)) = 0$$

The optimal compensation functions for this incentive problem are characterized in the following proposition.

Proposition 8.10.

If $C^* = (M_1^*(x_0), M_2^*(x_0, x), q^*(x_0), a^*(x_0))$ is a Pareto-optimal contract for the incentive problem with cash flow information reported by an independent auditor, then

$$(8.3) \quad 1/u_1'(M_1^*(x_0)) = 1/\lambda$$

$$(8.4) \quad 1/u_2'(M_2^*(x_0, x)) = 1/\lambda \cdot \left\{ 1 + \mu(x_0) \cdot \frac{f_a(x|x_0, q^*(x_0), a^*(x_0))}{f(x|x_0, q^*(x_0), a^*(x_0))} \right\}$$

where λ and $\mu(x_0)$ are optimal Lagrange-multipliers.⁶

Proof: see the appendix.

It is clear from the construction of the models that the fact that the cash flow is reported by an independent auditor instead of letting the manager report the cash flow without an audit is weakly Pareto preferred. A sufficient condition for a strict positive gain is that the manager has to take some risk at $t=1$ to make his report credible when there is no audit. This follows from the fact that he is not taking any risk at $t=1$ when there is an audit. Example 8.2 in section 8.1.1 can be used to illustrate this. In that example, the manager has a square-root utility function, and he gets perfect information at $t=1$. The production function is

$$x(x_{01}, q, a) = \begin{cases} 2 + a & \text{if } q \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(x_{02}, q, a) = \begin{cases} 4 + a & \text{if } q \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

where the cash flow of the previous period is $x_{01} = 1$ or $x_{02} = 2$ with equal probability. The optimal solution with communication is

$$M_1^{**}(x_{01}) = M_2^{**}(x_{01}) = 0.046 \quad d^{**}(x_{01}) = 0 \quad x^{**}(x_{01}) = 2.805$$

$$M_1^{**}(x_{02}) = M_2^{**}(x_{02}) = 0.164 \quad d^{**}(x_{02}) = 0 \quad x^{**}(x_{02}) = 4.617$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 0.210$$

$$V = 3.5$$

In this case the manager takes some risk to make the report credible, i.e., his compensation differs for x_{01} and x_{02} . If there is an audit, then the cash flow of the previous period and the capital investment can be directly observed by the investors at $t=1$. Hence, when the investors observe the dividend at $t=2$, they are also able to calculate the effort supplied by the manager. In effect, the manager is able to precommit to an optimally determined effort function, $a(x_0)$, and the first best solution can be obtained, i.e.,

$$M^{**}(x_{01}) = 0.348 \quad d^{**}(x_{01}) = 0 \quad x^{**}(x_{01}) = 2.848$$

$$M^{**}(x_{02}) = 0.348 \quad d^{**}(x_{02}) = 0 \quad x^{**}(x_{02}) = 4.848$$

$$E[u_1(\cdot) + u_2(\cdot) - H(\cdot)] = 0.921$$

$$V = 3.5$$

When there is an audit, the manager takes no risk at all and he supplies the same effort for both levels of the cash flow of the previous period. The value of the audit of the manager's report is $0.921 - 0.210 = 0.711$.

When the manager's report is audited, the cash flow of the previous period can be observed directly by the investors, and the manager is able to pre-commit to an optimally determined capital investment function. Hence, there is no moral hazard problem associated with the capital investment decision. However, there is still a moral hazard problem associated with the effort decision, and this affects the optimal capital investment function. This is demonstrated by the following proposition.

Proposition 8.11.

If $C^* = (M_1^*(x_0), M_2^*(x_0, x), q^*(x_0), a^*(x_0))$ is a Pareto-optimal contract for the incentive problem with cash flow information reported by an independent auditor, then $q^*(x_0)$ is determined such that

$$\begin{aligned} (8.5) \quad 1 - \int_X x \cdot f_q(x|x_0, q^*(x_0), a^*(x_0)) dx \\ + \int_X M_2^*(x_0, x) \cdot f_q(x|x_0, q^*(x_0), a^*(x_0)) dx \\ = \quad 1/\lambda \cdot \int_X u_2(M_2^*(x_0, x)) \cdot f_q(x|x_0, q^*(x_0), a^*(x_0)) dx \\ + 1/\lambda \cdot \mu(x_0) \cdot \int_X u_2(M_2^*(x_0, x)) \cdot f_{aq}(x|x_0, q^*(x_0), a^*(x_0)) dx \end{aligned}$$

Proof: see the appendix.

The left-hand side of (8.5) is the marginal change in the cum-dividend market value of the firm at $t=1$ given the level of cash flow of the previous period. The first two terms on the left-hand side is the difference between the marginal input and the expected marginal output. A first best solution for the capital investment given the level of effort is characterized by this difference being equal to zero. As it appears from (8.5) the optimal capital investment is, in general, different from the first best solution (i.e., when the risk sharing and the production decisions are full Pareto efficient). There are two reasons for this. First, a change in the capital investment affects the probability distribution of the dividend at $t=2$. Since the manager's compensation depends on the dividend at $t=2$, the probability distribution of his compensation is also affected by the capital investment given the compensation function. The third term on the left-hand side and the first term on the right-hand side reflect this. In a first best solution, the manager's compensation does not depend on the dividend at $t=2$, and hence, these two terms will be equal to zero.

The second term on the right-hand side reflects that a change in the capital investment also affects the likelihood ratio, f_a/f , and thereby the form of the optimal compensation function.

Both of these two effects on the optimal capital investment decision rule are due to the fact that the manager's compensation depends on the dividend at $t=2$. This dependence is caused by the moral hazard problem associated with the effort decision.

The following example illustrates that the optimal capital investment can be lower, higher or equal to the first best solution given the optimal level of effort depending on the production function.

Example 8.6.

Assume that, conditional on the cash flow of the previous period, the dividend at $t=2$ is exponentially distributed with mean $m(x_0, q, a)$, i.e.,

$$f(x|x_0, q, a) = m(x_0, q, a)^{-1} \cdot \exp\{ - x / m(x_0, q, a) \}$$

The first best solution for the capital investment given the cash flow of the previous period and the optimal level of effort, $q^{**}(x_0, a^*(x_0))$ is determined as the capital investment where the marginal investment is equal to the conditional expected marginal dividend at $t=2$. That is,

$$\frac{\partial}{\partial q} \left\{ q - \int_X x \cdot f(x|x_0, q, a^*(x_0)) dx \right\} \Big|_{q = q^{**}(x_0, a^*(x_0))} = 0$$

which is equivalent to

$$(8.6) \quad m_q(x_0, q^{**}(x_0), a^*(x_0)) = 1$$

To ensure that an interior optimal solution for the first best capital investment exists we assume that

$$m_q(x_0, q, a) > 0 \quad \text{and} \quad m_q(x_0, q, a) < 0 \quad \forall x_0, q, a$$

and also that the marginal productivity of effort is positive,

$$m_a(x_0, q, a) > 0 \quad \forall x_0, q, a.$$

The first order condition for the capital investment, (8.5), can be written as

$$\begin{aligned} m_q(x_0, q^*(x_0), a^*(x_0)) &= 1 + \frac{\partial}{\partial q} \int_X \{ M_2^*(x_0, x) \\ &\quad - u_2(M_2^*(x_0, x)) / u_2'(M_2^*(x_0, x, q)) \} \cdot f(x|x_0, q, a^*(x_0)) dx \Big|_{q=q^*(x_0)} \\ &= 1 + \int_X \{ M_2^*(x_0, x) - u_2(M_2^*(x_0, x)) / u_2'(M_2^*(x_0, x)) \} \cdot f_q(x|x_0, q^*(x_0), a^*(x_0)) dx \\ &\quad + \int_X \{ u_2(M_2^*(x_0, x)) \cdot u_2''(M_2^*(x_0, x)) \cdot M_{2q}(x_0, x, q^*(x_0)) / (u_2'(M_2^*(x_0, x)))^2 \} \\ &\quad \cdot f(x|x_0, q^*(x_0), a^*(x_0)) dx \end{aligned}$$

where the function $M_2(x_0, x, q)$ is defined by the first order condition for the compensation function taking the capital investment as a variable,

$$1/u_2'(M_2(x_0, x, q)) = 1/\lambda \cdot \left\{ 1 + \mu(x_0) \cdot \frac{f_a(x|x_0, q, a^*(x_0))}{f(x|x_0, q, a^*(x_0))} \right\},$$

such that $M_2(x_0, x, q^*(x_0)) = M_2^*(x_0, x)$.

If we assume that the manager has a square-root utility function,

$$u_2(M_2) = 2 \cdot (M_2)^{\frac{1}{2}},$$

then the first order condition for the capital investment takes the form

$$m_q(x_0, q^*(x_0), a^*(x_0)) =$$

$$\begin{aligned} 1 & - \int_X M_2^*(x_0, x) \cdot f_q(x|x_0, q^*(x_0), a^*(x_0)) \, dx \\ & - \int_X M_{2q}^*(x_0, x, q^*(x_0)) \cdot f(x|x_0, q^*(x_0), a^*(x_0)) \, dx \end{aligned}$$

$$= 1 - \frac{\partial}{\partial q} \int_X M_2(x_0, x, q) \cdot f(x|x_0, q, a^*(x_0)) \, dx \Big|_{q=q^*(x_0)}$$

Since the marginal productivity of q is decreasing with q , i.e. $m_{qq}(x_0, q, a) < 0$, we have that the relation between the optimal capital investment, $q^*(x_0)$, and the first best capital investment given the optimal level of effort, $q^{**}(x_0, a^*(x_0))$, is determined by the sign of

$$\frac{\partial}{\partial q} \int_X M_2(x_0, x, q) \cdot f(x|x_0, q, a^*(x_0)) \, dx \Big|_{q=q^*(x_0)}$$

such that if the sign is negative then $q^*(x_0) < q^{**}(x_0, a^*(x_0))$, and visa versa.

The assumption of exponentially distributed dividends at $t=2$ implies that

$$\frac{f_a(x|x_0, q, a)}{f(x|x_0, q, a)} = \frac{\{x - m(x_0, q, a)\} \cdot m_a(x_0, q, a)}{(m(x_0, q, a))^2}$$

such that

$$(M_2(x_0, x, q))^{\frac{1}{2}} = 1/\lambda \cdot \left\{ 1 + \mu(x_0) \frac{\{ x - m(x_0, q, a^*(x_0)) \} \cdot m_a(x_0, q, a^*(x_0))}{(m(x_0, q, a^*(x_0)))^2} \right\}$$

Provided that the first order condition for the compensation function is valid for all $x \in [0, \infty)$, i.e.,

$$1 + \mu(x_0) \cdot \frac{\{ x - m(x_0, q, a^*(x_0)) \} \cdot m_a(x_0, q, a^*(x_0))}{(m(x_0, q, a^*(x_0)))^2} > 0 \quad \text{all } x \in [0, \infty)$$

we get that

$$\begin{aligned} & \int_X M_2(x_0, x, q) \cdot f(x|x_0, q, a^*(x_0)) \, dx \\ &= 1/\lambda^2 \cdot [1 + \mu(x_0)^2 \cdot \{ m_a(x_0, q, a^*(x_0)) / m(x_0, q, a^*(x_0)) \}^2] \end{aligned}$$

Hence,

$$\begin{aligned} & \frac{\partial}{\partial q} \int_X M_2(x_0, x, q) \cdot f(x|x_0, q, a^*(x_0)) \, dx \Big|_{q=q^*(x_0)} \\ &= 1/\lambda^2 \cdot [2 \cdot \mu(x_0)^2 \cdot \{ m_a(x_0, q^*(x_0), a^*(x_0)) / m(x_0, q^*(x_0), a^*(x_0)) \}] \cdot \\ & \quad \{ m_{aq}(x_0, q^*(x_0), a^*(x_0)) \cdot m(x_0, q^*(x_0), a^*(x_0)) \\ & \quad - m_a(x_0, q^*(x_0), a^*(x_0)) \cdot m_q(x_0, q^*(x_0), a^*(x_0)) \} / m(x_0, q^*(x_0), a^*(x_0))^2 \end{aligned}$$

The term in the $[\cdot]$ -parenthesis is always positive. Hence, the relation between the optimal capital investment, $q^*(x_0)$, and the first best capital investment given the optimal level of effort, $q^{**}(x_0, a^*(x_0))$, is determined by the sign of the term in the $\{\cdot\}$ -parenthesis such that

$$(8.7) \quad \text{sign} \{ q^*(x_0) - q^{**}(x_0, a^*(x_0)) \} =$$

$$\begin{aligned} & \text{sign} \{ m_{aq}(x_0, q^*(x_0), a^*(x_0)) \cdot m(x_0, q^*(x_0), a^*(x_0)) \\ & \quad - m_a(x_0, q^*(x_0), a^*(x_0)) \cdot m_q(x_0, q^*(x_0), a^*(x_0)) \} \end{aligned}$$

This sign depends on how the capital investment and the level of effort interrelates in determining the conditional mean of the dividends at $t=2$. We consider the following three cases.

Case 1: $m(x_0, q, a) = h(x_0, q) + k(x_0, a)$

In this case the capital investment and the level of effort enters additively in the conditional mean of the dividends at $t=2$. From (8.7) we get that

$$\begin{aligned} \text{sign} \{ q^*(x_0) - q^{**}(x_0, a^*(x_0)) \} = \\ \text{sign} \{ 0 \cdot (h(x_0, q^*(x_0)) + k(x_0, a^*(x_0))) \\ - k_a(x_0, a^*(x_0)) \cdot h_q(x_0, q^*(x_0)) \} = \text{negative} \end{aligned}$$

since the marginal productivity of both the capital investment and the level of effort is strictly positive. Hence, the manager underinvests in this case relative to the first best capital investment. In this case, the first best capital investment is independent of the level of effort, such that the manager also underinvest relative to the first best level of effort.

Case 2: $m(x_0, q, a) = h(x_0, q) \cdot k(x_0, a)$

When the capital investment and the level of effort enter multiplicatively we get from (8.7) that

$$\begin{aligned} \text{sign} \{ q^*(x_0) - q^{**}(x_0, a^*(x_0)) \} = \\ \text{sign} \{ h_q(x_0, q^*(x_0)) \cdot k_a(x_0, a^*(x_0)) \\ - k_a(x_0, a^*(x_0)) \cdot h_q(x_0, q^*(x_0)) \} = \text{zero}. \end{aligned}$$

Hence, in this case the optimal capital investment is equal to the first best capital investment given the optimal level of effort.

$$\text{Case 3: } m(x_0, q, a) = h(x_0, q)^{k(x_0, a)}$$

Here we get from (8.7) that

$$\begin{aligned} & \text{sign} \{ q^*(x_0) - q^{**}(x_0, a^*(x_0)) \} \\ &= \text{sign} \{ m(x_0, q^*(x_0), a^*(x_0))^2 \cdot h_q(x_0, q^*(x_0)) \cdot k_a(x_0, a^*(x_0)) / h(x_0, q^*(x_0)) \} \\ &= \text{positive.} \end{aligned}$$

In this case it is optimal to overinvest relative to the first best investment given the optimal level of effort.

These examples illustrate that the optimal capital investment is affected by the moral hazard problem associated with the effort decision, even when the capital investment is observable to the investors. The examples also show that the relationship between the optimal capital investment and the first best capital investment given the optimal level of effort depends on the production function. Hence, there is no general relationship which states whether the moral hazard problem associated with the effort decision results in over- or underinvestment.

To get some economic intuition into what causes over- or underinvestment, note that when we compare the optimal investment to the first best investment given the optimal level of effort, then we have removed the effect on the dividends to the shareholders. That is, over- or underinvestment is only optimal if it becomes cheaper to motivate the manager or if it creates preferred incentives for the manager's supply of effort. That is, the scale of production may affect the incentive problem with respect to the effort decision.

In our example an increase in the capital investment increases the variance of the second period dividend as well and, thus, it becomes more difficult to draw inference about the manager's supply of effort from the observed dividend. That is, it becomes more expensive to motivate the manager to supply a given level of effort. Hence, this tends to reduce the optimal capital investment. The other effect is due to the fact that the capital investment may also affect the marginal productivity of effort. If an increase in the capital investment increases the marginal productivity of effort, then this effect tends to increase the optimal capital investment.

In our example with exponentially distributed dividends and a square-root utility function it follows from (8.7) that the net-effect on the optimal capital investment can be measured by the derivative of the ratio of the marginal productivity of effort to the standard deviation of the dividends, i.e.,

$$\text{sign} \{ q^*(x_0) - q^{**}(x_0, a^*(x_0)) \} = \text{sign} \left\{ \frac{\partial}{\partial q} \frac{m_a(x_0, q, a^*(x_0))}{m(x_0, q, a^*(x_0))} \bigg|_{q=q^*(x_0)} \right\}$$

In the first case, where the capital investment is additively related to the supply of effort, the marginal productivity of effort does not depend on the capital investment. Hence, the only effect of an increase in the capital investment is that it makes it more difficult to motivate the manager. Therefore, it is optimal to underinvest. In the second case, the ratio of the marginal productivity of effort to the standard deviation of the dividends is independent of the capital investment, i.e., the two effects on the optimal capital investment offset each other in this case. In the third case, the effect on the marginal productivity of effort of an increase in the capital investment is positive, and it dominates the effect on the standard deviation of the dividends, i.e.,

$$\frac{\partial}{\partial q} \frac{m_a(x_0, q, a^*(x_0))}{m(x_0, q, a^*(x_0))} = \frac{\partial}{\partial q} \{ k_a(x_0, a^*(x_0)) \cdot \ln(h(x_0, q)) \} > 0,$$

such that it is optimal to overinvest in this case. However, these results are specific results for our example.

8.3 Conclusion.

In this chapter, we have considered a model in which the manager's private information is the cash flow of the previous period. In section 8.1 we analyzed a model, in which the manager makes an accounting report without an audit, whereas the accounting report is audited in section 8.2.

When the manager makes an accounting report based on his observation of the cash flow of the previous period, then manager and investors can contract on the dividends as well as on the accounting report. Before the manager observes the cash flow of the previous period, the accounting principle is determined by the manager such that his expected utility is maximized, given that the production decision rules as well as the accounting principle are incentive compatible.

It is shown in Proposition 8.1 that it is weakly Pareto-preferred that the manager makes the accounting report if there are no direct reporting costs. In Proposition 8.2 it is shown that it is weakly Pareto-preferred that the manager precommits himself to an optimally determined dividend function which is contingent on the accounting report. The common-knowledge of the accounting principle induces a partition on the set of possible cash flows of the previous period. Hence, when the investors receive the accounting report, then they are also able to tell in which subset of possible cash flows of the previous period the true cash flow is. This is used in Proposition 8.3 to make a re-labelling of the manager's compensation functions and the optimally determined dividend function. The re-labelling is such that the manager's and the investors' expected utilities are unchanged, but the accounting principle is simply that the manager reports the cash flow of the previous period truthfully, i.e. the Revelation Principle is used. Hence, in this model, there are no gains to income smoothing or accrual accounting. Any income statement the manager can make can also be made by the investors, given their knowledge of the optimal production decision rules and given that the manager reports his private information truthfully.

The main results of this section are contained in Propositions 8.6 and 8.7. Proposition 8.6 gives a sufficient condition for no value of the accounting

reports made by the manager. The condition is that the optimally determined dividend function with accounting information is an invertible function. Proposition 8.7 gives necessary and sufficient conditions for no value of the accounting reports made by the manager in the case, where the manager gets perfect information. This proposition also shows that one of the gains to revealing the manager's information is that it may facilitate a preferred intertemporal risk sharing. A number of examples are given in section 8.1.1 which illustrate these results. The interpretation of the results are the following. If the dividend function is invertible, then the investors are able to infer the manager's private information from the observed dividend and through their knowledge of the capital investment decision rule. If the optimal dividend function with accounting information is invertible, then it is also invertible without accounting information, because in that case the capital investment decision can serve as a communication device for the manager's private information without disturbing the production incentives. If the optimal dividend function with accounting information is not invertible, then the optimal dividend function without accounting information may or may not be invertible. Whether it will be invertible or not, depends on the gains to revealing the manager's information and the loss of productive efficiency in doing that.

It has been assumed in the previous analysis that the manager is not allowed to lend or to borrow in the capital market. Hence, intertemporal risk sharing has to be made through the compensation scheme. The examples in section 8.1.1 illustrates that communication may be of value, because it assures that the optimal intertemporal risk sharing can be obtained for any capital investment decision rule, whereas this is not always possible without communication. Therefore, we consider in section 8.1.2 the consequences of allowing the manager to obtain the optimal intertemporal risk sharing through borrowing and lending in the capital market. In the case, where the manager gets perfect information, it is shown in Proposition 8.8 that the manager can obtain the same gains from borrowing and lending in the capital market as he can get with communication of his private information. This is so, even if his trades are not observable to the investors. Hence, without communication, it is strictly preferred that the manager is able to borrow and lend if and only if communication is of strict positive value without trading opportunities. With communication, the manager is indifferent as to

whether or not he is able to trade. Of course, these results are due to the fact that communication is of no value for incentive purposes when the manager gets perfect information before he communicates.

In Proposition 8.9 it is shown that if the manager is able to communicate, and he does not get perfect information, then the manager strictly prefers that he is able to precommit himself not to borrow or lend in the capital market. The reason is that in this case the optimal incentives with respect to production decisions are obtained partly by leaving the manager with a residual desire to save some of his first period compensation. Communication may still derive value from permitting a preferred intertemporal risk sharing. However, it is optimal that the manager is not fully insured.

If communication is of strict positive value without trading opportunities, cases exist with no communication, where it is of value that the manager can trade. This will be the case if the gain from a fully efficient intertemporal risk sharing obtained through trading is higher than the loss due to the fact that the manager cannot precommit himself to trade in such a way that the optimal intertemporal risk sharing for incentive purposes is obtained. Of course, if the manager's trades were observable to the shareholders, then he would be able to make such a precommitment.

When the manager does not receive perfect information, communication can be valuable for intertemporal risk sharing purposes as well as for incentive purposes. Hence, communication might also be of value in the case where the manager is able to trade, because then the manager does not have to make inefficient capital investment decisions in order to reveal his private information for incentive purposes. However, because it is strictly preferred that the manager is not able to trade in the communication case, we continue the analysis without trading opportunities.

In section 8.2, we analyzed a model in which the manager's report is audited. It is assumed that the auditor has incentives to perform the audit truthfully. In effect, the cash flow of the previous period can be considered as public information. This implies that the moral hazard problem associated with the capital investment decision is eliminated, since the capital investment can be calculated by the investors as the difference

between the cash flow of the previous period and the current dividend. Hence, the incentive problem is reduced to that of the standard agency model. However, it is shown in Proposition 8.11 that the moral hazard problem associated with the effort decision affects the optimal capital investments. An example shows that the effect on the optimal capital investments may cause underinvestment as well as overinvestment, depending on how the capital investment and the supply of effort are related in the production function. It is also shown by an example that audited cash flow information may be valuable even if cash flow information reported by the manager is of no value. Of course, this is due to the fact that there are no incentive problems associated with audited information in our model.

Footnotes for Chapter 8.

1. If it is assumed that the manager's utility function is unbounded from below such penalties always exist. For example, if $u_i(c_i)$ goes to minus infinity as consumption goes to zero, then P must be a positive number sufficiently close to zero.
2. Note that income smoothing in this context only applies to the choice of accounting principle and not to "real" income smoothing as analyzed by Lambert (1984).
3. See Section 8.1.2 below.
4. Note that it is assumed that the manager has infinite marginal utility for zero consumption (i.e., (A5)) so that we do not have to consider the possibility of bankruptcy. It is also assumed that the riskless interest rate is equal to zero (i.e., (B2)). However, this is not a critical assumption as long as the manager and the investors can borrow and lend at the same rate.
5. Rogerson (1985a) gives a similar result for a two period model without private information.
6. Note that in this case the compensation scheme is also such that if the cash flow of the previous period does not provide perfect information, then it is strictly preferred that the manager is not able to borrow and lend in the capital market.

CHAPTER 9.

Additional Accounting Information

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In this chapter we return to the general case, where the manager has more private information than the cash flow of the previous period. We assume throughout this chapter that the cash flow of the previous period is reported by an independent auditor so that it can be treated as public information. The additional information is either not reported or reported by the manager through a communication structure without an audit. The reason for considering this model is that cash flows are quite easily checked by an auditor, while this might not be the case with managers' information about e.g. future productivity of capital. That is, for this type of information it might be cheaper to motivate the manager to report his information than to have an auditor making an investigation of all the information processed by the manager.

In section 9.1 we define the incentive problems without and with communication of the manager's additional information. For the communication case, we characterize the optimal compensation functions in Proposition 9.1 and the optimal capital investment function in Proposition 9.2. When the cash flow of the previous period is observable to the investors, then the capital investment is also observable to the investors. This is reflected in the characterizations of the compensation functions by the absence of the derivative of the log-likelihood function with respect to the capital investment which is present in (7.2) and (8.2). In Example 9.1 it is shown that the optimal capital investment is affected by the moral hazard problems associated with the supply of effort as well as with the communication of the manager's private information.

In section 9.2 we analyze the value of accounting information reported by the manager. The signals $\xi \in \Xi$ provide an abstract and complete representation of the manager's private information. Accounting reports use a certain language such as values of the assets and the liabilities. Hence, an accounting principle can be defined as a function from the set of signals received by the manager to some set of accounting reports. If an accounting principle is an invertible function on the set of signals received by the manager then the set of accounting reports is also a complete representation of the manager's information. Hence, the issue is whether a non-invertible accounting principle can achieve the same gains of communication as an accounting principle which provides a complete representation of the

manager's private information. Proposition 9.3 gives sufficient conditions for this. These conditions are as follows: First, the investors must be able to infer the relevant parts of the manager's private information from the combination of the accounting report and the observed capital investment through their knowledge of the accounting principle and the optimal capital investment function. Secondly, the optimal capital investment function must also be optimal if the manager can report all of his private information through an accounting report. That is, the capital investment decision must not be affected by the limited communication possibilities.

The main result of this chapter is contained in Proposition 9.3. It relates to the general problem of choosing between alternative information sources. The general result is that it is of no social value that the manager communicates additional information through accounting reports if that information can be inferred from observable production decisions and if these production decisions are not affected by limited communication in the accounting reports.

The chapter is concluded in section 9.3 with concluding remarks on the analysis in Part II.

9.1 The optimal contract.

First, consider the case where the manager does not report his private information ξ to the investors. In this case, the cash flow of the previous period and the dividend are public information at $t=1$. Since the capital investment is equal to the cash flow of the previous period minus the dividend at $t=1$ the admissible compensation functions can be written as

$$M_1 = M_1(x_0, q)$$

$$M_2 = M_2(x_0, q, x)$$

For each pair of signals to the manager at $t=1$, (x_0, ξ) , the manager chooses the capital investment, $q(x_0, \xi)$, and the level of effort, $a(x_0, \xi)$, which maximizes his conditional expected utility given the compensation functions. Hence, the incentive problem can be defined as follows.

Definition 9.1.

Pareto-optimal contract with cash flow information reported by an independent auditor and no communication of additional information.

The contract $C^* = (M_1^*(x_0, q), M_2^*(x_0, q, x), q^*(x_0, \xi), a^*(x_0, \xi))$ is a Pareto-optimal contract if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} & \int_{X_0} \int_{\Xi} u_1(M_1(x_0, q(x_0, \xi))) \cdot f(x_0, \xi) d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X u_2(M_2(x_0, q(x_0, \xi), x)) \cdot f(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) dx d\xi dx_0 \\ & - \int_{X_0} \int_{\Xi} H(a(x_0, \xi)) \cdot f(x_0, \xi) d\xi dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \int_{\Xi} \{ x_0 - q(x_0, \xi) - M_1(x_0, q(x_0, \xi)) \} \cdot f(x_0, \xi) d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X \{ x - M_2(x_0, q(x_0, \xi), x) \} \cdot f(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) dx d\xi dx_0 = \nabla \end{aligned}$$

$$\forall (x_0, \xi) \in X_0 \times \Xi : \{q(x_0, \xi), a(x_0, \xi)\} \in \operatorname{argmax}_{(q, a) \in Q \times A} \{ u_1(M_1(x_0, q))$$

$$+ \int_X u_2(M_2(x_0, q, x)) \cdot f(x|x_0, \xi, q, a) dx - H(a) \}$$

When the manager does not report his private information ξ he can only precommit to a capital investment function contingent on the cash flow of the previous period, i.e.,

$$q(x_0, \xi) = q(x_0) \quad \forall (x_0, \xi) \in X_0 \times \Xi.$$

This type of precommitment precludes the manager from varying the capital investment with his private information. Thus, the precommitment prevents the manager from shirking, but it also prevents him from using useful productivity information in his capital investment decision. It is therefore not, in general, optimal that the manager makes this precommitment.

As in Chapter 8 it can be shown that it is weakly Pareto-preferred that the manager issues an accounting report to the investors based on his private information,

$$m: X_0 \times \Xi \rightarrow M,$$

such that, for each value of the public signal x_0 and private signal ξ , he makes a report $m(x_0, \xi) = m$ (Proposition 8.1). This follows from the fact that the compensation functions can be chosen such that the manager's compensations do not depend on what he reports. Similarly, it can be shown that it is weakly Pareto-preferred that the manager precommits to an optimally determined capital investment function depending on the public signal x_0 and the manager's report m (Proposition 8.2),

$$q = q(x_0, m).$$

This follows from the manager's opportunity to precommit to the optimal capital investment function without precommitment. Finally, it can also be shown as in Chapter 8 that the Revelation Principle applies (Proposition 8.3), such that it can be assumed that the compensation functions are truth-inducing contracts, i.e.,

$$m(x_0, \xi) = \xi \quad \forall (x_0, \xi) \in X_0 \times \Xi.$$

Definition 9.2. Pareto-optimal contract with cash flow information reported by an independent auditor, communication of the manager's private information and precommitment to an optimally determined capital investment function.

The contract $C^* = (M_1^*(x_0, m), M_2^*(x_0, m, x), m^*(x_0, \xi), q^*(x_0, m), a^*(x_0, \xi, m))$ is a truth-inducing Pareto-optimal contract if it is a solution to the program:

$$\begin{aligned} \max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} & \int_{X_0} \int_{\Xi} u_1(M_1(x_0, \xi)) \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X u_2(M_2(x_0, \xi, x)) \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 \\ & - \int_{X_0} \int_{\Xi} H(a(x_0, \xi)) \cdot f(x_0, \xi) \, d\xi dx_0 \end{aligned}$$

subject to

$$\begin{aligned} & \int_{X_0} \int_{\Xi} \{ x_0 - q(x_0, \xi) - M_1(x_0, \xi) \} \cdot f(x_0, \xi) \, d\xi dx_0 \\ & + \int_{X_0} \int_{\Xi} \int_X \{ x - M_2(x_0, \xi, x) \} \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\ & \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 = 0 \end{aligned}$$

$$\forall (x_0, \xi) \in X_0 \times \Xi: \xi \in \operatorname{argmax}_{m \in \Xi} \{ u_1(M_1(x_0, m))$$

$$+ \int_X u_2(M_2(x_0, m, x)) \cdot f(x | x_0, \xi, q(x_0, m), a(x_0, \xi, m)) \, dx - H(a(x_0, \xi, m)) \}$$

$$\forall (x_0, \xi, m) \in X_0 \times \Xi \times \Xi:$$

$$a(x_0, \xi, m) \in \operatorname{argmax}_{a \in A} \{ \int_X u_2(M_2(x_0, m, x)) \cdot f(x | x_0, \xi, q(x_0, m), a) \, dx - H(a) \}$$

As indicated by the arguments preceding this definition it can be shown as in Chapter 8 that it is weakly Pareto preferred that the manager communicates his private information to the investors and that he precommits to an optimally determined capital investment function. That is, the optimal value of the program in Definition 9.2 is greater than or equal to the optimal value of the program in Definition 9.1.

The manager's private information ξ is in general an element of a subset of R^n , where n is some positive integer. In order to characterize the optimal contract with communication and precommitment we assume that ξ is one-dimensional. However, there would be no conceptual difference in the characterization of the contract if a multi-dimensional ξ were allowed, but the notation would be more complicated. We also assume that the incentive compatibility constraints can be substituted with the respective first order conditions which are

$$\begin{aligned} \forall (x_0, \xi) \in X_0 \times \Xi: & u_1'(M_1(x_0, \xi)) \cdot M_{1\xi}(x_0, \xi) \\ & + \int_X u_2'(M_2(x_0, \xi, x)) \cdot M_{2\xi}(x_0, \xi, x) \cdot f(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) dx \\ & + q_\xi(x_0, \xi) \cdot \int_X u_2(M_2(x_0, \xi, x)) \cdot f_q(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) dx = 0 \\ \forall (x_0, \xi) \in X_0 \times \Xi: & \int_X u_2(M_2(x_0, \xi, x)) \cdot f_a(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) dx \\ & - H'(a(x_0, \xi)) = 0 \end{aligned}$$

where $a(x_0, \xi) \equiv a(x_0, \xi, \xi)$.

The optimal compensation functions are characterized in the following proposition.

Proposition 9.1.

If the contract $C^* = (M_1^*(x_0, m), M_2^*(x_0, m, x), m^*(x_0, \xi), q^*(x_0, m), a^*(x_0, \xi, m))$ is a truth-inducing Pareto-optimal contract for the incentive problem in Definition 9.2, then

$$(9.1) \quad 1/u_1'(M_1^*(x_0, \xi)) = 1/\lambda \cdot \left\{ 1 - \rho_\xi(x_0, \xi) - \rho(x_0, \xi) \cdot \frac{f_\xi(x_0, \xi)}{f(x_0, \xi)} \right\}$$

$$(9.2) \quad 1/u_2'(M_2^*(x_0, \xi, x)) \\ = 1/\lambda \cdot \left\{ 1 - \rho_\xi(x_0, \xi) - \rho(x_0, \xi) \cdot \frac{f_\xi(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right. \\ \left. + [\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)] \cdot \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right\}$$

where λ , $\rho(x_0, \xi)$ and $\mu(x_0, \xi)$ are optimal Lagrange-multipliers.

Proof: see the appendix.

The characterizations of the optimal compensation function has a similar structure as the characterizations of the optimal compensation functions in the case where the cash flow of the previous period is the only information processed by the manager, and where he communicates this information to the investors (i.e., (8.1) and (8.2)). The main difference is that the manager now communicates the additional information ξ while the cash flow of the previous period is public information.

If we compare the characterizations, then there is a term in (8.2) which is not present in (9.2), namely the derivative of the log-likelihood function with respect to the capital investment q ,

$$\rho(x_0) \cdot \frac{f_q(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))}$$

This term can be interpreted as a measure of the likelihood that the manager implemented the capital investment $q^*(x_0)$ based on the observation of the reported cash flow at $t=1$ and the dividend at $t=2$. A similar term is not present in (9.2), since the capital investment can now be inferred directly through the observable cash flow of the previous period and the dividend at $t=1$. This is reflected by the fact that due to the observability of the cash flow of the previous period the manager is now able to precommit himself to a capital investment function while in Chapter 8 he was only able to precommit to a dividend function.

The optimal capital investment function to which the manager precommits himself is characterized in the following proposition.

Proposition 9.2.

If the contract $C^* = (M_1^*(x_0, m), M_2^*(x_0, m, x), m^*(x_0, \xi), q^*(x_0, m), a^*(x_0, \xi, m))$ is a truth-inducing Pareto-optimal contract for the incentive problem in Definition 9.2, then the optimal capital investment $q^*(x_0, \xi)$ is determined such that

$$\begin{aligned}
 (9.3) \quad & \int_X x \cdot f_q(x|x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx \\
 & = 1 + \frac{\partial}{\partial q} \int_X \{ M_2^*(x_0, \xi, x) - u_2(M_2^*(x_0, \xi, x)) / u_2'(M_2(x_0, \xi, q, x)) \} \\
 & \quad \cdot f(x|x_0, \xi, q, a^*(x_0, \xi)) dx \Big|_{q = q^*(x_0, \xi)}
 \end{aligned}$$

where the function $M_2(x_0, \xi, q, x)$ is defined by the first order condition for the compensation at $t=2$ taking the capital investment as a variable,

$$\begin{aligned}
1/u'_2(M_2(x_0, \xi, q, x)) = 1/\lambda \cdot & \left\{ 1 - \rho_\xi(x_0, \xi) \right. \\
& - \rho(x_0, \xi) \cdot \frac{f_\xi(x_0, \xi, x|q, a^*(x_0, \xi))}{f(x_0, \xi, x|q, a^*(x_0, \xi))} \\
& - \rho(x_0, \xi) \cdot a^*_\xi(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x|q, a^*(x_0, \xi))}{f(x_0, \xi, x|q, a^*(x_0, \xi))} \\
& \left. + \mu(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x|q, a^*(x_0, \xi))}{f(x_0, \xi, x|q, a^*(x_0, \xi))} \right\}
\end{aligned}$$

such that $M_2(x_0, \xi, q^*(x_0, \xi), x) = M_2^*(x_0, \xi, x)$.

Proof: see the appendix.

In Chapter 8 it was shown that the optimal capital investment deviated from the first best capital investment given the optimal level of effort (Proposition 8.9). This was caused by the moral hazard problem associated with the manager's supply of effort. The second term on the right hand side of (9.3) reflects that this is also the case for the incentive problem of this chapter. Example 8.6 showed that whether or not it is optimal to overinvest or to underinvest relative to the first best capital investment given the optimal level of effort, depends on how the capital investment and the supply of effort are related in the production function, but not on how the cash flow of the previous period is related to the capital investment. The following example shows that optimality of over- or underinvestment for the incentive problem of this chapter depends on how the capital investment is related both to the supply of effort and to the manager's private information. This, of course, is due to the fact that there is a moral hazard problem associated with the supply of effort as well as with the manager's communication of his private information.

Example 9.1.

Assume that x_0 , ξ and x are exponentially distributed such that

$$f(\xi) = \xi^{-1} \cdot \exp\{-\xi / \xi\}$$

$$f(x_0|\xi) = m_1(\xi)^{-1} \cdot \exp\{-x_0 / m_1(\xi)\}$$

$$f(x|x_0, \xi, q, a) = m_2(x_0, \xi, q, a)^{-1} \cdot \exp\{-x / m_2(x_0, \xi, q, a)\}$$

where $m_{2q}(x_0, \xi, q, a) > 0$, $m_{2a}(x_0, \xi, q, a) > 0$ and $m_{2\xi}(x_0, \xi, q, a) > 0$. Furthermore, assume that the marginal productivity of capital is decreasing, $m_{2qq}(x_0, \xi, q, a) < 0$, such that a first best capital investment given the optimal level of effort exists and is given by

$$m_{2q}(x_0, \xi, q^{**}(x_0, \xi, a^*(x_0, \xi)), a^*(x_0, \xi)) = 1.$$

Assume also that the manager has a square-root utility function:

$$u_1(M_1) = 2 \cdot (M_1)^{\frac{1}{2}}, \quad u_2(M_2) = 2 \cdot (M_2)^{\frac{1}{2}}$$

The first order condition for the optimal capital investment (9.3) can then be formulated as follows.

$$\begin{aligned} & \int_X x \cdot f_q(x|x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx \\ &= 1 - \int_X M_2^*(x_0, \xi, x) \cdot f_q(x|x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx \\ & \quad - \int_X M_{2q}^*(x_0, \xi, q^*(x_0, \xi), x) \cdot f(x|x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx \\ &= 1 - \frac{\partial}{\partial q} \int_X M_2(x_0, \xi, q, x) \cdot f(x|x_0, \xi, q, a^*(x_0, \xi)) dx \Big|_{q = q^*(x_0, \xi)} \end{aligned}$$

Since the marginal productivity of capital is decreasing, the relation between the optimal capital investment and the first best capital investment given the optimal level of effort is then determined such that

$$(9.4) \text{ sign } \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\ = \text{ sign } \left\{ \frac{\partial}{\partial q} \int_X M_2(x_0, \xi, q, x) \cdot f(x|x_0, \xi, q, a^*(x_0, \xi)) dx \right|_{q = q^*(x_0, \xi)} \right\}$$

The distributional assumptions give that

$$\frac{f_\xi(x_0, \xi)}{f(x_0, \xi)} = \frac{\{x_0 - m_1(\xi)\} \cdot m_{1\xi}(\xi)}{m_1(\xi)^2} - \frac{1}{\xi^2}$$

$$\frac{f_\xi(x_0, \xi, x|q, a)}{f(x_0, \xi, x|q, a)} = \frac{f_\xi(x|x_0, \xi, q, a)}{f(x|x_0, \xi, q, a)} + \frac{f_\xi(x_0, \xi)}{f(x_0, \xi)}$$

$$\frac{f_\xi(x|x_0, \xi, q, a)}{f(x|x_0, \xi, q, a)} = \frac{\{x - m_2(x_0, \xi, q, a)\} \cdot m_{2\xi}(x_0, \xi, q, a)}{m_2(x_0, \xi, q, a)^2}$$

$$\frac{f_a(x_0, \xi, x|q, a)}{f(x_0, \xi, x|q, a)} = \frac{\{x - m_2(x_0, \xi, q, a)\} \cdot m_{2a}(x_0, \xi, q, a)}{m_2(x_0, \xi, q, a)^2}$$

Using these it is shown in the appendix that (9.4) is equivalent to

$$(9.5) \text{ sign } \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\ = \text{ sign } \{ [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2aq}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\ - \rho(x_0, \xi) \cdot m_{2\xi q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\ - [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\ - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_{2q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \}$$

It follows from this equation that the relationship between the capital investment and the level of effort as well as the relationship between the

capital investment and the manager's private information determine whether it is optimal to overinvest or to underinvest. We consider three cases similar to those analyzed in Example 8.6. We also assume that the level of effort and the manager's private information are related to the capital investment in the same functional form.

$$\text{Case 1: } m_2(x_0, \xi, q, a) = h(x_0, q) + k(x_0, \xi, a)$$

In the case where q is additively related to ξ and a we get that

$$m_{2aq}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) = 0$$

$$m_{2a\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) = 0$$

such that

$$\begin{aligned} & \text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\ &= \text{sign} \{ - [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\ & \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_{2q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \} \end{aligned}$$

Using the equation (A9.2) in the appendix, this is equivalent to

$$\begin{aligned} & \text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\ &= \text{sign} \{ - \lambda \cdot a^*(x_0, \xi) \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))^2 \cdot \\ & \quad m_{2q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) / m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \} \\ &= \text{negative} \end{aligned}$$

since the marginal productivity of capital as well as of effort is positive. Hence, in the case where q is additively related to ξ and a it is optimal to underinvest.

Case 2: $m_2(x_0, \xi, q, a) = h(x_0, q) \cdot k(x_0, \xi, a)$

In this case we get that

$$\begin{aligned} & m_{2aq}(x_0, \xi, q, a) \cdot m_2(x_0, \xi, q, a) \\ &= h_q(x_0, q) \cdot k_a(x_0, \xi, a) \cdot h(x_0, q) \cdot k(x_0, \xi, a) \\ &= m_{2q}(x_0, \xi, q, a) \cdot m_{2a}(x_0, \xi, q, a) \end{aligned}$$

and

$$\begin{aligned} & m_{2\xi q}(x_0, \xi, q, a) \cdot m_2(x_0, \xi, q, a) \\ &= h_q(x_0, q) \cdot k_\xi(x_0, \xi, a) \cdot h(x_0, q) \cdot k(x_0, \xi, a) \\ &= m_{2q}(x_0, \xi, q, a) \cdot m_{2\xi}(x_0, \xi, q, a) \end{aligned}$$

Inserting this in (9.5) we get that

$$\text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} = \text{zero},$$

such that the optimal capital investment is equal to the first best capital investment given the optimal level of effort.

Case 3: $m_2(x_0, \xi, q, a) = h(x_0, q)^{k(x_0, \xi, a)}$

In this case, we get that

$$\begin{aligned} m_{2a}(x_0, \xi, q, a) &= k_a(x_0, \xi, a) \cdot \ln(h(x_0, q)) \cdot h(x_0, q)^{k(x_0, \xi, a)} \\ m_{2\xi}(x_0, \xi, q, a) &= k_\xi(x_0, \xi, a) \cdot \ln(h(x_0, q)) \cdot h(x_0, q)^{k(x_0, \xi, a)} \\ m_{2q}(x_0, \xi, q, a) &= k(x_0, \xi, a) \cdot h_q(x_0, q) \cdot h(x_0, q)^{k(x_0, \xi, a)} / h(x_0, q) \end{aligned}$$

$$\begin{aligned}
m_{2aq}(x_0, \xi, q, a) = & [k_a(x_0, \xi, a) \cdot h_q(x_0, q)) \\
& + k_a(x_0, \xi, a) \cdot \ln(h(x_0, q)) \cdot k(x_0, \xi, a) \cdot h_q(x_0, q)] \\
& \cdot h(x_0, q)^{k(x_0, \xi, a)} / h(x_0, q)
\end{aligned}$$

$$\begin{aligned}
m_{2\xi q}(x_0, \xi, q, a) = & [k_\xi(x_0, \xi, a) \cdot h_q(x_0, q)) \\
& + k_\xi(x_0, \xi, a) \cdot \ln(h(x_0, q)) \cdot k(x_0, \xi, a) \cdot h_q(x_0, q)] \\
& \cdot h(x_0, q)^{k(x_0, \xi, a)} / h(x_0, q)
\end{aligned}$$

$$m_{2aq}(x_0, \xi, q, a) \cdot m_2(x_0, \xi, q, a) - m_{2a}(x_0, \xi, q, a) \cdot m_{2q}(x_0, \xi, q, a)$$

Inserting these expressions in (9.5) give that

$$\begin{aligned}
& \text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\
& = \text{sign} \{ [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot k_a(x_0, \xi, a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot k_\xi(x_0, \xi, a^*(x_0, \xi))] \cdot h(x_0, q)^{k(x_0, \xi, a)} \\
& \quad \cdot h_q(x_0, q) \cdot h(x_0, q)^{k(x_0, \xi, a)} / h(x_0, q) \} \\
& = \text{sign} \{ [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot k_a(x_0, \xi, a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot k_\xi(x_0, \xi, a^*(x_0, \xi))] \\
& \quad \cdot \ln(h(x_0, q^{*(x_0, \xi)})) \cdot h(x_0, q)^{k(x_0, \xi, a)} \} \\
& = \text{sign} \{ (\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \}
\end{aligned}$$

Using equation (A9.2) in the appendix, we get that

$$\begin{aligned}
& \text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\
& = \text{sign} \{ \lambda \cdot a^*(x_0, \xi) \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))^2 / \\
& \qquad \qquad \qquad m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \} \\
& = \text{positive},
\end{aligned}$$

such that it is optimal to overinvest in this case.

These three cases show that overinvestment as well as underinvestment can be optimal.¹ Hence, the moral hazard problems associated with the supply of effort and the communication of the manager's private information have no general effect on the optimal capital investments. It depends crucially on the production functions.

9.2 The value of accounting information.

The value of communication can be analyzed in a similar way as in Propositions 8.5, 8.6 and 8.7 of the previous chapter. A sufficient condition for no value of communication is that the optimal capital investment function with communication is an invertible function of ξ given the cash flow of the previous period. Hence, a necessary condition for strict positive value of communication is that there exists a set of cash flows of the previous period which has positive probability, for which the manager's private information cannot be inferred from the optimal capital investment function with communication. This, on the other hand, will always occur if ξ is multi-dimensional and $q^*(x_0, \cdot)$ is a smooth function.

Communication has been analyzed such that the manager can choose among all communication structures, which are functions from the set of signals, $X_0 \times \Xi$, to a set of messages. There exist an infinite number of such functions that maximize the manager's expected utility. The Revelation Principle was then applied such that it can be assumed without loss of generality that the manager reports his private information truthfully and nothing more, i.e.,

$$m(x_0, \xi) = \xi \quad \forall (x_0, \xi) \in X_0 \times \Xi.$$

It is easy to see that any invertible function of ξ will result in the same expected utilities to both the manager and the investors and that it induces the same production decision rules. To see this, let $m(x_0, \xi)$ be an arbitrary invertible function of ξ for all x_0 . If the compensation functions are chosen such that

$$(9.6) \quad M_1(x_0, m(x_0, \xi)) = M_1^*(x_0, \xi) \quad \forall (x_0, \xi) \in X_0 \times \Xi$$

$$(9.7) \quad M_2(x_0, m(x_0, \xi), x) = M_2^*(x_0, \xi, x) \quad \forall (x_0, \xi) \in X_0 \times \Xi$$

and the capital investment function to which the manager precommits himself is given by

$$(9.8) \quad q(x_0, m(x_0, \xi)) = q^*(x_0, \xi) \quad \forall (x_0, \xi) \in X_0 \times \Xi$$

then the manager gets the same (x_0, ξ) -contingent compensations whether he reports $m(x_0, \xi) = m$ or he reports ξ for the optimal truth-inducing contract. It is therefore optimal to report $m(x_0, \xi) = m$, and the optimal production decision rules remain unchanged. Hence, any invertible function of ξ (for all x_0) is an optimal communication structure for the incentive problem where the class of communication structures is unrestricted. We refer to this as full communication of the manager's private information, since the common-knowledge of the communication structure allows the investors to infer the manager's private information from the report.

The signals $\xi \in \Xi$ provide an abstract and complete representation of the manager's private information. Accounting reports, on the other hand, use a certain language such as some measure of the values of the assets and the liabilities. From an informational perspective, the language used is of no concern as long as the manager is able to communicate the relevant parts of his private information using this language. In other words, if the investors are able to infer the relevant parts of the information received by the manager, then it does not matter what accounting principle the manager is using.

The crucial question is then whether a given accounting principle allows the investors to infer that information. Any well-defined accounting principle can be represented as a function $n: X_0 \times \Xi \rightarrow N$. If this function takes the form of the intrinsic values of the assets and liabilities given the manager's information (i.e. the expected cash flows conditional on (x_0, ξ)), then the accounting principle would be intrinsic economic earnings (see, e.g., Beaver (1981)). From the argument above, the investors are able to infer the manager's information, if intrinsic economic earnings are an invertible function of ξ for all x_0 . In the following, we analyze general conditions which are sufficient to ensure that a given accounting principle permits the investors to infer the relevant parts of the manager's private information.

First, we define the incentive problem where the accounting principle is pre-specified. That is, the manager is no longer able to choose among different communication structures. Hence, full communication of the manager's private information is not guaranteed. We refer to this as limited communication, since in this case the investors are not necessarily able to infer the information received by the manager from the observed accounting report and the common-knowledge of the accounting principle. The compensation functions are determined such that it is optimal for the manager to report $n(x_0, \xi)$ if he receives the information (x_0, ξ) . The function $n(\cdot)$ represents the pre-specified accounting principle.

Definition 9.3. Pareto-optimal contract with cash flow information reported by an independent auditor and limited communication of the manager's private information.

The contract $C^* = (M_1^*(x_0, n), M_2^*(x_0, n, x), n(x_0, \xi), q^*(x_0, \xi, n), a^*(x_0, \xi, n))$ is a Pareto-optimal contract given the accounting principle $n(\cdot)$, if it is a solution to the program:

$$\max_{\substack{M_1(\cdot), M_2(\cdot) \\ q(\cdot), a(\cdot)}} \int_{X_0} \int_{\Xi} u_1(M_1(x_0, n(x_0, \xi)) \cdot f(x_0, \xi)) d\xi dx_0$$

$$\begin{aligned}
& + \int_{X_0} \int_{\Xi} \int_X u_2(M_2(x_0, n(x_0, \xi), x)) \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\
& \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 \\
& - \int_{X_0} \int_{\Xi} H(a(x_0, \xi)) \cdot f(x_0, \xi) \, d\xi dx_0
\end{aligned}$$

subject to

$$\begin{aligned}
& \int_{X_0} \int_{\Xi} \{ x_0 - q(x_0, \xi) - M_1(x_0, n(x_0, \xi)) \} \cdot f(x_0, \xi) \, d\xi dx_0 \\
& + \int_{X_0} \int_{\Xi} \int_X \{ x - M_2(x_0, n(x_0, \xi), x) \} \cdot f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \\
& \quad \cdot f(x_0, \xi) \, dx d\xi dx_0 = \nabla
\end{aligned}$$

$$\begin{aligned}
& \forall (x_0, \xi) \in X_0 \times \Xi: n(x_0, \xi) \in \operatorname{argmax}_{n \in N} \{ u_1(M_1(x_0, n)) \\
& + \int_X u_2(M_2(x_0, n, x)) \cdot f(x | x_0, \xi, q(x_0, \xi, n), a(x_0, \xi, n)) \, dx - H(a(x_0, \xi, n)) \}
\end{aligned}$$

$$\begin{aligned}
& \forall (x_0, \xi, n) \in X_0 \times \Xi \times N: (q(x_0, \xi, n), a(x_0, \xi, n)) \in \\
& \quad \operatorname{argmax}_{(q, a) \in Q \times A} \{ \int_X u_2(M_2(x_0, n, x)) \cdot f(x | x_0, \xi, q, a) \, dx - H(a) \}
\end{aligned}$$

Note that the manager does not precommit to a capital investment function contingent on x_0 and the report n . The reason is that this type of precommitment may be of negative value in the case where the accounting principle is not optimally determined and not invertible with respect to the manager's private information.

The following proposition gives sufficient conditions for no incremental value of full communication.

Proposition 9.3.

Let the optimal contract for the incentive problem with full communication be $C^{**} = (M_1^{**}(x_0, m), M_2^{**}(x_0, m, x), m^{**}(x_0, \xi), q^{**}(x_0, m), a^{**}(x_0, \xi, m))$ and let $C^* = (M_1^*(x_0, n), M_2^*(x_0, n, x), n(x_0, \xi), q^*(x_0, \xi, n), a^*(x_0, \xi, n))$ be the optimal contract for the incentive problem with limited communication. If the function $h: X_0 \times \Xi \rightarrow N \times Q^*$ defined by

$$h(x_0, \xi) \equiv \{ n(x_0, \xi), q^*(x_0, \xi) \}$$

is an invertible function of ξ for all x_0 and

$$q^*(x_0, \xi) = q^{**}(x_0, m^{**}(x_0, \xi))$$

then full communication is of no incremental value over limited communication.

Proof:

From the arguments preceding the proposition we can assume without loss of generality that

$$m^{**}(x_0, \xi) = \xi \quad \forall (x_0, \xi) \in X_0 \times \Xi.$$

For the full communication case the manager precommits himself to the capital investment function $q^{**}(x_0, m)$. It can also be assumed without loss of generality that the manager precommits himself to making an accounting report using the accounting principle $n(x_0, \cdot)$, such that if the cash flow of the previous period is x_0 and the manager reports m then this additional accounting report is $n(x_0, m)$. The compensation functions for the full communication case can then be written as

$$M_1^{**}(x_0, q, n, m) = \begin{cases} M_1^{**}(x_0, m) & \text{if } q = q^{**}(x_0, m) \text{ and } n = n(x_0, m) \\ P & \text{otherwise} \end{cases}$$

$$M_2^{**}(x_0, q, n, m, x) = \begin{cases} M_2^{**}(x_0, m, x) & \text{if } q = q^{**}(x_0, m) \text{ and } n = n(x_0, m) \\ P & \text{otherwise} \end{cases}$$

We can now define a limited communication contract C by

$$M_1(x_0, q, n) = \begin{cases} \max_p m & M_1^{**}(x_0, q, n, m) \text{ if } (q, n) \in N \times Q^{**}(x_0) \\ & \text{otherwise} \end{cases}$$

$$M_2(x_0, q, n, x) = \begin{cases} \max_p m & M_2^{**}(x_0, q, n, m, x) \text{ if } (q, n) \in N \times Q^{**}(x_0) \\ & \text{otherwise} \end{cases}$$

$$q(x_0, \xi) = q^{**}(x_0, \xi)$$

$$a(x_0, \xi) = a^{**}(x_0, \xi)$$

where

$$N \times Q^{**}(x_0) \equiv \{ n, q \mid \exists \xi \in \Xi: q^{**}(x_0, \xi) = q, n(x_0, \xi) = n \}$$

Let (x_0, ξ) be given and consider two arbitrary production decisions $\hat{q}$, $\hat{a}$ and an accounting report $\hat{n}$. Then

$$u_1(M_1(x_0, \hat{q}, \hat{n})) + \int_X u_2(M_2(x_0, \hat{q}, \hat{n}, x)) \cdot f(x|x_0, \xi, \hat{q}, \hat{a}) dx - H(\hat{a})$$

$$= u_1(M_1^{**}(x_0, \hat{q}, \hat{n}, \hat{m})) + \int_X u_2(M_2^{**}(x_0, \hat{q}, \hat{n}, \hat{m}, x)) \cdot f(x|x_0, \xi, \hat{q}, \hat{a}) dx - H(\hat{a})$$

since there is a unique message, $\hat{m}$, for $(\hat{q}, \hat{n}) \in N \times Q^{**}(x_0)$ which maximizes both $M_1^{**}(x_0, \hat{q}, \hat{n}, \cdot)$ and $M_2^{**}(x_0, \hat{q}, \hat{n}, \cdot, x)$ for any x when $h(x_0, \cdot)$ is invertible and $q^{**}(x_0, \xi) = q^{**}(x_0, \xi)$. For $(\hat{q}, \hat{n})$ not in $N \times Q^{**}(x_0)$ the relation is trivial. The compensation functions $M_1^{**}(\cdot)$ and $M_2^{**}(\cdot)$ induce the manager to report $m^{**}(x_0, \xi) = \xi$ and to implement the production decisions $q^{**}(x_0, \xi)$ and $a^{**}(x_0, \xi)$. Hence,

$$u_1(M_1^{**}(x_0, \hat{q}, \hat{n}, \hat{m})) + \int_X u_2(M_2^{**}(x_0, \hat{q}, \hat{n}, \hat{m}, x)) \cdot f(x|x_0, \xi, \hat{q}, \hat{a}) dx - H(\hat{a})$$

$$\leq u_1(M_1^{**}(x_0, q^{**}(x_0, \xi), n(x_0, \xi), \xi))$$

$$+ \int_X u_2(M_2^{**}(x_0, q^{**}(x_0, \xi), n(x_0, \xi), x)) \cdot f(x|x_0, \xi, q^{**}(x_0, \xi), a^{**}(x_0, \xi)) dx$$

$$- H(a^{**}(x_0, \xi))$$

$$\begin{aligned}
&\leq u_1(M_1(x_0, q^{**}(x_0, \xi), n(x_0, \xi))) \\
&\quad + \int_X u_2(M_2(x_0, q^{**}(x_0, \xi), n(x_0, \xi), x)) \cdot f(x|x_0, \xi, q^{**}(x_0, \xi), a^{**}(x_0, \xi)) dx \\
&\qquad\qquad\qquad - H(a^{**}(x_0, \xi)) \\
&= u_1(M_1(x_0, q(x_0, \xi), n(x_0, \xi))) \\
&\quad + \int_X u_2(M_2(x_0, q(x_0, \xi), n(x_0, \xi), x)) \cdot f(x|x_0, \xi, q(x_0, \xi), a(x_0, \xi)) dx \\
&\qquad\qquad\qquad - H(a(x_0, \xi))
\end{aligned}$$

where the second inequality follows from the definition of $M_1(\cdot)$ and $M_2(\cdot)$ and the last equality follows from the definition of $q(\cdot)$ and $a(\cdot)$. This establishes that the contract C is an incentive compatible contract with limited communication.

From the invertibility of $h(x_0, \cdot)$ and $q^*(x_0, \xi) = q^{**}(x_0, \xi)$ it follows that

$$\begin{aligned}
M_1^{**}(x_0, q^{**}(x_0, \xi), n(x_0, \xi), m) &= \begin{cases} M_1^{**}(x_0, \xi) & \text{if } m = \xi \\ P & \text{otherwise} \end{cases} \\
M_2^{**}(x_0, q^{**}(x_0, \xi), n(x_0, \xi), m, x) &= \begin{cases} M_2^{**}(x_0, x) & \text{if } m = \xi \\ P & \text{otherwise} \end{cases}
\end{aligned}$$

and from the definition of $M_1(\cdot)$ and $M_2(\cdot)$ this implies that

$$\begin{aligned}
M_1(x_0, q(x_0, \xi), n(x_0, \xi)) &= M_1^{**}(x_0, \xi) \\
M_2(x_0, q(x_0, \xi), n(x_0, \xi), x) &= M_2^{**}(x_0, \xi, x)
\end{aligned}$$

Thus, the (x_0, x) -contingent compensations to the manager are the same for the contracts C and C^{**} . Since the production decision rules are also identical, both the manager and the investors get the same expected utility for the two contracts. An optimal contract for the incentive problem with limited communication is therefore weakly Pareto-preferred to the optimal contract with full communication. However, any contract with limited communication is also a feasible contract with full communication. Hence, the optimal contracts for full and limited communication give the manager the same expected utility. **Q.E.D.**

The proposition states that accounting principles, which do not allow the manager to communicate all of his private information, achieve the same gains as full communication if the following two conditions are satisfied. The first condition is that the investors are able to infer the manager's private information from the combination of the observed capital investment and the accounting report. The second condition is that the optimal capital investment function with limited communication is also optimal with full communication.

The first condition is more restrictive than necessary. Suppose both conditions in the proposition are satisfied and that the optimal compensations in the contract with limited communication do not depend on whether the report is n or n' . Then it is easily verified that the optimal contract for an otherwise identical pre-specified accounting principle, which does not distinguish between the reports n and n' , gives the same optimal production decisions and the same expected utility to the manager. This will be true even if the investors will not be able to infer all of the manager's private information from the combination of the observed capital investment and the accounting report. The condition can therefore be interpreted in such a way that the investors must be able to infer that part of the manager's private information which is relevant to the manager's optimal compensations.

The condition reflects the fact that accounting information is not the only type of information available to the investors. Observable productive decisions, here the capital investment decision, also provide the investors with information.

The second condition states that the capital investment decision must not be affected by limited communication. When the accounting principle is pre-specified and does not allow the manager to communicate all of his private information, then it might be optimal to change the capital investment decision rule such that more of his private information will be revealed through the observed capital investment. This reduces the productive efficiency of the capital investment decision. It is the same argument as that underlying the examples 8.1 - 8.4 in the previous chapter. The accounting principle must therefore allow the manager to communicate so much of his

private information that the capital investment decision is not affected by its role as a communication device for the manager's private information.

9.3 Conclusion.

The analysis in Part II gives both negative and positive conclusions about the value of accounting information. First, the analysis shows that other sources of information, such as observable capital investments, may be as efficient as accounting information in communicating the manager's private information to the investors. If there are direct reporting costs, it may even be cheaper to communicate that information through observable production decisions than through accounting reports. Secondly, the analysis shows that if the accounting principle cannot be used to communicate all of the manager's private information, which is relevant to the optimal compensations, then the observable production decisions might be distorted in order to communicate more information. Whether the observable production decisions will be distorted or not depends on the gains of communicating more information and the loss of production efficiency in doing so, cf. Example 8.4. The tradeoff to be made, when evaluating different accounting principles, is therefore between the direct reporting cost and the loss of productive efficiency for alternative accounting principles.

The analysis of the value of accounting information in section 9.2 can be extended to an analysis of some of the economic consequences of accounting regulation. Accounting regulation can be considered as specifying admissible accounting principles. Given the characteristics of a firm, such as the production function and the internal information structures, these accounting principles can be represented as a pre-specified class of communication structures. The manager can then choose an optimal communication structure from this class. If there exists a communication structure in the class of pre-specified communication structures, which fulfils the conditions in Proposition 9.3, then it will be optimal to use this communication structure, since it achieves the same gains as full communication. Hence, the issue is whether the regulation permits accounting principles, which enables the manager to communicate the relevant part of his private infor-

mation through the combination of the accounting report and the observable production decisions without affecting the productive efficiency of those decisions. Otherwise, accounting regulation might lead to sub-optimal production decisions.

One of the main implications of the analysis in Part I is that delegation of production responsibility to managers leads to social gains if managers are better informed than investors. This is also true for the economy analyzed in Part II. With full communication, the manager can, without loss of social value, precommit himself to an optimally determined capital investment function measurable with respect to public information about the cash flow of the previous period and his accounting report. Hence, delegation of the capital investment decision is not an issue in this case.² With limited communication, however, delegation of the capital investment decision can be strictly preferred even if limited communication achieves the same gains as full communication. This is due to the fact that if the capital investment decision is not delegated, then the capital investment must be measurable with respect to public information about the cash flow of the previous period and the manager's accounting report, which in this case does not reveal all of the manager's private information. Hence, additional restrictions are imposed on the capital investment decision rule. In fact, the reason why limited communication might achieve the same gains as full communication is that the capital investment decision is delegated to the manager. This is so both because the manager is able to vary the capital investment with useful private information, and because he can then reveal additional information which is useful for incentive purposes.³

In order to simplify the analysis we have assumed that all events are firm-specific and that the managers are not allowed to trade in the capital market, i.e., the assumptions (B1) - (B3). In section 8.1.2 we considered some of the consequences of allowing the manager to lend and borrow in the capital market. Under the assumption that the manager's trades are not observable to the investors it was shown that in the case with communication of the manager's private information it is weakly preferred that the manager is not able to borrow and lend. In the cases, where the manager does not receive perfect information, it is, in fact, strictly preferred that he is not allowed to trade. This is due to the fact that in order to

achieve the optimal incentives, the manager is left with a desire to save some of his first period compensation. Therefore, we continued the analysis without trading opportunities.

On the other hand, it was also shown that, even if the manager's trades are not observable to the investors, cases exist without communication in which it is strictly preferred that the manager is able to borrow and lend due to a preferred intertemporal risk sharing. Also, if the manager is not able to precommit himself not to trade, communication might still be valuable for incentive purposes.

Another interesting topic for further research is related to another aspect of the managers' trading opportunities in the capital market. Due to the insider trading problem, it was shown in Part I that it is optimal to restrict the managers' trading opportunities to trading in well-diversified portfolios, even if their trading is observable to the investors. The conjecture is that this might not be the case, when there is a moral hazard problem and a value of conveying firm-specific information to the investors. If the managers' trading in the shares of their own firms is observable to the investors then their trading behavior conveys information to the investors about the private information received by the managers. Hence, insider trading might be an alternative source of information for the investors to the information conveyed by observable production decisions and accounting reports.

The assumption of no economy-events is only made for notational ease. The qualitative results of the analysis remain unchanged as long as the economy-wide information is publicly available, and the manager is not able to trade. The differences are that the managers must participate in sharing the economy-wide risks and that economy-wide information is used to draw inference about unobservable production decisions in a number of firms. However, the no-trading assumption is still critical for the analysis.

Atkinson and Feltham (1983) consider a model with firm-specific events as well as economy-wide events. One of their main results is that if there are no firm-specific events then there is no moral hazard problem. The argument is that it is optimal to sell the firm to the manager. The optimal risk

sharing can be obtained through the manager's trading in a complete market for economy-wide events. Hence, a necessary condition for the existence of a moral hazard problem is that there are firm-specific events or that the market for economy-wide events is incomplete.

In general, the economy-wide information influences the compensations to the managers of a number of firms. If part of the economy-wide information is not publicly available, then the dividends are not independent conditional on public information across firms. In this case we are not able to separate the allocation problem into an incentive problem for each firm. When the managers act in correlated environments, the compensation to a manager depends not only on the dividends of his own firm but also on the dividends of all firms, since these dividends convey information about the production conditions of the firm (see Holmström (1982) and Atkinson and Feltham (1983)). Interesting topics for further research are analyses of optimal communication strategies for economy-wide events and of the value of economy-wide accounting information reported by the managers.

Footnotes for Chapter 9.

1. The same economic intuition of the causes of under- and overinvestment as that given in Example 8.6 applies also in this example.
2. See Melumad and Reichelstein (1987).
3. Note that, even if it is preferred that the capital investment decision is based on the manager's information, it might be of negative value that the manager gets better information about firm-specific events contrary to what was the case for the analysis in Part I. This is due to the fact that additional private information to the manager might allow better production decisions as well as make it more difficult to make the production decision rules credible to the shareholders (i.e., additional shirking possibilities). Christensen (1981) demonstrates this important point by an example.

Appendix to Part II

Proof of Proposition 7.1:

If the contract is an optimal solution to the incentive problem, then the Lagrangian has a stationary point at (M_1^*, M_2^*, q^*, a^*) (Luenberger (1969; Theorem 9.3.1)). The Lagrangian for the incentive problem in Definition 7.3 is:

$$L(M_1, M_2, q, a, \lambda, \zeta, \mu)$$

$$\begin{aligned}
 = & \int_{X_0} \int_{\Xi} [u_1(M_1(x_0 - q(x_0, \xi))) \cdot f(x_0, \xi) \\
 & + \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \\
 & - H(a(x_0, \xi)) \cdot f(x_0, \xi)] d\xi dx_0 \\
 & + \lambda \cdot \{ \int_{X_0} \int_{\Xi} [\{ x_0 - q(x_0, \xi) - M_1(x_0 - q(x_0, \xi)) \} \cdot f(x_0, \xi) \\
 & + \int_X \{ x - M_2(x_0 - q(x_0, \xi), x) \} \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx] d\xi dx_0 \\
 & - \nabla \} \\
 & + \int_{X_0} \int_{\Xi} \zeta(x_0, \xi) [u_1'(M_1(x_0 - q(x_0, \xi))) \cdot M_{1q}(x_0 - q(x_0, \xi)) \cdot f(x_0, \xi) \\
 & + \int_X u_2'(M_2(x_0 - q(x_0, \xi), x)) \cdot M_{2q}(x_0 - q(x_0, \xi), x) \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \\
 & + \int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f_q(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx] d\xi dx_0
 \end{aligned}$$

$$\begin{aligned}
& + \int_{X_0} \int_{\Xi} \mu(x_0, \xi) \left[\int_X u_2(M_2(x_0 - q(x_0, \xi), x)) \cdot f_a(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \right. \\
& \quad \left. - H'(a(x_0, \xi)) \cdot f(x_0, \xi) \right] d\xi dx_0
\end{aligned}$$

where λ , $\zeta(x_0, \xi)$ and $\mu(x_0, \xi)$ are the Lagrange-multipliers for the constraints in the incentive problem. Furthermore, it is used that

$$f(x | x_0, \xi, q(x_0, \xi), a(x_0, \xi)) \cdot f(x_0, \xi) = f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi))$$

since the signals (x_0, ξ) do not depend on the production decisions.

A stationary point for the Lagrangian is characterized by the Fréchet differential with respect to each of the control variables being equal to 0 for all admissible variations. The set of admissible variations are those functions, $\delta(\cdot)$, for which $\delta(\cdot) \equiv 0$ at the boundary of its domain (see e.g. Luenberger (1969; section 7.5)).

The manager's compensation at time $t=1$ is a function of the dividend at time $t=1$, $d = x_0 - q^*(x_0, \xi)$, and hence, so are the admissible variations in the compensation. The Fréchet differential of the Lagrangian with respect to M_1 is therefore:

$$\begin{aligned}
& \frac{\partial}{\partial M_1} L(M_1^*, M_2^*, q^*, a^*, \lambda, \zeta, \mu; \delta) \\
& = \int_{X_0} \int_{\Xi} \left[u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \delta(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi) \right. \\
& \quad \left. - \lambda \cdot \delta(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi) \right. \\
& \quad \left. - \zeta(x_0, \xi) \cdot \{ u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot M_{1d}^*(x_0 - q^*(x_0, \xi)) \cdot \delta(x_0 - q^*(x_0, \xi)) \right. \\
& \quad \left. + u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \delta_d(x_0 - q^*(x_0, \xi)) \} \cdot f(x_0, \xi) \right] d\xi dx_0
\end{aligned}$$

where the third term comes from

$$M_{1q}(x_0 - q^*(x_0, \xi)) = -M_{1d}(x_0 - q^*(x_0, \xi))$$

and taking the Fréchet differential of the function

$$h(M_1(x_0 - q^*(x_0, \xi)), M_{1d}(x_0 - q^*(x_0, \xi))) \equiv -u_1(M_1(x_0 - q^*(x_0, \xi))) \cdot M_{1d}(x_0 - q^*(x_0, \xi))$$

Since $\delta_{x_0}(x_0 - q^*(x_0, \xi)) = \delta_d(x_0 - q^*(x_0, \xi)) \cdot d_{x_0}^*(x_0, \xi)$ we get that

$$\delta_d(x_0 - q^*(x_0, \xi)) = \delta_{x_0}(x_0 - q^*(x_0, \xi)) / d_{x_0}^*(x_0, \xi)$$

for $d_{x_0}^*(x_0, \xi)$ different from zero. Using this, and defining

$$\gamma(x_0, \xi) \equiv \zeta(x_0, \xi) / d_{x_0}^*(x_0, \xi)$$

the Fréchet differential can be written as

$$\frac{\partial}{\partial M_1} L(M_1^*, M_2^*, q^*, a^*, \lambda, \zeta, \mu; \delta)$$

$$= \int_{X_0} \int_{\Xi} [u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \delta(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi)$$

$$- \lambda \cdot \delta(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi)$$

$$- \{ \gamma(x_0, \xi) \cdot d_{x_0}^*(x_0, \xi) \cdot u_1', (M_1^*(x_0 - q^*(x_0, \xi))) \cdot M_{1d}^*(x_0 - q^*(x_0, \xi)) \cdot \delta(x_0 - q^*(x_0, \xi))$$

$$+ \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \delta_{x_0}(x_0 - q^*(x_0, \xi)) \} \cdot f(x_0, \xi)] d\xi dx_0$$

The term including $\delta_{x_0}(x_0 - q^*(x_0, \xi))$ can now from an integration by parts be formulated as:

$$\int_{X_0} \int_{\Xi} \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \delta_{x_0}(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi) d\xi dx_0$$

$$= \int_{\Xi} \int_{X_0} \delta_{x_0}(x_0 - q^*(x_0, \xi)) \cdot [\gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot f(x_0, \xi)] dx_0 d\xi$$

$$\begin{aligned}
&= \int_{\Xi} [\delta(x_0 - q^*(x_0, \xi)) \cdot \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot f(x_0, \xi)]_{X_0} d\xi \\
&\quad - \int_{\Xi} \int_{X_0} \delta(x_0 - q^*(x_0, \xi)) \cdot \frac{d}{dx_0} [\gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot f(x_0, \xi)] dx_0 d\xi \\
&= - \int_{\Xi} \int_{X_0} \delta(x_0 - q^*(x_0, \xi)) \cdot [\gamma_{x_0}(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot f(x_0, \xi) \\
&\quad + \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot M_{1d}^*(x_0 - q^*(x_0, \xi)) \cdot d_{x_0}^*(x_0, \xi) \cdot f(x_0, \xi) \\
&\quad + \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot f_{x_0}(x_0, \xi)] dx_0 d\xi
\end{aligned}$$

where the third equality comes from $\delta(x_0 - q^*(x_0, \xi)) = 0$ at the boundary of X_0 . Substitution of this term gives that

$$\begin{aligned}
&\frac{\partial}{\partial M_1} L(M_1^*, M_2^*, q^*, a^*, \lambda, \gamma, \mu; \delta) \\
&= \int_{X_0} \int_{\Xi} [u_1'(M_1^*(x_0 - q^*(x_0, \xi))) - \lambda \\
&\quad + \gamma_{x_0}(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \\
&\quad + \gamma(x_0, \xi) \cdot u_1'(M_1^*(x_0 - q^*(x_0, \xi))) \cdot \frac{f_{x_0}(x_0, \xi)}{f(x_0, \xi)}] \cdot \delta(x_0 - q^*(x_0, \xi)) \cdot f(x_0, \xi) d\xi dx_0 \\
&= \int_D \int_{X_0 \times \Xi} [u_1'(M_1^*(d)) - \lambda \\
&\quad + \gamma_{x_0}(x_0, \xi) \cdot u_1'(M_1^*(d)) \\
&\quad + \gamma(x_0, \xi) \cdot u_1'(M_1^*(d)) \cdot \frac{f_{x_0}(x_0, \xi)}{f(x_0, \xi)}] \cdot f(x_0, \xi | d, q^*(x_0, \xi)) d\xi dx_0 \delta(d) f(d | q^*) dd
\end{aligned}$$

where

$$f(d|q^*) = \int_{X_0 \times \Xi} f(x_0, \xi | d, q^*(x_0, \xi)) d\xi dx_0$$

which is the "probability" that the manager's signals (x_0, ξ) belong to the set of signals that gives the same dividend, d , given the optimal capital investment decision rule:

$$X_0 \times \Xi | d = \{ (x_0, \xi) \in X_0 \times \Xi \mid x_0 - q^*(x_0, \xi) = d, d \in D \}$$

Since the Fréchet differential must be equal to 0 for all admissible variations, $\delta(d)$, the inner integral must be equal to 0 for all dividends $d \in D$:

$$\begin{aligned} \int_{X_0 \times \Xi} [u_1'(M_1^*(d)) - \lambda + \gamma_{x_0}(x_0, \xi) \cdot u_1'(M_1^*(d)) \\ + \gamma(x_0, \xi) \cdot u_1'(M_1^*(d)) \cdot \frac{f_{x_0}(x_0, \xi)}{f(x_0, \xi)}] \cdot f(x_0, \xi | d, q^*(x_0, \xi)) d\xi dx_0 = 0 \end{aligned}$$

which is equivalent to:

$$\begin{aligned} 1/u_1'(M_1^*(d)) &= 1/\lambda \cdot \left\{ 1 + \int_{X_0 \times \Xi} \gamma_{x_0}(x_0, \xi) \cdot f(x_0, \xi | d, q^*(x_0, \xi)) d\xi dx_0 \right. \\ &\quad \left. + \int_{X_0 \times \Xi} \gamma(x_0, \xi) \cdot \frac{f_{x_0}(x_0, \xi)}{f(x_0, \xi)} \cdot f(x_0, \xi | d, q^*(x_0, \xi)) d\xi dx_0 \right\} \\ &= 1/\lambda \cdot \left\{ 1 + \frac{\int_{X_0 \times \Xi | d} \gamma_{x_0}(x_0, \xi) \cdot f(x_0, \xi) d\xi dx_0}{f(d|q^*)} \right\} \end{aligned}$$

$$+ \frac{\int_{X_0 \times \Xi | d} \gamma(x_0, \xi) \cdot f_{x_0}(x_0, \xi) d\xi dx_0}{f(d | q^*)} \left. \right\}$$

since

$$f(x_0, \xi | d, q^*(x_0, \xi)) = \begin{cases} \frac{f(x_0, \xi)}{f(d | q^*)} & \text{for } (x_0, \xi) \in X_0 \times \Xi | d \\ 0 & \text{otherwise.} \end{cases}$$

This completes the proof of (7.1)

The manager's compensation at time $t=2$ is defined over the dividends at time $t=1$ and time $t=2$. Thus we obtain:

$$\begin{aligned} & \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, q^*, a^*, \lambda, \gamma, \mu; \delta) \\ &= \int_{X_0} \int_{\Xi} \int_X [u_2'(M_2^*(d, x)) - \lambda] \cdot \delta(d, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0 \\ & - \int_{X_0 \times \Xi} \int_X [\gamma(x_0, \xi) \cdot d_{x_0}^*(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot M_{2d}^*(d, x) \cdot \delta(d, x) \\ & \quad \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\ & + \gamma(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot \delta_{x_0}(d, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\ & - \gamma(x_0, \xi) \cdot d_{x_0}^*(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot \delta(d, x) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\ & - \mu(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot \delta(d, x) \cdot f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx d\xi dx_0 \end{aligned}$$

where it is used (like before) that $M_{2q} = -M_{2d}$, $\delta_d = \delta_{x_0}/d_{x_0}^*$ and the definition of $\gamma(x_0, \xi)$.

The term including δ_{x_0} is again integrated by parts:

$$\begin{aligned}
& \int_{\Xi} \int_X \left[\delta(d, x) \cdot \gamma(x_0, \xi) u_2'(M_2^*(d, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \right]_{X_0} dx d\xi \\
& - \int_{\Xi} \int_X \int_{X_0} \delta(d, x) \cdot \frac{d}{dx_0} (\gamma(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))) dx_0 dx d\xi \\
& = - \int_{\Xi} \int_X \int_{X_0} \delta(d, x) \cdot [\gamma_{x_0}(x_0, \xi) u_2'(M_2^*(d, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \gamma(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot M_{2d}^*(d, x) \cdot d_{x_0}^*(x_0, \xi) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \gamma(x_0, \xi) u_2'(M_2^*(d, x)) \cdot \{ f_{x_0}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot q_{\delta}^*(x_0, \xi) \\
& \quad + f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot a_{\delta}^*(x_0, \xi) \}] dx_0 dx d\xi
\end{aligned}$$

Substitution of this term yields:

$$\begin{aligned}
& \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, q^*, a^*, \lambda, \gamma, \mu; \delta) \\
& = \int_{X_0} \int_{\Xi} \int_X [u_2'(M_2^*(d, x)) - \lambda + \gamma_{x_0}(x_0, x) \cdot u_2'(M_2^*(d, x)) \\
& \quad + \gamma(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot \{ \frac{f_{x_0}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \\
& \quad + \frac{f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \} \\
& \quad + u_2'(M_2^*(d, x)) \cdot (\mu(x_0, \xi) + \gamma(x_0, \xi) \cdot a_{\delta}^*(x_0, \xi)) \cdot \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}]
\end{aligned}$$

$$\cdot \delta(d, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0$$

$$= \int_D \int_X \int_{X_0 \times \Xi} [u_2'(M_2^*(d, x)) - \lambda + \gamma_{x_0}(x_0, x) \cdot u_2'(M_2^*(d, x)) \\ + \gamma(x_0, \xi) \cdot u_2'(M_2^*(d, x)) \cdot \{ \frac{f_{x_0}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \\ + \frac{f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \} \\ + u_2'(M_2^*(d, x)) \cdot (\mu(x_0, \xi) + \gamma(x_0, \xi) \cdot a_{x_0}^*(x_0, \xi)) \cdot \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}]$$

$$\cdot f(x_0, \xi | d, x, q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0 \delta(d, x) \cdot f(d, x | q^*, a^*) dx dd$$

where

$$f(d, x | q^*, a^*) = \int_{X_0} \int_{\Xi} f(x_0, \xi | d, x, q^*(x_0, \xi), a^*(x_0, \xi)) d\xi dx_0$$

Again the inner integral must be equal to 0 for all pairs of dividends $(d, x) \in D \times X$. This integral is 0 if and only if (7.2) is true, since

$$f(x_0, \xi | d, x, q^*(x_0, \xi), a^*(x_0, \xi)) = \frac{f(x_0, \xi, d, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(d, x | q^*, a^*)} \\ = \begin{cases} \frac{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(d, x | q^*, a^*)} & \text{for } (x_0, \xi) \in X_0 \times \Xi | d \\ 0 & \text{otherwise} \end{cases}$$

This completes the proof of the proposition. **Q.E.D.**

Proof of Proposition 8.4:

As in the proof of Proposition 7.1 the problem is to characterize a stationary point of the Lagrangian, which is

$$\begin{aligned}
 & L(M_1, M_2, d, a, \lambda, \rho, \mu, \tau) \\
 &= \int_{X_0} [u_1(M_1(x_0)) \cdot f(x_0) \\
 &\quad + \int_X u_2(M_2(x_0, x)) \cdot f(x_0, x | q(x_0), a(x_0)) dx \\
 &\quad - H(a(x_0)) \cdot f(x_0)] dx_0 \\
 &+ \lambda \cdot \left\{ \int_{X_0} [\{ d(x_0) - M_1(x_0) \} \cdot f(x_0) \right. \\
 &\quad \left. + \int_X \{ x - M_2(x_0, x) \} \cdot f(x_0, x | q(x_0), a(x_0)) dx] dx_0 - \nabla \right\} \\
 &+ \int_{X_0} \rho(x_0) [u_1^*(M_1(x_0)) \cdot M_{1x_0}(x_0) \cdot f(x_0) \\
 &\quad + \int_X u_2^*(M_2(x_0, x)) \cdot M_{2x}(x_0, x) \cdot f(x_0, x | q(x_0), a(x_0)) dx \\
 &\quad - d_{x_0}(x_0) \cdot \int_X u_2(M_2(x_0, x)) \cdot f_q(x_0, x | q(x_0), a(x_0)) dx] dx_0 \\
 &+ \int_{X_0} \mu(x_0) \cdot \left[\int_X u_2(M_2(x_0, x)) \cdot f_a(x_0, x | q(x_0), a(x_0)) dx \right. \\
 &\quad \left. - H'(a(x_0)) \cdot f(x_0) \right] dx_0
 \end{aligned}$$

$$+ \int_{X_0} \tau(x_0) \cdot \{ d(x_0) - x_0 + q(x_0) \} \cdot f(x_0) dx_0$$

where λ , $\pi(x_0)$, $\mu(x_0)$ and $\tau(x_0)$ are the respective Lagrange-multipliers for the constraints in the incentive problem.

Only (8.2) will be proved, since (8.1) is a special case of (8.2). Thus, we take the Fréchet differential of the Lagrangian with respect to M_2 at the optimal contract:

$$\begin{aligned} & \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, d^*, a^*, \lambda, \rho, \mu; \delta) \\ &= \int_{X_0} \int_X [u'_2(M_2^*(x_0, x)) - \lambda] \cdot \delta(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) dx dx_0 \\ &+ \int_{X_0} \rho(x_0) \cdot \left\{ \int_X u'_2(M_2^*(x_0, x)) \cdot M_{2x_0}^*(x_0, x) \cdot \delta(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) dx \right. \\ &+ \int_X u'_2(M_2^*(x_0, x)) \cdot \delta_{x_0}(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) dx \\ &\left. - d_{x_0}^*(x_0) \cdot \int_X u'_2(M_2^*(x_0, x)) \cdot \delta(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) dx \right\} dx_0 \\ &+ \int_{X_0} \mu(x_0) \cdot \int_X u'_2(M_2^*(x_0, x)) \cdot \delta(x_0, x) \cdot f_a(x_0, x | q^*(x_0), a^*(x_0)) dx dx_0 \end{aligned}$$

The term including $\delta_{x_0}(x_0, x)$ is integrated by parts:

$$\begin{aligned}
& \int_X [\delta(x_0, x) \cdot \rho(x_0) u_2'(M_2^*(x_0)) \cdot f(x_0, x | q^*(x_0), a^*(x_0))]_{X_0} dx \\
& - \int_X \int_{X_0} \delta(x_0, x) \cdot \frac{d}{dx_0} (\rho(x_0) \cdot u_2'(M_2^*(x_0, x)) \cdot f(x_0, x | q^*(x_0), a^*(x_0))) dx_0 dx \\
& = - \int_{X_0} \int_X \delta(x_0, x) \cdot [\rho_{x_0}(x_0) u_2'(M_2^*(x_0, x)) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) \\
& + \rho(x_0) \cdot u_2'(M_2^*(x_0, x)) \cdot M_{2x_0}^*(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) \\
& + \rho(x_0) u_2'(M_2^*(x_0, x)) \cdot \{ f_{x_0}(x_0, x | q^*(x_0), a^*(x_0)) \\
& + f_q(x_0, x | q^*(x_0), a^*(x_0)) \cdot q_{x_0}^*(x_0) \\
& + f_a(x_0, x | q^*(x_0), a^*(x_0)) \cdot a_{x_0}^*(x_0) \}] dx_0 dx
\end{aligned}$$

since $\delta(x_0, x) = 0$ at the the boundary of X_0 . Inserting this expression gives that

$$\begin{aligned}
& \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, d^*, a^*, \lambda, \rho, \mu; \delta) \\
& = \int_{X_0} \int_X [u_2'(M_2^*(x_0, x)) - \lambda + \rho_{x_0}(x_0) \cdot u_2'(M_2^*(x_0, x)) \\
& - \rho(x_0) \cdot u_2'(M_2^*(x_0, x)) \cdot \frac{f_{x_0}(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))} \\
& - \rho(x_0) \cdot u_2'(M_2^*(x_0, x)) \cdot \frac{f_q(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))}
\end{aligned}$$

$$\begin{aligned}
& + u_2'(M_2^*(x_0, x)) \cdot [\mu(x_0, \xi) - \rho(x_0) \cdot a_{x_0}^*(x_0)] \cdot \frac{f_a(x_0, x | q^*(x_0), a^*(x_0))}{f(x_0, x | q^*(x_0), a^*(x_0))}] \\
& \cdot \delta(x_0, x) \cdot f(x_0, x | q^*(x_0), a^*(x_0)) \, dx dx_0
\end{aligned}$$

At the optimal contract the Fréchet differential of the Lagrangian is 0 for admissible variations $\delta(x_0, x)$. Hence, the term in the $[\cdot]$ -parenthesis must be equal to 0. This is equivalent to (8.2). **Q.E.D.**

Proof of Proposition 8.10:

The Lagrangian for the incentive problem is:

$$\begin{aligned}
 & L(M_1, M_2, q, a, \lambda, \mu) \\
 &= \int_{X_0} [u_1(M_1(x_0)) \cdot f(x_0) \\
 &\quad + \int_X u_2(M_2(x_0, x)) \cdot f(x | x_0, q(x_0), a(x_0)) \, dx \\
 &\quad - H(a(x_0)) \cdot f(x_0)] \, dx_0 \\
 &+ \lambda \cdot \left\{ \int_{X_0} [\{ x_0 - q(x_0) - M_1(x_0) \} \cdot f(x_0) \right. \\
 &\quad \left. + \int_X \{ x - M_2(x_0, x) \} \cdot f(x | x_0, q(x_0), a(x_0)) \, dx] \, dx_0 - \nabla \right\} \\
 &+ \int_{X_0} \mu(x_0) \cdot \left[\int_X u_2(M_2(x_0, x)) \cdot f_a(x | x_0, q(x_0), a(x_0)) \, dx \right. \\
 &\quad \left. - H'(a(x_0)) \cdot f(x_0) \right] \, dx_0
 \end{aligned}$$

where λ and $\mu(x_0)$ are the respective Lagrange-multipliers.

Using the technique of pointwise optimisation the partial derivative of the Lagrangian with respect to $M_1(x_0)$ for some x_0 is

$$\frac{\partial}{\partial M_1(x_0)} L(M_1^*, M_2^*, q^*, a^*, \lambda, \mu) = u_1'(M_1(x_0)) \cdot f(x_0) - \lambda \cdot f(x_0)$$

A stationary point of the Lagrangian is therefore characterized by (8.3).

Similar, the compensation in the second period is characterized by

$$[u_2'(M_2^*(x_0, x)) - \lambda] \cdot f(x|x_0, q^*(x_0), a^*(x_0)) \cdot f(x_0) \\ + \mu(x_0) \cdot u_2'(M_2^*(x_0, x)) \cdot f_a(x|x_0, q^*(x_0), a^*(x_0)) \cdot f(x_0) = 0$$

which is equivalent to (8.4). Q.E.D.

Proof of Proposition 8.11:

(8.5) is the first order condition with respect to the capital investment for a stationary point for the Lagrangian for the incentive problem given in the proof of Proposition 8.10.

Proof of Proposition 9.1:

The Lagrangian for the incentive problem in Definition 9.2 is

$$L(M_1, M_2, q, a, \lambda, \rho, \mu) \\ = \int_{X_0} \int_{\Xi} [u_1(M_1(x_0, \xi)) \cdot f(x_0, \xi) \\ + \int_X u_2(M_2(x_0, \xi, x)) \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \\ - H(a(x_0, \xi)) \cdot f(x_0, \xi)] d\xi dx_0 \\ + \lambda \cdot \{ \int_{X_0} \int_{\Xi} [\{ x_0 - q(x_0, \xi) - M_1(x_0, \xi) \} \cdot f(x_0, \xi)$$

$$\begin{aligned}
& + \int_X \{ x - M_2(x_0, \xi, x) \} \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \} d\xi dx_0 - \nabla \} \\
& + \int_{x_0} \int_{\Xi} \rho(x_0, \xi) [u'_1(M_1(x_0, \xi)) \cdot M_{1\xi}(x_0, \xi) \cdot f(x_0, \xi) \\
& + \int_X u'_2(M_2(x_0, \xi, x)) \cdot M_{2\xi}(x_0, \xi, x) \cdot f(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \\
& + q_\xi(x_0, \xi) \cdot \int_X u_2(M_2(x_0, \xi, x)) \cdot f_q(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \} d\xi dx_0 \\
& + \int_{x_0} \int_{\Xi} \mu(x_0, \xi) [\int_X u_2(M_2(x_0, \xi, x)) \cdot f_a(x_0, \xi, x | q(x_0, \xi), a(x_0, \xi)) dx \\
& - H'(a(x_0, \xi)) \cdot f(x_0, \xi)] d\xi dx_0
\end{aligned}$$

where λ , $\rho(x_0, \xi)$ and $\mu(x_0, \xi)$ are the Lagrange-multipliers for the constraints in the incentive problem.

The Fréchet differential of the Lagrangian with respect to M_2 is:

$$\begin{aligned}
& \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, q^*, a^*, \lambda, \rho, \mu; \delta) \\
& = \int_{x_0} \int_{\Xi} \int_X [u'_2(M_2^*(x_0, \xi, x)) - \lambda] \cdot \delta(x_0, \xi, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0 \\
& + \int_{x_0} \int_{\Xi} \int_X [\rho(x_0, \xi) \cdot u'_2(M_2^*(x_0, \xi, x)) \cdot M_{2\xi}^*(x_0, \xi, x) \cdot \delta(x_0, \xi, x) \\
& \quad \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& + \rho(x_0, \xi) \cdot u'_2(M_2^*(x_0, \xi, x)) \cdot \delta_\xi(x_0, \xi, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))
\end{aligned}$$

$$\begin{aligned}
& + \rho(x_0, \xi) \cdot q_\xi^*(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x)) \cdot \delta(x_0, \xi, x) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& + \mu(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x)) \cdot \delta(x_0, \xi, x) \cdot f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx d\xi dx_0
\end{aligned}$$

The term including δ_ξ can be integrated by parts:

$$\begin{aligned}
& \int_{X_0} \int_X [\delta(x_0, \xi, x) \cdot \rho(x_0, \xi) u_2'(M_2^*(x_0, \xi, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx d\xi dx_0 \\
& - \int_{\Xi} \int_X \int_{X_0} \delta(x_0, \xi, x) \\
& \quad \cdot \frac{d}{d\xi} [\rho(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx_0 dx d\xi \\
& = - \int_{\Xi} \int_X \int_{X_0} \delta(x_0, \xi, x) \cdot [\rho_\xi(x_0, \xi) u_2'(M_2^*(x_0, \xi, x)) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \rho(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x)) \cdot M_{2\xi}^*(x_0, \xi, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \rho(x_0, \xi) u_2'(M_2^*(x_0, \xi, x)) \cdot \{ f_\xi(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot q_\xi^*(x_0, \xi) \\
& \quad + f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot a_\xi^*(x_0, \xi) \}] dx_0 dx d\xi
\end{aligned}$$

Substitution of this term yields:

$$\begin{aligned}
& \frac{\partial}{\partial M_2} L(M_1^*, M_2^*, q^*, a^*, \lambda, \rho, \mu; \delta) \\
& = \int_{X_0} \int_{\Xi} \int_X [u_2'(M_2^*(x_0, \xi, x)) - \lambda - \rho_\xi(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x))
\end{aligned}$$

$$\begin{aligned}
& - \rho(x_0, \xi) \cdot u'_2(M_2^*(x_0, \xi, x)) \cdot \left\{ \frac{f_\xi(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right. \\
& + u'_2(M_2^*(x_0, \xi, x)) \cdot (\mu(x_0, \xi) - \rho(x_0, \xi)) \cdot \frac{a^*(x_0, \xi)}{\xi} \cdot \left. \frac{f_a(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))}{f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))} \right\} \\
& \cdot \delta(x_0, \xi, x) \cdot f(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0
\end{aligned}$$

If M_2^* is a stationary point for the Lagrangian this integral must be equal to zero for all admissible variations $\delta(x_0, \xi, x)$. Hence, the term in the [·]-parenthesis must be equal to zero, and this is equivalent to (9.2). The first order condition for the optimal compensation function at $t=1$ follows from similar calculations. **Q.E.D.**

Proof of Proposition 9.2:

The Fréchet differential of the Lagrangian for the incentive problem (see the proof of Proposition 9.1.) with respect to q is:

$$\begin{aligned}
& \frac{\partial}{\partial q} L(M_1^*, M_2^*, q^*, a^*, \lambda, \rho, \mu; \delta) \\
& = \int_{X_0} \int_{\Xi} \int_X u_2(M_2^*(x_0, \xi, x)) \cdot \delta(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0 \\
& - \int_{X_0} \int_{\Xi} \lambda \cdot \delta(x_0, \xi) \cdot f(x_0, \xi) d\xi dx_0 \\
& + \int_{X_0} \int_{\Xi} \int_X \lambda \cdot \{ x - M_2^*(x_0, \xi, x) \} \cdot \delta(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx d\xi dx_0
\end{aligned}$$

$$\begin{aligned}
& + \int_{X_0} \int_{\Xi} \int_X [\rho(x_0, \xi) \cdot \{ u_2'(M_2^*(x_0, \xi, x)) \cdot M_{2\xi}^*(x_0, \xi, x) \cdot \delta(x_0, \xi) \\
& \quad \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + u_2(M_2^*(x_0, \xi, x)) \cdot \delta_\xi(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + q_\xi^*(x_0, \xi) \cdot u_2(M_2^*(x_0, \xi, x)) \cdot \delta(x_0, \xi) \cdot f_{q\xi}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \} \\
& \quad + \mu(x_0, \xi) \cdot u_2(M_2^*(x_0, \xi, x)) \cdot \delta(x_0, \xi) \cdot f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx d\xi dx_0
\end{aligned}$$

The term including δ_ξ can be integrated by parts:

$$\begin{aligned}
& \int_{X_0} \int_{\Xi} \int_X [\delta(x_0, \xi) \cdot \rho(x_0, \xi) u_2(M_2^*(x_0, \xi, x)) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx dx_0 \\
& - \int_{\Xi} \int_X \int_{X_0} \delta(x_0, \xi) \\
& \quad \cdot \frac{d}{d\xi} [\rho(x_0, \xi) \cdot u_2(M_2^*(x_0, \xi, x)) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))] dx_0 dx d\xi \\
& = - \int_{\Xi} \int_X \int_{X_0} \delta(x_0, \xi) \cdot [\rho_\xi(x_0, \xi) u_2(M_2^*(x_0, \xi, x)) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \rho(x_0, \xi) \cdot u_2'(M_2^*(x_0, \xi, x)) \cdot M_{2\xi}^*(x_0, \xi, x) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + \rho(x_0, \xi) u_2(M_2^*(x_0, \xi, x)) \cdot \{ f_{\xi q}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad + f_{qq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot q_\xi^*(x_0, \xi) \\
& \quad + f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \cdot a_\xi^*(x_0, \xi) \}] dx_0 dx d\xi
\end{aligned}$$

Substitution of this term yields:

$$\frac{\partial}{\partial q} L(M_1^*, M_2^*, q^*, a^*, \lambda, \rho, \mu; \delta)$$

$$\begin{aligned}
&= \int_{X_0} \int_{\Xi} \left\{ - \lambda \cdot f(x_0, \xi) \right. \\
&\quad + \lambda \cdot \int_X \{ x - M_2^*(x_0, \xi, x) \} \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) dx \\
&\quad + \int_X u_2(M_2^*(x_0, \xi, x)) \cdot [f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad - \rho_\xi(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad - \rho(x_0, \xi) \cdot f_{\xi q}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi) \cdot f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad \left. + \mu(x_0, \xi) \cdot f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \right] dx \quad \left. \right\} \\
&\quad \cdot \delta(x_0, \xi) \cdot d\xi dx_0
\end{aligned}$$

If q^* is a stationary point for the Lagrangian this integral must be equal to zero for all admissible variations $\delta(x_0, \xi)$ in the capital investment. Hence, the term in the $\{\cdot\}$ -parenthesis must be equal to zero. This is equivalent to the following condition:

$$\begin{aligned}
&\int_X x \cdot f_q(x | x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx = 1 \\
&+ \int_X M_2^*(x_0, \xi, x) \cdot f_q(x | x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx \\
&- \int_X u_2(M_2^*(x_0, \xi, x)) \cdot \frac{1}{\lambda} \cdot [f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad - \rho_\xi(x_0, \xi) \cdot f_q(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
&\quad - \rho(x_0, \xi) \cdot f_{\xi q}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi))
\end{aligned}$$

$$\begin{aligned}
& - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \\
& + \mu(x_0, \xi) \cdot f_{aq}(x_0, \xi, x | q^*(x_0, \xi), a^*(x_0, \xi)) \bigg] / f(x_0, \xi) \, dx \\
= & 1 + \frac{\partial}{\partial q} \int_X M_2^*(x_0, \xi, x) \cdot f(x | x_0, \xi, q, a^*(x_0, \xi)) \, dx \bigg|_{q = q^*(x_0, \xi)} \\
& - \frac{\partial}{\partial q} \int_X u_2(M_2^*(x_0, \xi, x)) \cdot \\
& 1/\lambda \cdot \left\{ 1 - \rho(x_0, \xi) \cdot \frac{f_{\xi}(x_0, \xi, x | q, a^*(x_0, \xi))}{f(x_0, \xi, x | q, a^*(x_0, \xi))} \right. \\
& - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x | q, a^*(x_0, \xi))}{f(x_0, \xi, x | q, a^*(x_0, \xi))} \\
& + \mu(x_0, \xi) \cdot \frac{f_a(x_0, \xi, x | q, a^*(x_0, \xi))}{f(x_0, \xi, x | q, a^*(x_0, \xi))} \left. \right\} \\
& \cdot f(x_0, \xi, x | q, a^*(x_0, \xi)) / f(x_0, \xi) \, dx \bigg|_{q = q^*(x_0, \xi)} \\
= & 1 + \frac{\partial}{\partial q} \int_X \{ M_2^*(x_0, \xi, x) - u_2(M_2^*(x_0, \xi, x)) / u_2'(M_2(x_0, \xi, q, x)) \} \\
& \cdot f(x | x_0, \xi, q, a^*(x_0, \xi)) \, dx \bigg|_{q = q^*(x_0, \xi)}
\end{aligned}$$

which is (9.3). **Q.E.D.**

Proof of equation (9.5):

If we insert the likelihood-ratios in the first order conditions for the compensation functions we get that

$$\begin{aligned} (M_2(x_0, \xi, q, x))^{\frac{1}{2}} = & \left\{ (M_1^*(x_0, \xi))^{\frac{1}{2}} \right. \\ & + \frac{1}{\lambda} \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q, a^*(x_0, \xi)) \\ & - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q, a^*(x_0, \xi))] \\ & \left. \cdot \frac{x - m_2(x_0, \xi, q, a^*(x_0, \xi))}{m_2(x_0, \xi, q, a^*(x_0, \xi))^2} \right\} \end{aligned}$$

Provided that this characterization is valid for all values of the dividend at $t=2$ we get that

$$\begin{aligned} & \int_X M_2(x_0, \xi, q, x) \cdot f(x|x_0, \xi, q, a^*(x_0, \xi)) dx \\ & = M_1^*(x_0, \xi) \\ & + \left\{ \frac{1}{\lambda} \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q, a^*(x_0, \xi)) \right. \\ & \quad \left. - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q, a^*(x_0, \xi))] / m_2(x_0, \xi, q, a^*(x_0, \xi)) \right\}^2 \end{aligned}$$

Differentiating this expression with respect to q gives that

$$\begin{aligned} (A9.1) \quad & \frac{\partial}{\partial q} \int_X M_2(x_0, \xi, q, x) \cdot f(x|x_0, \xi, q, a^*(x_0, \xi)) dx \Big|_{q = q^*(x_0, \xi)} \\ & = 2 \cdot \left\{ \frac{1}{\lambda} \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \right. \\ & \quad \left. - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] / m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \right\} \\ & \quad \cdot \frac{\partial}{\partial q} \left\{ \frac{1}{\lambda} \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_{\xi}^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q, a^*(x_0, \xi)) \right. \end{aligned}$$

$$\begin{aligned}
& - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q, a^*(x_0, \xi)) \bigg] / m_2(x_0, \xi, q, a^*(x_0, \xi)) \bigg\} \bigg|_{q = q^*(x_0, \xi)} \\
& = 2 \cdot \{ 1/\lambda \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] / m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \} \\
& \quad \cdot \{ 1/\lambda \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2aq}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - 1/\lambda \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_{2q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \} \\
& \quad \quad \quad / \{ m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \}^2
\end{aligned}$$

From the characterization of the optimal compensation function $M_2^*(x_0, \xi)$ and the act selection constraint we get that

$$\begin{aligned}
& \int_X u_2(M_2^*(x_0, \xi, x)) \cdot f_a(x | x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx - H'(a^*(x_0, \xi)) \\
& = \int_X 2 \cdot (M_2^*(x_0, \xi, x))^{\frac{1}{2}} \cdot [x - m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \\
& \quad m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) / m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))^2 \\
& \quad \quad \quad \cdot f(x | x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) dx - 2 \cdot a^*(x_0, \xi) \\
& = 2 \cdot 1/\lambda \cdot \{ (\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad \quad \quad / m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))^2 - 2 \cdot a^*(x_0, \xi) \\
& = 0
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
(A9.2) \quad & a^*(x_0, \xi) \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))^2 / m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& = 1/\lambda \cdot [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))]
\end{aligned}$$

The term on the right hand side is the first term in (A9.1). Since the marginal productivity of effort is strictly positive, $m_{2a} > 0$, the left hand side is strictly positive. This implies that

$$\begin{aligned}
& \text{sign} \{ q^*(x_0, \xi) - q^{**}(x_0, \xi, a^*(x_0, \xi)) \} \\
& = \text{sign} \{ [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2aq}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_2(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - [(\mu(x_0, \xi) - \rho(x_0, \xi) \cdot a_\xi^*(x_0, \xi)) \cdot m_{2a}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \\
& \quad - \rho(x_0, \xi) \cdot m_{2\xi}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi))] \cdot m_{2q}(x_0, \xi, q^*(x_0, \xi), a^*(x_0, \xi)) \}
\end{aligned}$$

which is (9.5). Q.E.D.

Implications for Accounting

In this dissertation we have analyzed the economic consequences of reporting firm-specific information in a general capital market setting. We will now synthesize the implications of the analysis for accounting.

In Part I it was shown that in order to achieve efficient consumption/production plans which are based on the managers' superior information about firm-specific events, it does not matter whether the managers report their information to the investors or not. The efficiency of the resource allocation depends only on the managers' information about firm-specific productivity events. The only information the investors need is information relevant to their portfolio plans. If well-diversified portfolios exist which have enough flexibility such that any financially feasible and desired consumption plan can be implemented, then the investors will restrict their portfolio plans to these portfolios. They will do this, because they can insure themselves against both firm-specific risk and mispricing of individual firms costlessly by following this portfolio rule. Firm-specific information is not relevant to the composition and the market values of well-diversified portfolios, and therefore, the reporting of this information to the investors is of no social value.

In order to ensure that the managers implement their part of efficient risk sharing, their trading opportunities in the capital market are in their self-interest restricted to well-diversified portfolios. Also, to ensure that the managers have no incentives not to implement an efficient production plan, the managers are given compensation schemes which are related to the average dividends in the economy.

Given the managers' trading opportunities and their compensation schemes, it was shown that their private information structure could be considered to be endogenously determined. However, although there are no social gains connected with reporting firm-specific information, the investors have strong private incentives to acquire firm-specific information for speculation purposes. If the investors' information structures for firm-specific events are considered to be endogenously determined, it might have social value that the managers report firm-specific information to the investors. Suppose that the managers report all firm-specific information which is relevant to the market values of firms, i.e. the information that makes the

intrinsic values of firms measurable with respect to that information. If there are no reporting costs, the managers have no incentives not to report this information given their trading opportunities and their compensation schemes. In this case, the investors will not spend resources on private information acquisition and not take firm-specific risks from speculative positions. In aggregate, the economy is better off in terms of saved resources used for private information acquisition and in terms of more efficient risk sharing.

The reporting of firm-specific information becomes valuable due to the conflict between the investors' social and private incentives to acquire that information. However, not all of the managers' private information has to be reported. It is of no value to report information which does not influence the market values of firms (such as project-specific information), since the investors cannot obtain any gains from speculation based on this information; i.e. the investors have no social or private incentives to spend resources on the acquisition of that type of information.

In this dissertation we have considered the firms as being financed entirely by equity. Feltham and Christensen (1988) investigate a similar model with equity as well as debt financing. Firms who have received good news have incentives to reveal their information while firms with bad news have incentives not to reveal their information upon issuing debt claims. The debtholders, on the other hand, realize that the firms have these incentives. Hence, the gains to withholding bad news might be eliminated. But what is good news and what is bad news? The application of option pricing analysis to the valuation of corporate liabilities (see Smith (1979) for a survey) suggests that this is determined by the value of the firm. In nearly all of those models, the current value of the firm is a sufficient statistic for the value of the corporate liabilities. Hence, only information which is relevant to the value of the firm, is relevant to the value of the corporate liabilities. In other words, the type of information to be reported in this setting is the same as the information which eliminates the conflict between the investors' social and private incentives to acquire information. Of course, if individual assets are put up as a collateral for a debt issue, project-specific information might also be relevant.

Brennan and Kraus (1987) investigate a model in which the financing strategies themselves reveal the managers' private information costlessly. The information that can be inferred from the financing strategies is therefore an alternative source of information for the investors to the information in accounting reports. As long as the information can be revealed costlessly in terms of risk sharing and production incentives, it might be an efficient alternative to accounting information.

In Part II we analyzed an agency model in which the managers have a non-pecuniary return for one of the production factors, namely their own supply of effort. The other factor of production was taken to be the amount of capital invested. Due to the unobservability of the supply of effort, a moral hazard problem was created. The results in Part I concerning the sharing of risks among investors still apply, but the managers have to take some firm-specific risk related to the sequence of the dividends of their own firms in order to make the production decision rules credible to the investors. This creates an additional decision-influencing demand for firm-specific accounting information. The demand for firm-specific information due to the conflict between the investors' social and private incentives to acquire information is of course still present, although it is not explicitly analyzed in the model.

The optimal compensation schemes in the presence of moral hazard problems are such that the managers are not indifferent as to what information they report through accounting reports. This occurs because the accounting reports are used to evaluate the managers. The Revelation Principle was then used to show that it can be assumed without reducing the social value that the compensation schemes are such that the managers in their self-interest report all of their private information truthfully. Hence, both sources of demand for firm-specific accounting information can be fulfilled at the same time without creating less efficient consumption/production plans than what is due to the moral hazard problem.

One of the main results of the analysis in Part II is that accounting principles which do not allow the managers to communicate all of their private information to the investors through the accounting report might achieve the same gains as a full communication. One reason is that not all of the

information might be relevant to the evaluation of managers. The other reason is that the investors can infer some of the information received by the managers from the (either indirectly or directly) observed capital investments; i.e., for accounting principles which do not allow full communication, the capital investment decision plays the dual role of a productive decision and a communication device for the managers' private information.

It was also shown that if the accounting principle does not allow the managers to communicate enough of their private information then the capital investments might be different from what they would have been if more information could be communicated through the accounting report. The capital investment decision rules are changed in order to communicate more of the managers' private information through the observed capital investments. In this case, there is a conflict between the production role and the communication role of the capital investment decision, and it leads to sub-optimal capital investments.

Generally, accounting principles are stated in terms of how the value of the assets and the liabilities are determined. If intrinsic values are the valuation concept, then the accounting principle fulfils the demand for firm-specific information that is due to the conflict between the investors' social and private incentives to acquire information. Of course, this is true of all accounting principles, which makes it possible for the investors to infer the information that is relevant to the market value of the firm. However, these accounting principles may or may not fulfil the decision-influencing demand for information depending on the characteristics of the firm. It may therefore be unwise to standardize the accounting principles across firms. If the accounting principles do not fulfil the decision-influencing demand for information, the consequence might be a sub-optimal allocation of investment capital in the economy due to the communication role of the capital investment decisions.

Danish Summary

Formålet med denne afhandling har været at give en økonomisk analyse af, på hvilken måde virksomhedsspecifik information påvirker ressource allokeringen i efficiente kapitalmarkeder.

Der er to typer af økonomiske agenter i den beskrevne økonomi, investorer og virksomhedsledere. Investorernes beslutningsproblem består i at allokere deres formue til forbrug og til investering i andele af produktive virksomheder. Virksomhedslederne skal udover disse to beslutninger fastlægge produktionsplanerne i deres respektive virksomheder.

I tidligere generelle ligevægtsanalyser af allokeringsproblemet i økonomier af denne type træffer investorerne og virksomhedslederne deres beslutninger på grundlag af den samme information. Dette medfører, at delegering af produktionsbeslutningerne til virksomhedslederne ikke spiller nogen betydende rolle i modellen.

I økonomien, der analyseres i denne afhandling, er virksomhedslederne bedre informeret om produktionsbetingelserne i deres respektive virksomheder end investorerne. De to typer af agenter har dog samme information om de generelle økonomiske betingelser i økonomien. Delegering af produktionsbeslutningerne kan i dette tilfælde føre til en bedre fordeling af ressourcerne mellem virksomhederne og til en bedre udnyttelse af ressourcerne internt i virksomhederne.

Analysen giver således en rationel baggrund for delegering af beslutningsansvar gennem antagelsen om den asymmetriske information mellem investorer og virksomhedsledere.

Antagelsen om den asymmetriske information giver anledning til i det mindste to yderligere spørgsmål. Det første, der analyseres, er, hvorvidt der kan opnås en bedre fordeling af ressourcerne, såfremt virksomhedsledernes private information rapporteres til investorerne. Det andet er, hvorledes virksomhedsledernes incitamenter med hensyn til produktionsbeslutninger og med hensyn til rapportering af deres private information kan bestemmes på en optimal måde gennem fastsættelsen af aflønningsformen. Dette er de centrale spørgsmål, der analyseres i afhandlingen.

Afhandlingen består af to dele. Part I giver en generel ligevægtsanalyse af efficient ressource allokering i en økonomi med asymmetrisk information. I Part II benyttes resultaterne i Part I, for såvidt angår betingelserne for en efficient risikodeling mellem investorerne, til at lave en partiel ligevægtsanalyse af værdien af, at virksomhedslederne rapporterer deres private information gennem f.eks. eksterne regnskaber. Dette spørgsmål behandles med baggrund i en antagelsen om, at der eksisterer et moral hazard problem både med hensyn til produktionsbeslutningerne og med hensyn til rapporteringen af den private information. Afhandlingen afsluttes med en sammenfatning af de implikationer, der kan uddrages af analysen for det eksterne regnskabsvæsen.

**Part I: Efficient Resource Allocation in Capital Markets with
Asymmetric Information.**

I tidligere generelle ligevægtsanalyser af ressource allokering under usikkerhed er naturtilstandene og informationen, der karakteriserer økonomien, generelt specificeret. Den usikkerhed, der har betydning for investorernes beslutningsproblem og virksomhedsledernes produktionsbeslutninger, er fuldstændigt beskevet ved et antal gensidigt udelukkende naturtilstande. Denne generelle formulering opretholdes i afhandlingen, men specifikationen udvides til at beskrive en stor økonomi med mange virksomheder og investorer. Der foretages således en eksplicit opdeling af naturtilstandene i generelle tilstande og i virksomhedsspecifikke tilstande.

Virksomhedsspecifikke tilstande er de naturtilstande, der kun har betydning for de enkelte virksomheders afkast, og som er uafhængige givet de generelle tilstande. Naturtilstande, der ikke opfylder disse to betingelser, karakteriseres som generelle tilstande. Virksomhedsspecifik information er på tilsvarende måde defineret som information, der reducerer usikkerheden på de enkelte virksomheders fremtidige afkast, og som ikke reducerer usikkerheden på andre virksomheders fremtidige afkast givet den generelle information.

Denne formulering af usikkerheden i økonomien er valgt af to grunde. For

det første er et af de væsentligste bidrag i finansieringsteorien, at det er optimalt for investorerne at diversificere deres investering i værdipapirer. Capital Asset Pricing Modellen, den forbrugsbaserede Capital Asset Pricing Model og Arbitrage Pricing Teorien har alle en optimal diversifikation som det grundlæggende element. Baggrunden for disse resultater er, at ligevægtspriserne bliver fastsat på en sådan måde, at investorerne omkostningsfrit kan forsikre sig mod virksomhedsspecifik risiko ved at vælge en veldiversificeret portefølje. Det vises i afhandlingen, at disse resultater er af central betydning for den rolle, virksomhedsspecifik information har for ressource allokeringen i bytteøkonomier såvel som i produktionsøkonomier.

For det andet har den empiriske forskning indenfor sammenhængen mellem regnskabsinformation og aktiemarkeder analyseret regnskabsinformation som virksomhedsspecifik information.

I kapitel 2 beskrives økonomien i afsnit 2.1. Pareto efficiente forbrugs- og produktionsplaner karakteriseres i afsnit 2.2. Det antages, at investorernes og virksomhedsledernes præferencer kan beskrives ved tids-additive nyttefunktioner defineret over deres respektive forbrugsplaner. Det antages endvidere, at investorernes og virksomhedsledernes forventninger kan beskrives ved den samme sandsynlighedsfordeling på mængden af naturtilstande. Forskelle i forventninger skyldes således alene, at virksomhedslederne har bedre information om deres respektive virksomhedsspecifikke tilstande. Store tals lov for uafhængige stokastiske variable medfører, at i en økonomi med mange virksomheder og investorer afhænger per capita forbrugsmulighederne ikke af de uafhængige virksomhedsspecifikke tilstande. Brugbare forbrugsplaner defineres derfor på baggrund af antagelsen, at de aggregerede forbrugsmuligheder er lig med de forventede aggregerede forbrugsmuligheder givet informationen om den generelle naturtilstand. Denne definition af brugbare forbrugsplaner afspejler det forhold, at de økonomiske agenter har mulighed for at bortdiversificere virksomhedsspecifik risiko.

Det vises i Proposition 2.3, at investorernes såvel som virksomhedsledernes efficiente forbrugsplaner kan repræsenteres som voksende funktioner af de aggregerede forbrugsmuligheder. Efficiente forbrugsplaner afhænger således ikke af virksomhedsspecifik information. I Proposition 2.4 vises det, at

efficiente produktionsplaner maksimerer den sande værdi af virksomhedernes afkast. Denne værdi bestemmes af de betingede forventede afkast givet de generelle tilstande og virksomhedsledernes bedre information om de specifikke tilstande. Nutidsværdien af disse betingede forventede afkast bestemmes ved hjælp af implicitte tilstandsafhængige priser hørende til de generelle tilstande. En efficient allokering af ressourcerne mellem virksomhederne og internt i virksomhederne forudsætter således, at produktionsbeslutningerne baseres på virksomhedsledernes information om de specifikke tilstande. Det har derimod ingen værdi, at virksomhedslederne rapporterer denne information til investorerne, idet efficiente forbrugsplaner ikke afhænger af virksomhedsspecifik information. En umiddelbar konsekvens af de to ovennævnte propositioner er, at bedre information om generelle eller specifikke tilstande har værdi, hvis og kun hvis den efficiente produktionsplan ændres. Dette betyder specielt, at maksimering af virksomhedernes sande værdi foretrækkes fremfor maksimering af virksomhedernes markedsværdi, såfremt virksomhedsledernes bedre information er relevant for optimale produktionsbeslutninger.

I afsnit 2.2.1 gives der en række eksempler, der illustrerer disse generelle resultater. Specielt vises det, at virksomhedsledernes bedre information om de specifikke tilstande kan benyttes til at foretage en bedre allokering af de aggregerede investeringer i økonomien mellem virksomhederne, således at det aggregerede fremtidige afkast bliver større for den samme aggregerede investering. Dette er baggrunden for definitionen af produktive informationsstrukturer i afsnit 2.2.2. I afsnit 2.2.3 pålægges der yderligere struktur på produktionsmulighederne, således at det bliver muligt at identificere de karakteristika, der giver informationen værdi i en produktionsøkonomi. Der skelnes mellem information om produktiviteten af de nuværende investeringer, d.v.s. om det fremtidige marginalafkast, og information om fremtidige ekstraordinære afkast, der ikke afhænger af produktionsplanen. Det vises i Proposition 2.7, at produktivitetsinformation har værdi, uanset om det er generel information eller virksomhedsspecifik information. Derimod har information om ekstraordinære tilstande kun værdi, hvis det er information om generelle tilstande. Information om generelle ekstraordinære tilstande har værdi, idet informationen reducerer usikkerheden på de fremtidige aggregerede forbrugsmuligheder. Det er således optimalt, at de aggregerede investeringer er negativt relateret til informationen om de frem-

tidlige ekstraordinære afkast. Information om virksomhedsspecifikke ekstraordinære afkast har ingen værdi, idet forventningerne til de fremtidige aggregerede forbrugsmuligheder og forventningerne til marginalafkastet af investeringerne ikke ændres som følge af denne information. Produktivitetsinformation har værdi, fordi forventningerne til marginalafkastet ændres. Ved generel produktivitetsinformation ændres også forventningerne til de fremtidige aggregerede forbrugsmuligheder. Da bedre information har værdi, hvis og kun hvis produktionsplanen ændres, har det værdi, at virksomhedslederne har virksomhedsspecifik produktivitetsinformation, men det er ikke nødvendigt for at opnå den bedre allokering af de aggregerede investeringer at denne information rapporteres til investorerne.

Analysen i kapitel 2 karakteriserer Pareto efficiente forbrugs- og produktionsplaner for givne informationsstrukturer. Endvidere karakteriseres de økonomiske konsekvenser af ændrede informationsstrukturer. Analysen behandler således ikke spørgsmålet om, hvilken rolle informationen spiller for implementeringen af disse planer i en decentraliseret markedsøkonomi. I en sådan økonomi er de opnåelige forbrugsplaner bestemt af de afkastmuligheder, der kan opnås gennem investering i eksisterende værdipapirer på markedet. Produktionsplanerne er bestemt ud fra virksomhedsledernes præferencer med hensyn til alternative produktionsbeslutninger. I kapitel 3 defineres en sekventiel ligevægt for den beskrevne økonomi i afsnit 3.1. Ligevægten er karakteriseret ved, at forbrugs-, portefølje- og produktionsplanerne er individuelt rationelle givet initialbeholdningerne og de enkelte økonomiske agents information på beslutningstidspunkterne, samt ved at udbud er lig med efterspørgsel på alle markeder i alle tidspunkter og tilstande. I en markedsøkonomi er der to principielle grunde til, at ligevægtsplanerne ikke er efficiente. For det første kan der være for få værdipapirer til at implementere en efficient risikodeling, og for det andet er det ikke givet, at virksomhedslederne har incitament til at implementere efficiente produktionsplaner. Disse to forhold analyseres i henholdsvis afsnit 3.2 og i kapitel 4.

I den beskrevne økonomi kan der handles på kapitalmarkedet før og efter, at informationen bliver kendt. Handlen, før informationen bliver kendt, skal sikre, at risikodelingen er efficient uden information, idet offentlig information ellers kan have en negativ værdi. Handlen, efter informationen

bliver kendt, giver mulighed for, at der kan opnås flere forbrugsplaner gennem dynamiske porteføljestrategier. Forudsætningen for implementeringen af en efficient risikodeling er et tilstrækkeligt antal forskellige værdipapirer. Et komplet kapitalmarked med lige så mange lineært uafhængige tilstandsafhængige afkastvektorer som antallet af mulige informationssignaler sikrer, at det er muligt at opnå en efficient risikodeling. Det nødvendige antal forskellige afkastvektorer kan imidlertid reduceres betydeligt, idet efficiente forbrugsplaner er målelige med hensyn til informationen om de generelle tilstande og som følge af muligheden for dynamiske handelsstrategier. Et dynamisk quasi-komplet kapitalmarked defineres som et marked, hvor enhver forbrugsplan, der er finansielt brugbar og målelig med hensyn til generel information, kan implementeres ved dynamisk handel i veldiversificerede porteføljer. Investorerne opnår en optimal diversifikation af den virksomhedsspecifikke risiko ved at restringere deres porteføljestrategier til veldiversificerede porteføljer. Ydermere opnår investorerne en optimal forsikring mod den eventuelt manglende information om de virksomhedsspecifikke tilstande. Enkeltvirksomheder kan være over- eller undervurderede relativt til virksomhedsledningens information. Markedsværdien af veldiversificerede porteføljer er derimod lig med den sande værdi af porteføljerne givet virksomhedsledningens bedre information om de specifikke tilstande (Proposition 3.2). Det vises i Proposition 3.3, at betingelserne på ligevægtspriserne for et dynamisk quasi-komplet kapitalmarked er tilstrækkelige betingelser for en efficient risikodeling mellem investorerne.

Under antagelsen om tids-additive nyttefunktioner og homogene forventninger er det vist i kapitel 2, at efficiente forbrugsplaner er målelige med hensyn til de aggregerede forbrugsmuligheder på ethvert tidspunkt. Der behøver således ikke at eksistere veldiversificerede porteføljer, der ved dynamisk handel kan implementere enhver finansielt brugbar forbrugsplan, der er målelig med hensyn til generel information. På denne baggrund reduceres det nødvendige antal af veldiversificerede porteføljer yderligere i afsnit 3.2.2. Det vises i Proposition 3.4, at risikodelingen er efficient, hvis der eksisterer veldiversificerede porteføljer, som har afkastvektorer, der udspænder det lineære underrum af ønskede investeringsafkast. Dette lineære underrum er bestemt af variationen i de aggregerede forbrugsmuligheder i den følgende periode og variationen i den betingede sandsynlighedsfordeling for de aggregerede forbrugsmuligheder i fremtidige perioder.

Den relevante information på et givet tidspunkt for efficiente forbrugsplaner er kun informationen om de aggregerede forbrugsmuligheder på dette tidspunkt. For at kunne implementere disse forbrugsplaner gennem dynamisk handel i veldiversificerede porteføljer behøver investorerne imidlertid også information om den betingede sandsynlighedsfordeling for de fremtidige aggregerede forbrugsmuligheder. Dette behov for yderligere information opstår som følge af, at i fler-periode modeller er ikke kun afkastet i den efterfølgende periode relevant for porteføljevalget. Det optimale porteføljevalg bestemmes også af, hvorledes afkastet kan geninvesteres. Virksomhedsspecifik information ændrer ikke på forventningerne til de aggregerede forbrugsmuligheder. Denne information er derfor ikke af betydning for hverken efficiente forbrugsplaner eller implementeringen af disse.

Det er vist i kapitel 2, at virksomhedsledningens efficiente forbrugsplaner er målelige med hensyn til den generelle information, samt at efficiente produktionsplaner maksimerer den sande værdi af virksomhedernes afkast. For at opnå en Pareto-optimal allokering må virksomhedslederne derfor udelukkes fra at handle i deres respektive virksomheders værdipapirer baseret på deres private information om de specifikke tilstande. De må ydermere motive-res til at vælge de produktionsbeslutninger, der maksimerer den sande værdi af virksomhedernes afkast. Disse to spørgsmål behandles i kapitel 4, afsnit 4.1 og afsnit 4.2, respektivt.

Problemet med insider trading kan elimineres ved at restingere virksomhedsledningens handel med værdipapirer til veldiversificerede porteføljer. Virksomhedernes markedsværdi afhænger kun af offentlig information. Nogle virksomheder er derfor overvurderede, mens andre er undervurderede relativt til virksomhedsledningens information. Veldiversificerede porteføljer er derimod korrekt vurderede. Forudsat, at der eksisterer et tilstrækkeligt antal forskellige veldiversificerede porteføljer, tillader denne restriktion på virksomhedernes porteføljevalg stadig en efficient risikodeling. Restriktionen sikrer imidlertid, at virksomhedslederne ikke kan udnytte deres private information på bekostning af investorerne. Idet investorerne må antages at kunne forudse, at virksomhedslederne kan opnå anormale afkast ved insider trading, vises det, at det er i virksomhedsledningens egen interesse, at der findes en sådan restriktion på deres porteføljevalg.

Virksomhedslederne må motiveres til at maksimere den sande værdi af virksomhedernes afkast. Dette gøres gennem fastsættelsen af virksomhedsledernes aflønningsform. En sådan aflønningsform eksisterer, hvis virksomhedsledernes præferencer med hensyn til alternative produktionsbeslutninger kun afhænger af det resulterende økonomiske afkast. Ligevægtsaflønningen består af en tilstandsafhængig del og en andel af det gennemsnitlige afkast for alle virksomheder. Denne type aflønningsform giver såvel optimale incitamenter med hensyn til produktionsbeslutningerne samt en efficient risikodeling mellem investorerne og virksomhedslederne.

Ligevægtsaflønningen motiverer virksomhedslederne til at implementere Pareto-efficiente produktionsplaner. De produktionsbeslutninger, der maksimerer virksomhedsledernes egne forventede nytte, er således de samme som de produktionsbeslutninger, der maksimerer den sande værdi af virksomhedernes afkast. Den sande værdi af en virksomhed er defineret som den sande værdi af virksomhedens afkast minus værdien af virksomhedslederens aflønning. Det er vist i Propositionerne 4.2 og 4.3, at Pareto-efficiente produktionsplaner også maksimerer virksomhedernes sande værdi givet ligevægtsaflønningen, samt at disse produktionsplaner er optimale for alle investorer.

Formen på ligevægtsaflønningen strider mod aflønningsformer, der er tættere knyttet til de enkelte virksomheders afkast, som f.eks. aflønningssystemer baseret på tildeling af aktier eller aktieoptioner. Der er to grunde til, at aflønningsformer af denne type ikke er optimale i den analyserede model. For det første vil en aflønning, der er baseret på den enkelte virksomheds markedsværdi, motivere virksomhedslederen til at maksimere markedsværdien af den pågældende virksomhed. Virksomhedslederen benytter således ikke sin private information til at sikre en optimal allokering af de aggregerede investeringer mellem virksomhederne. For det andet afhænger en virksomheds markedsværdi af virksomhedsspecifik information. Virksomhedslederen vil derfor kræve en risikopræmie for at påtage sig denne virksomhedsspecifikke risiko. Idet aktionærerne kan eliminere virksomhedsspecifik risiko gennem diversifikation af deres investeringer, er det ikke optimalt for dem at betale virksomhedslederen en sådan risikopræmie.

Virksomhedsspecifikke tilstande er blevet defineret som de naturtilstande, der kun har betydning for de enkelte virksomheders afkast, og som er uaf-

hængige givet de generelle tilstande. Ellers karakteriseres tilstandene som generelle tilstande. Mange generelle tilstande kan derfor også være irrelevante for investorernes forbrugs- og porteføljeplaner. Det er også blevet antaget i den tidligere analyse, at investorerne og virksomhedslederne har den samme information om de generelle tilstande. I kapitel 5, afsnit 5.1, sammenlignes denne økonomi med en iøvrigt identisk økonomi, hvor investorerne kun modtager information om de nuværende aggregerede forbrugsmuligheder og sandsynlighedsfordelingen for de fremtidige aggregerede forbrugsmuligheder. Der identificeres tilstrækkelige betingelser for, at ligevægten i den oprindelige økonomi kan opretholdes som en ligevægt i økonomien med mindre information om de generelle tilstande til investorerne. Generelt er kun information, der har betydning for sammensætningen og for markedsværdierne af veldiversificerede porteføljer, relevant for investorernes implementering af efficiente forbrugsplaner. Der kan ikke opnås bedre allokeringer ved at rapportere virksomhedsledernes yderligere private information om såvel generelle som specifikke tilstande.

Part I afsluttes i afsnit 5.2 med en diskussion af en mulig årsag til, at investorerne foretrækker, at virksomhedslederne rapporterer deres private information om de specifikke tilstande. I den tidligere analyse er virksomhedsledernes og investorernes informationsstrukturer blevet analyseret som eksogent givne. I en mere generel analyse må disse informationsstrukturer naturligvis behandles som beslutningsvariable. Det vises, at restriktionen på virksomhedsledernes adgang til handel i værdipapirer samt formen på ligevægtsaflønnningen medfører, at virksomhedsledernes præferencer bliver sammenfaldende med investorernes såvel med hensyn til produktionsbeslutninger som med hensyn til valget mellem informationsstrukturer for specifikke tilstande. Virksomhedsledernes informationsstrukturer kan derfor betragtes som en del af ligevægten i økonomien.

Investorernes informationsstrukturer for virksomhedsspecifikke tilstande kan derimod ikke opretholdes som en del af ligevægten. Det har ingen værdi for økonomien som helhed, at investorerne modtager virksomhedsspecifik information. Investorerne har derimod stærke private incitamenters til at søge denne type information. Modtager en investor privat information om en eller flere virksomheder, vil han ikke restringere sit porteføljevalg til veldiversificerede porteføljer. Han vil derimod være villig til at påtage sig

virksomhedsspecifik risiko fra spekulative positioner baseret på denne information, samt være villig til at afholde omkostninger for at opnå den private information. Alle investorer må antages at forudse, at der kan opnås anormale afkast på basis af privat information. I det ekstreme tilfælde vil dette betyde, at gennem ændringerne i udbud og efterspørgsel vil al den information, der er relevant for den sande værdi af virksomhederne, blive reflekteret i markedspriserne. Herved elimineres det private incitament til at søge information. Dette er imidlertid en inoptimal tilstand for økonomien som helhed både som følge af en ineffICIENT risikodeling og som følge af, at der anvendes ressourcer til at søge privat information. Rapportering af virksomhedsspecifik information kan således være af værdi, idet de private incitamenter til at søge denne information herved elimineres.

Analysen i Part I har fokuseret på den rolle informationen spiller for implementeringen af efficiente forbrugs- og produktionsplaner. Informationen spiller således ingen rolle for motiveringen af virksomhedslederne i den analyserede model. Analysen har således kun behandlet et aspekt af de økonomiske konsekvenser af en rapportering af virksomhedsledernes private information.

Part II: Accounting Information and Moral Hazard in Efficient Capital Markets.

I denne del af afhandlingen ændres en af antagelserne i Part I, således at det bliver muligt at studere efterspørgslen efter ekstern regnskabsinformation i en økonomi, hvor det ikke er muligt at opnå fuldstændig overensstemmelse mellem virksomhedsledernes og investorernes præferencer med hensyn til alternative produktionsbeslutninger. Spørgsmålet om informationens rolle for motiveringen og kontrollen af virksomhedslederne bliver derved en central del af analysen.

Det antages i modsætning til i Part I, at virksomhedsledernes præferencer med hensyn til alternative produktionsbeslutninger ikke kun afhænger af det resulterende økonomiske afkast. Præferencerne afhænger også af den indsats virksomhedslederne må yde for at opnå dette afkast. Det antages endvidere,

at virksomhedernes produktionsfunktioner kan simplificeres således, at produktionen er bestemt af den investerede kapital, naturtilstanden samt af virksomhedslederernes indsats. Såfremt virksomhedsledernes indsats ikke kan observeres af investorerne, og virksomhedslederne har en negativ nytte af at yde en større indsats, da eksisterer der et såkaldt moral hazard problem. Dette problem består i, at en virksomhedsleder kun kan gøre en given indsats troværdig, hvis han gennem sin aflønningsform får incitamenter til at yde denne indsats i sin egen interesse. Disse incitamenter opnås ved, at virksomhedslederne påtager sig mere risiko, end det er optimalt for risikodelingen. Yderligere information får som konsekvens, at den inoptimale risikodeling kan reduceres samtidig med, at incitamenterne med hensyn til virksomhedslederens indsats ikke forringes.

I tidligere analyser af moral hazard problemer, d.v.s. de såkaldte principal/agent modeller, er der kun én produktionsfaktor, nemlig agentens (virksomhedslederens) indsats. Denne formulering er for restriktiv til, at man kan analysere det kontraktmæssige forhold mellem virksomhedsledelsen og de eksterne investorer. Finansieringen af produktionsprocessen må antages at spille en central rolle for dette forhold. Når dette forhold inddrages i en udvidet principal/agent model, kan analysen bidrage til at besvare centrale spørgsmål i det eksterne regnskabsvæsen. Det fremgår bl.a. af analysen, at en revideret opgørelse af virksomhedens nettoindbetalinger i den foregående periode samt en indkomstopgørelse spiller en central rolle for en optimal kommunikation mellem virksomhedsledelsen og de eksterne investorer.

Betragtes regnskabsinformation som virksomhedsspecifik information, da har ekstern regnskabsinformation ingen værdi ifølge modellen i Part I. Investorerne behøver kun information om det nuværende udbytte, der er defineret som nettoindbetalingen i den foregående periode minus beløbet, der anvendes til investering i produktionsprocessen. Investorerne behøver således ikke information om hverken nettoindbetalingen i den foregående periode eller investeringens størrelse. De behøver kun information om forskellen. En revideret opgørelse af nettoindbetalingen i den foregående periode er derfor uden værdi.

I den udvidede principal/agent model, hvor virksomhedslederen har en negativ nytte af en af produktionsfaktorerne, nemlig hans egen indsats, vil der

være en efterspørgsel efter en sådan revideret opgørelse. Når virksomhedslederens indsats ikke kan observeres af investorerne, og hans indsats kan substitueres med yderligere investeringer, da vil han naturligvis gøre dette, medmindre han får incitamenter gennem sin aflønningsform, der afholder ham fra at foretage denne substitution. Uden regnskabsinformation kan disse incitamenter kun opnås ved, at virksomhedslederen påtager sig en del af den virksomhedsspecifikke risiko, der er relateret til sekvensen af udbytter til investorerne. Dette repræsenterer imidlertid en inefficient risikodeling. Der analyseres tre typer af regnskabsinformation, der kan reducere denne inefficiens, nemlig information om nettobetalingen rapporteret af virksomhedslederen (kapitel 8, afsnit 8.1), en revideret opgørelse af nettobetalingerne (kapitel 8, afsnit 8.2), samt yderligere privat information rapporteret af virksomhedslederen (kapitel 9).

I kapitel 7 defineres incitament problemet for tilfældet uden regnskabsinformation, og de optimale aflønningsformer for dette problem karakteriseres. De eneste variable, der kan observeres af investorerne, er udbytterne i de to perioder. Investorerne kender imidlertid de optimale beslutningsregler for produktionsbeslutningerne, d.v.s. de funktionelle sammenhænge mellem virksomhedslederens private information, den optimale investering og den optimale indsats. Idet det nuværende udbytte er lig med nettoindbetalingen i den foregående periode minus investeringen, inducerer udbyttet en opdeling af mængden, der beskriver virksomhedslederens mulige private informationssignaler. Med andre ord, investorerne kan uddrage information udfra udbyttens størrelse og kendskabet til den optimale investeringsregel. Når opdelingen er fuldstændig, er det muligt for investorerne at uddrage præcist, hvilket informationssignal virksomhedslederen har modtaget. Når virksomhedslederen skal vælge den optimale investeringsregel, må han således tage i betragtning, at forskellige investeringsregler giver anledning til forskellige informationsstrukturer for investorerne. Med andre ord, investeringsbeslutningen har to roller, nemlig både som en produktionsbeslutning og som en beslutning, der kommunikerer noget af virksomhedslederens private information til investorerne. Det er denne duale rolle for investeringsbeslutningen, der er grundlaget for analysen af værdien af ekstern regnskabsinformation i kapitlerne 8 og 9.

I kapitel 8 analyseres en model, hvor virksomhedslederen kun har privat information om virksomhedens nettoindbetaling i den foregående periode. Regnskabsinformation om nettoindbetalingen i den foregående periode introduceres i modellen, først som rapporteret af virksomhedslederen (afsnit 8.1) og dernæst som revideret regnskabsinformation (afsnit 8.2).

Spørgsmålet, der stilles i afsnit 8.1, er, hvorvidt det har positiv værdi, at virksomhedslederen rapporterer størrelsen af nettoindbetalingerne i den foregående periode til investorerne. Først benyttes det såkaldte Revelation Principle til at vise, at det ikke er nogen bindende begrænsning at antage, at aflønningsformen skal være således, at virksomhedslederen rapporterer nettoindbetalingen sandfærdigt, samt at der ikke rapporteres yderligere information. Det periodeisolerede regnskab spiller således ikke nogen betydende rolle i denne situation.

Det vises dernæst i Proposition 8.6, at en nødvendig betingelse for en positiv værdi af, at virksomhedslederen rapporterer nettoindbetalingen, er, at den optimale investeringsregel med rapportering er således, at den optimale udbyttefunktion ikke er en invertibel funktion af nettoindbetalingen. Er den optimale udbyttefunktion med rapportering en invertibel funktion, da er den optimale investeringsregel også optimal uden rapportering af nettoindbetalingen. I dette tilfælde er investorerne i stand til at uddrage virksomhedslederens private information udfra det observerede udbytte og kendskabet til den optimale investeringsregel. Der er således ingen konflikt mellem investeringsbeslutningens to roller i denne situation. I den situation, hvor den optimale investeringsregel uden rapportering medfører, at den optimale udbyttefunktion bliver invertibel, er det imidlertid ikke sikkert, at den samme investeringsregel er optimal med rapportering af nettoindbetalingen. Dette skyldes, at investeringsreglen uden rapportering kan blive påvirket af rollen som en beslutning, der kommunikerer noget af virksomhedslederens private information. Med andre ord, det kan være optimalt, at virksomhedslederen vælger inefficiente investeringer blot for at blive i stand til at afspejle mere information gennem de observerbare udbytter. I dette tilfælde har det værdi, at virksomhedslederen rapporterer nettoindbetalingen i den foregående periode, idet han derved kan vælge mere effektive investeringsbeslutninger.

I afsnit 8.1.1 gives der en række eksempler, der illustrerer disse resultater. Eksemplerne viser, at en af gevinsterne ved, at virksomhedslederens private information kommunikerer til investorerne, er, at det herved bliver muligt at opnå en bedre intertemporal risikodeling. Uden rapportering af den private information skal omkostningen ved en mindre efficient investeringsbeslutning således afvejes med gevinsten ved en bedre intertemporal risikodeling. Med rapportering af den private information kan der opnås en optimal intertemporal risikodeling samtidig med, at investeringsbeslutningerne er efficiente.

I den tidligere analyse i kapitel 8 er det antaget, at virksomhedslederen ikke kan investere på kapitalmarkedet, og specielt, at han ikke kan låne eller investere risikofrit. Virksomhedslederen kan derfor kun opnå den optimale intertemporale risikodeling gennem sin aflønning, og som vist i eksemplerne spiller kommunikationen af hans private information en central rolle for denne risikodeling. I afsnit 8.1.2 analyseres derfor konsekvenserne af, at virksomhedslederen kan opnå den optimale intertemporale risikodeling gennem lån og investering i et risikofrit aktiv. For det tilfælde, hvor virksomhedslederen opnår perfekt information om det fremtidige afkast inden valget af produktionsbeslutningerne, vises det i Proposition 8.8, at virksomhedslederen kan opnå de samme gevinster ved lån og investering i det risikofrie aktiv, som han kan ved kommunikation af sin private information. Dette er tilfældet selv i den situation, hvor virksomhedslederens handel i det risikofrie aktiv ikke kan observeres af investorerne. Når der ikke kan kommunikerer, har det derfor positiv værdi, at virksomhedslederen kan låne og investere i det risikofrie aktiv i de situationer, hvor kommunikation har positiv værdi uden handel i det risikofrie aktiv. Omvendt, når der kan kommunikerer, er virksomhedslederen indifferent mellem, om han kan handle i det risikofrie aktiv eller ej.

Antagelsen om perfekt information er kritisk for disse resultater, idet kommunikation er uden værdi til incitamentformål i dette tilfælde. I Proposition 8.9 vises det, at uden perfekt information og med kommunikation fortrækker virksomhedslederen, at han er i stand til at forpligtige sig til ikke at handle i det risikofrie aktiv. Dette skyldes, at de optimale incitamentter med hensyn til sandfærdig kommunikation og optimale produktionsbeslutninger delvis opnås ved, at virksomhedslederen for den optimale af-

lønningsform har et incitament til opspare noget af aflønningen i den første periode. Kommunikation kan dog stadig have værdi som følge af en bedre intertemporal risikodeling. For at skabe de optimale incitamenter er det imidlertid ikke optimalt med en fuld intertemporal risikodeling. Sammenholdes resultaterne for tilfældene med perfekt og ikke-perfekt information, indicerer disse, at der findes tilfælde, hvor det har positiv værdi, at virksomhedslederne kan handle i det risikofrie aktiv, når han ikke er i stand til at kommunikere sin private information. Dette blev vist i eksempel 8.5. Det, der skal afvejes i forhold til hinanden, er gevinsten ved en fuld intertemporal risikodeling gennem handel i det risikofrie aktiv og det eventuelle tab i produktionsincitamenterne, der følger af, at virksomhedslederen ikke kan forpligtige sig til den optimale intertemporale risikodeling.

I afsnit 8.2 analyseres tilfældet med en revideret opgørelse af nettoindbetalingen i den foregående periode. Det antages, at revisoren har incitamenter til altid at rapportere sandfærdigt. Nettoindbetalingen kan derfor behandles som offentlig information. En revideret opgørelse af nettoindbetalingen kan være af positiv værdi selv i de situationer, hvor det ikke har nogen værdi, at virksomhedslederen kommunikerer nettoindbetalingen. Dette skyldes naturligvis, at der ikke er incitament problemer forbundet med reviderede opgørelser, som der er med opgørelser rapporteret af virksomhedslederen. Når nettoindbetalingen i den foregående periode er offentlig information, kan investorerne også direkte observere investeringens størrelse, nemlig som forskellen mellem nettoindbetalingen og den betalte udbytte. En revideret opgørelse af nettoindbetalingen fjerner dermed moral hazard problemet med hensyn til virksomhedslederens fastsættelse af de optimale investeringer. Moral hazard problemet med hensyn til virksomhedslederens indsats påvirker imidlertid den optimale investeringsregel. Investeringens størrelse kan være betemmende for, hvor dyrt det er at motivere virksomhedslederen til at yde en given indsats. Det vises ved hjælp af et eksempel, at dette moral hazard problem kan resultere i såvel over- som underinvesteringer afhængig af, hvorledes investeringens størrelse og virksomhedslederens indsats er relateret i produktionsfunktionen.

I kapitel 9 vendes der tilbage til det generelle tilfælde, hvor virksomhedslederen observerer nettoindbetalingen i den foregående periode såvel

som et yderligere informationssignal. Det antages i dette kapitel, at investorerne modtager en revideret opgørelse af nettoindbetalingen, mens virksomhedslederen kommunikerer den yderligere information. Denne model er motiveret af, at det må forventes, at en revisor er i stand til at foretage en revision af betalingerne i virksomheden, mens det vil være betydeligt vanskeligere at foretage en revision af informationer om f.eks. den fremtidige produktivitet af den investerede kapital.

I afsnit 9.1 formuleres incitament problemet og den optimale kontrakt karakteriseres. Det vises i Proposition 9.2, at selvom der ikke er noget moral hazard problem forbundet med investeringsbeslutningen, påvirkes investeringsbeslutningen både af moral hazard problemet med hensyn til virksomhedslederens indsats og af incitament problemet med hensyn til kommunikationen af virksomhedslederens private information. Det vises ved et eksempel, at overinvesteringer såvel som underinvesteringer kan være optimale afhængig af, hvorledes investeringen, virksomhedslederens indsats og private information er relaterede i produktionsfunktionen.

I afsnit 9.2 analyseres værdien af regnskabsinformation rapporteret af virksomhedslederen. Dette kan analyseres på samme måde som i afsnit 8.1. En tilstrækkelig betingelse for ingen værdi af kommunikationen er således, at den optimale investeringsregel med kommunikation er en invertibel funktion af virksomhedslederens private information givet den reviderede opgørelse af nettoindbetalingen i den foregående periode. Generelt kan dette imidlertid kun være tilfældet, hvis virksomhedslederens private informationssignal, ξ , er en-dimensional.

Signalerne $\xi \in \Xi$ giver en abstrakt og fuldstændig repræsentation af virksomhedslederens private information. Information i eksterne regnskaber tager derimod form som indkomstopgørelser, der angiver værdier af aktiver og passiver baseret på et sæt af vurderingsprincipper. Et vurderingsprincip kunne f.eks. være den sande nutidsværdi af de fremtidige afkast givet den offentlige information og virksomhedslederens private information. Regnskabsprincipper analyseres derfor som bestående af en revideret opgørelse af nettoindbetalingen i den foregående periode samt en indkomstopgørelse rapporteret af virksomhedslederen givet et sæt af vurderingsprincipper. Et veldefineret regnskabsprincip kan generelt repræsenteres som en funktion

fra mængden af virksomhedslederens informationssignaler til en mængde af mulige regnskaber. Er denne funktion invertibel, da giver de mulige regnskaber også en fuldstændig repræsentation af virksomhedslederens information. De samme gevinster kan derfor opnås som ved en direkte kommunikation af den private information ξ . Det interessante spørgsmål er derfor, under hvilke betingelser de samme gevinster kan opnås med regnskabsprincipper, der ikke tillader fuld kommunikation af virksomhedslederens private information.

Proposition 9.3 giver tilstrækkelige betingelser for, at et givet regnskabsprincip resulterer i de samme gevinster som fuld kommunikation. Regnskabsprincippet må opfylde følgende to betingelser. For det første skal investorerne være i stand til at uddrage de relevante dele af virksomhedslederens private information fra kombinationen af det foreliggende regnskab og af den observerede investering. Informationen uddrages gennem investorerens kendskab til regnskabsprincippet og til den optimale investeringsregel. For det andet skal den optimale investeringsregel også være optimal for tilfældet med fuld kommunikation. Er disse to betingelser opfyldt, da har det ingen værdi at ændre regnskabsprincippet til et mere informativt regnskabsprincip.

Den første betingelse afspejler det forhold, at regnskabsinformation ikke er den eneste form for information, der er til rådighed for investorerne. Investorerne kan også uddrage information af observerbare produktionsbeslutninger, og af andre observerbare beslutninger truffet af virksomhedslederen. Den anden betingelse betyder, at regnskabsprincippet må være så informativt, at virksomhedslederen kan kommunikere så meget af sin private information gennem regnskabet, at den observerbare investeringsbeslutning ikke påvirkes af rollen som et kommunikationsinstrument for virksomhedslederens private information.

Analysen i Part II har fokuseret på rollen, som virksomhedsspecifik regnskabsinformation spiller for incitamentstrukturen. Det er blevet vist, at en revideret opgørelse af nettoindbetalingerne eliminerer moral hazard problemet med hensyn til virksomhedsledernes fastsættelse af de optimale investeringer. Indkomstopgørelser rapporteret af virksomhedslederne, kan være af værdi, fordi relevant information for incitamentproblemet ikke kan kom-

munikeres gennem observerbare produktionsbeslutninger, men også som følge af, at virksomhedslederne ikke behøver at foretage inefficiente investeringer blot for at kunne kommunikere relevant information gennem de observerbare investeringer. Med andre ord, indkomstopgørelser kan være af værdi som følge af forbedrede incitamenters og som følge af en mere efficient allokering af investeringerne.

Konklusion.

I denne afhandling er de økonomiske konsekvenser af rapportering af virksomhedsspecifik information blevet analyseret i en generel kapitalmarkedsmodel. I dette afsnit foretages en sammenfatning af de implikationer for det eksterne regnskabsvæsen, der kan udledes af analysen i afhandlingens to dele.

Analysen i Part I viste, at det ikke er nødvendigt, at virksomhedslederne rapporterer deres private information til investorerne for at opnå effiente forbrugs- og produktionsplaner, der er baseret på virksomhedsledernes bedre information. Graden af resource allokeringens efficiens afhænger kun af virksomhedsledernes information om de virksomhedsspecifikke tilstande. Den eneste information, investorerne behøver, er generel information, der er relevant for porteføljeplanerne. Eksisterer der veldiversificerede porteføljer, der har så meget fleksibilitet, at de kan implementere enhver finansielt brugbar og efficient forbrugsplan, da vil investorerne restringere deres porteføljevalg til disse porteføljer. Dette er optimalt, idet investorerne herved kan opnå en omkostningsfri forsikring både mod virksomhedsspecifik risiko og mod den eventuelt forkerte prisfastsættelse af de enkelte virksomheder relativt til virksomhedsledernes bedre information. Virksomhedsspecifik information er ikke relevant for hverken sammensætningen af eller markedspriserne på veldiversificerede porteføljer, og rapporteringen af denne type information til investorerne er derfor uden værdi.

For at sikre at virksomhedslederne implementerer deres del af en efficient risikodeling, blev det vist, at det er i både investorernes og virksomhedsledernes egen interesse, at virksomhedsledernes porteføljeplaner begrænses

til veldiversificerede porteføljer. Restriktioner på insider trading er imidlertid ikke alene en nødvendig betingelse for at opnå en efficient risikodeling, men også en nødvendig betingelse for at opnå efficiente produktionsplaner. Under antagelse af, at virksomhedsledernes præferencer med hensyn til alternative produktionsplaner kun afhænger af de økonomiske afkast, blev det vist, at en aflønningsform, der er baseret på det gennemsnitlige afkast for alle virksomheder, sikrer, at virksomhedslederne ikke har incitamenter til ikke at implementere efficiente produktionsplaner. Givet denne aflønningsform og restriktionen på virksomhedsledernes porteføljeplaner blev det også vist, at virksomhedslederne har incitamenter til at foretage optimale beslutninger for økonomien som helhed med hensyn til valg af deres egne informationsstrukturer.

Det har ingen værdi for økonomien som helhed, at investorerne modtager virksomhedsspecifik information. Investorerne har derimod stærke private incitamenter til at søge denne type information i spekulationsøjemed. Betragtes investorernes informationsstrukturer derfor som endogent bestemte, kan det have en værdi for økonomien som helhed, at virksomhedslederne rapporterer deres private information. Herved elimineres investorernes incitamenter til at søge privat information og dermed også den inefficiente risikodeling, der følger af spekulation på basis af privat information. Det har imidlertid ingen værdi at rapportere information, der ikke er relevant for de sande værdier af virksomhederne, idet investorerne ikke har incitamenter til at søge denne type information i spekulationsøjemed.

De enkelte virksomheders afkast og virksomhedsspecifik information spiller ingen betydende rolle for motiveringen eller evalueringen af virksomhedslederne. Kun information om de generelle økonomiske betingelser i økonomien og de gennemsnitlige afkast for alle virksomheder har betydning. Antagelsen om, at virksomhedsledernes præferencer med hensyn til alternative produktionsplaner kun afhænger af de økonomiske afkast, er en meget kritisk antagelse for dette resultat. I Part II blev der derfor analyseret en generaliseret principal/agent model. I denne model har virksomhedslederne en negativ nytte af en af produktionsfaktorerne, nemlig deres egen indsats. Den anden produktionsfaktor er den investerede kapital i produktionsprocessen. Idet virksomhedsledernes indsats ikke kan observeres af investorerne, og en større indsats kan substitueres af større investeringer, eksisterer der i

denne model et moral hazard problem, både med hensyn til indsatsens størrelse og med hensyn til substitutionen af indsats med større investeringer. Resultaterne i Part I med hensyn til risikodelingen mellem investorerne gælder stadig, men for at gøre produktionsbeslutningerne troværdige for investorerne må virksomhedslederne nu tage noget af den virksomhedsspecifikke risiko, der er relateret til sekvensen af afkast for de enkelte virksomheder. Dette skaber en efterspørgsel efter virksomhedsspecifik regnskabsinformation, idet denne inefficiente risikodeling mellem investorerne og virksomhedslederne herved kan reduceres.

Det blev vist, at en revideret opgørelse af nettoindbetalingerne har værdi, idet investeringens størrelse dermed kan observeres af investorerne som forskellen mellem nettoindbetalingen i den foregående periode og det observerede udbytte. Moral hazard problemet med hensyn til substitutionen af indsats med større investeringer bliver således elimineret som følge af en revideret opgørelse af nettoindbetalingen.

Yderlige regnskabsinformation, der rapporteres af virksomhedslederne, kan også være af værdi. De optimale aflønningskontrakter er i dette tilfælde bestemt således, at virksomhedslederne ikke er indifferente overfor, hvad de rapporterer i regnskabet. Dette følger af, at virksomhedslederne delvis bliver aflønnet på basis af, hvad de selv rapporterer. Det såkaldte Revelation Principle blev derfor benyttet til at vise, at det ikke er nogen restriktion at antage, at aflønningskontrakterne bestemmes på en sådan måde, at virksomhedslederne rapporterer al deres private information sandfærdigt. Efterspørgslen efter information, der skyldes investorernes private incitament til at søge information, og efterspørgslen efter information, der skyldes moral hazard problemet, kan således opfyldes på samme tid, uden at forbrugs- og produktionsplanerne bliver mindre efficiente end det, der følger af moral hazard problemet.

Et af hovedresultaterne i Part II er, at regnskabsprincipper, der ikke tillader virksomhedslederne at rapportere al deres private information gennem regnskabet, under nogle betingelser kan give anledning til de samme gevinster fra kommunikation som en fuldstændig kommunikation. For det første er det ikke sikkert, at al den private information er relevant for evalueringen af virksomhedslederne. For det andet kan investorerne uddrage noget af

virksomhedsledernes private information af de enten direkte eller indirekte observerbare investeringsbeslutninger. Investeringsbeslutningen spiller således en dual rolle som en produktionsbeslutning og som et kommunikationsinstrument for virksomhedsledernes private information.

Det blev imidlertid også vist, at når regnskabsprincippet ikke tillader en tilstrækkelig kommunikation, da kan det være optimalt at foretage andre investeringer, end dem der er optimale med fuld kommunikation. Investeringsreglen ændres for at kommunikere mere information gennem de observerbare investeringer. I dette tilfælde bliver der en konflikt mellem investeringsbeslutningens rolle som en produktionsbeslutning og dens rolle som et kommunikationsinstrument. Denne konflikt fører til sub-optimale investeringer.

Generelt er regnskabsprincipper formuleret som måden, hvorpå værdien af aktiver og passiver fastsættes. Benyttes de sande nutidsværdier af de fremtidige afkast givet den offentlige information og virksomhedsledernes private information som vurderingsprincip, opfylder regnskabsprincippet efterspørgslen efter information, der skyldes investorernes private incitamenter til at søge information. Denne efterspørgsel vil være opfyldt for alle regnskabsprincipper, der gør det muligt for investorerne at udtrække al den information af regnskaberne og af de observerede investeringer, der er relevant for de sande værdier af virksomhederne givet virksomhedsledernes private information. Hvorvidt disse regnskabsprincipper også opfylder efterspørgslen efter information, der skyldes moral hazard problemet, afhænger af de enkelte virksomheders karakteristika. Det må derfor forventes, at det ikke vil være optimalt at standardisere regnskabsprincipperne på tværs af virksomhederne. Opfylder regnskabsprincipperne ikke denne efterspørgsel, bliver konsekvensen en inefficent allokering af investeringerne i økonomien på grund af investeringsbeslutningens rolle som et kommunikationsinstrument for virksomhedsledernes private information.

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